The Method of Local Perturbations in the Theory of Nanosystems

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PREFACE

Over the past few decades, studies of the properties of the nanosystems have occupied one of the central places in solid state physics. Instead of the traditional name "solid state physics", the term "nanophysics" is increasingly used. More and more attention is being paid to nanostructures such as quantum dots, quantum wells, quantum wires, nanotubes, mesoscopic rings, and two-dimensional electron gas. The increased interest in these systems is due to several reasons. They are functional elements of modern instruments and devices. Methods of preparing the nanostructures under laboratory conditions are steadily improving. They are interesting to theoreticians because they are a convenient testing ground for testing the new methods for studying the dynamics of quasiparticles in limited volumes, when the quantum effects are significant. Of particular interest are the properties of electron nanostructures in a magnetic field due to hybridization effects of dimensional and magnetic quantization of electron motion.

In modern solid state physics, along with traditional three-dimensional electron systems, the low-dimensional electron systems (0D – quantum dots, 1D – quantum wires and nanotubes, 2D – two-dimensional electron gas (2DEG) and quantum rings) are increasingly studied. The motion of conduction electrons in low-dimensional systems is limited by narrow quantum wells. This essentially determines both the energy spectrum of the electrons, unperturbed by the impurity potential, and the nature of the formation of bound electron states on impurity atoms.

The goal of this book is to present a theory of the localization of conduction electrons in the aforementioned nanosystems within the framework of the local perturbation method, considered as a general theoretical approach, that is valid in the description of electron systems of any dimension and type of confinement potential in the presence of a magnetic field. The generality of the approach used consists, in particular, in that the derivation of the equation for the spectrum of impurity levels is carried out from the "first principles", which include the use of the quantum theory of scattering as the initial equations. The book also uses methods of quantum field theory.

The universality of the approach proposed here is that the specific type of nanosystem and the nature of the confinement potential are not

specified from the very beginning. The choice of a particular nanosystem occurs only at the stage of calculating the intermediate expressions for the overlap integrals, which are included in the equation for impurity levels. Among all possible models of the scattering potential of an impurity atom, a potential of a special form is chosen is the first-rank operator of the theory of degenerate perturbations.

The magnetic localization of electrons at impurity atoms considered here, due to the quasi-one-dimensional motion of electrons in a strong magnetic field, stimulated studies of the physical manifestations of the so-called magnetoimpurity states in low-dimensional systems. The actual physical effects will be described in detail in the book: quantum oscillations of the thermodynamic quantities of 2DEG with impurity states, the new maxima in the frequency dependence of the high-frequency conductivity tensor 2DEG with impurity states, the new branches of magnetoplasma and electron spin waves in inversion layers, a cross situation in the spectrum of spin waves, joint the effect of impurity states of phonons and electrons on the spectra of elastic waves in 2DEG in inversion layers.

In the physics of disordered systems, the new physical phenomena have been discovered and are actively discussed: the Mott transition, interference additives to conductivity, Anderson electron localization, weak localization, and others. Since extensive scientific literature is devoted to these effects, we will not discuss them in this book. We confine ourselves to a description of the effects caused by the resonant scattering of quasiparticles by isolated impurity atoms, which can localize electrons and phonons in low-dimensional systems: quantum dots, quantum wires, nanotubes, mesoscopic rings, in a two-dimensional electron gas. The consideration is based on the I. Lifshits model, within the framework of which the short-range impurity atoms are located at random points in the system. The role of single impurity center in such systems increases with a decrease of their size.

In our book, for the first time in the world scientific literature, we plan to describe the application of the local perturbation method for a theoretical description of the physical properties of a fairly wide range of nanosystems.

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INTRODUCTION

The successes of the theory of a solid with an ideal crystal lattice are largely due to the existence of translational symmetry of the lattice. This symmetry makes it possible to introduce a quasimomentum of elementary excitation in a solid, to reveal the band character of the energy spectrum of bodies. Symmetry allows using the methods of quantum mechanics, statistical physics and kinetics to theoretically describe the properties of ideal solids. An extensive scientific literature is devoted to the properties of such systems [1 – 3]. However, the crystal lattice of a real solid is far from ideal. As a rule, it is distorted by alien impurity atoms, vacancies, dislocations, and other periodicity violations. In addition, with a change in temperature, a solid can undergo a phase transition and go into a liquid state. The idea of an ideal lattice in this state is meaningless. The needs of technology and the logic of the development of science compel physicists to deal with such systems as well.

Systems characterized by violations of the strict periodicity of the lattice are called disordered. In the last sixty years, the structure and properties of condensed disordered systems have attracted more and more attention from both physicists and representatives of related sciences. The reasons for this are, on the one hand, the successes of solid state physics (primarily the physics of semiconductors), and on the other hand, the fact that it is disordered systems (crystals with impurities, liquid metals, amorphous bodies, biopolymers, etc.) that are systems of general positions, and ordered structures such as a perfect crystal lattice are idealized objects. The rapid and ever-growing development of research on the most diverse aspects of the physics of disordered systems has led to the fact that now we can speak of an important, vast and meaningful field of solid state physics.

At the origins of the theory of disordered systems were such outstanding scientists as F. Anderson, I.M. Lifshits and N.F. Mott. It is thanks to their pioneering ideas and papers [4 – 7] that a rapidly growing stream of publications on the theory of disordered systems appeared in the mid-1950s. At present, the physics of disordered systems has deep and general theoretical concepts and developed mathematical apparatus, a large number of various experimental results, and an extremely rapidly growing field of applications. Over time, monographs appeared on its

various aspects [8 – 10]. Many concepts that have arisen in the theory (self-averaging, Anderson localization, Anderson's transition, large-scale localization theory, weak localization) gradually entered textbooks [2, 11].

An important section of the theory of disordered systems is associated with the study of the energy spectrum of single-particle excitations and their quantum states. The absence of translational invariance radically reconstructs the statistical picture of these excitations and the associated mechanism of kinetic phenomena. The systematics of single-particle states turns out to be less clear, and the structure of the spectrum is more diverse than in the ordered case. Therefore, even the one-particle approximation makes it possible to formulate a number of basic concepts, to see many characteristic features and qualitative features of disordered systems, and to study various interesting phenomena.

One of the most widespread versions of the one-particle approximation is the problem of the motion of a particle in a random field created by impurity atoms of finite concentration, randomly distributed over the sites of the crystal lattice. In this case, the Hamiltonian of the interaction of electrons with impurities has the form $U(\vec{r}) = \sum_i U(\vec{r} - \vec{R}_i)$, where $\vec{R}_i$ is the radius vector of the impurity located in the i-th site. The points $\vec{R}_i$ are random, located with an average density determined by the concentration of impurities. Thus, we are dealing with the problem of an electron in a random field of impurities, the state of which can be found by solving the Schrödinger equation with a random Hamiltonian

$$-\frac{\hbar^2}{2m}\Delta\psi + U(\vec{r})\psi = E\psi,$$

where m is the mass of an electron, E is its energy, ψ is the wave function of a stationary state, and $\hbar$ is a quantum constant.

The physics of disordered systems develops through a series of stages. In this case, the following are investigated: isolated impurity atoms in individual lattice sites, multi-impurity systems, amorphous state, many-particle systems with a non-ideal lattice (for example, doped superconductors). In passing to the study of low-dimensional systems, systems with a small number of particles, the role of impurity atoms increases. Even the embedding of one alien atom into such a system can lead to the appearance of a number of properties that have no analogues in massive samples.

The theory of localization of quasiparticles on an isolated impurity in a crystal was constructed in general form by I.M. Lifshits in Refs. [12 – 18]. The method of local perturbations developed by him was successfully used in the theory of the electron energy spectrum of metals and semiconductors [19 – 23], in the theory of the crystal lattice oscillation spectrum [24, 25], the spectrum of spin waves in magnetics [26], in the theory of liquid metals [27 – 30].

The method of local perturbations was further developed in the theory of electron and phonon energy spectra of nonideal metals and semiconductors in Refs. [31 – 54].

I. Lifshits showed that impurity atoms and other lattice defects significantly change the energy spectrum of the conductor. He predicted local oscillations of atoms around an isolated impurity atom. The frequencies of local oscillations lie outside the spectrum of an ideal crystal. Along with local states, the spectrum of quasiparticles can contain singular points located inside the unperturbed zone. If such a state falls into a region where the unperturbed density of states is low, then in its vicinity there is a strong localization of the spectral density of states. The states that have arisen are called quasilocal (resonant) states [43, 45]. The wave function of this state is localized near the impurity, decaying at a finite distance from it. With time, such a state decays, passing into states of a continuous spectrum. These states were found experimentally in experiments with neutron scattering by crystals [55]. Local and quasilocal states of phonons, electrons, and magnons in bulk samples are described in the literature cited above. This approach in the physics of disordered systems is called the Lifshits model.

Along with the Lifshits model, there is also Anderson's model [56] for studying the spectrum of disordered systems. In Anderson's model, various atoms and associated potential wells of electrons of different depths are located at the sites of a regular lattice. On the contrary, in the Lifshits model, the same potential wells are randomly located in space. It is assumed that the potential of the well is short-range, and the average distance between the wells is large in comparison with the radius of action of the potential and the radius of the wave function of the impurity state.

The effect of a quantizing magnetic field on impurity states of electrons in massive metals and semiconductors based on the Lifshits model is considered in papers [57 – 62] and in the dissertation of one of the authors [63].

It is known that not every impurity in a three-dimensional conductor that attracts electrons is capable of forming a bound state [64]. For such a state to appear, the potential well, into which an electron falls in the

impurity field, must be sufficiently deep and wide. In other words, it is required that the depth of the well exceeds the uncertainty of the particle energy $\hbar^2/mr_0^2$, where r_0 is the radius of the well. If this condition is not met, the bound state of the electron is not formed. Only potential scattering of electrons by an impurity center occurs, accompanied by an insignificant phase shift of the electron wave function. The situation changes in a magnetic field. The motion of a particle in this field is similar to one-dimensional, and in the one-dimensional case (and in the two-dimensional) a bound state arises in a well of any intensity. Thus, the magnetic field localizes electrons on an attractive impurity even in the case when localization is impossible without a magnetic field. In a magnetic field on weak impurities, specific bound and resonant states of electrons on donors and holes on acceptors appear, and are caused by the joint action of an attractive impurity and a magnetic field on the particle. For this reason, such states are called magnetoimpurity states.

The idea of magnetic localization of carriers on isolated impurity atoms goes back to the papers by Skobov [65], Bychkov [66], and Demkov and Drukarev [67, 68]. Skobov obtained an exact formula for the amplitude of electron scattering by a short-range impurity center in a magnetic field and discovered its resonant character. Bychkov predicted a bound state of an electron, split off by an attracting impurity from the ground Landau level by an amount [65, 66]

$$\Delta = \frac{1}{2}\hbar\omega_c\left(\frac{a}{l}\right)^2,$$

where $\omega_c = eH/mc$ is the cyclotron frequency [64], $l = \sqrt{c\hbar/eH}$ is the magnetic length [64], a is the scattering length. A more precise equation for the binding energy Δ was obtained by Demkov and Drukarev [67].

Bychkov noted that the Landau quantization leads to the "multiplication" of bound states. An attracting impurity of small radius, removing the degeneracy in the position of the center of the Larmor "orbit", splits off one impurity level from each Landau level. The levels split off from the second, third, etc. Landau levels fall into the region of the continuous spectrum and turn out to be quasi-local. Being in resonance with the Landau states, they acquire a finite width Γ, inversely proportional to the electron lifetime near the impurity. This time is associated with the possibility of the transition of electrons to the lower Landau subbands.

From the point of view of the theory of scattering, the magnetoimpurity states of electrons are the Breit-Wigner resonances known in quantum mechanics in the scattering of electrons by impurity center of attraction in a magnetic field [64]. They correspond to the complex poles of the scattering amplitude located on the nonphysical sheet of the Riemann surface. The positions of magnetoimpurity resonances arising on small-radius impurities are given by the formula [57, 58]:

$$\varepsilon_N = \hbar\omega_c\left(N+\frac{1}{2}\right)-\Delta,$$

where $N=0,1,...$ is the resonance number. Their half-widths are [58]:

$$\Gamma_N = 2\Delta\left(\frac{\Delta}{\hbar\omega_c}\right)^{1/2}\sum_n\left(N-n\right)^{-1/2}.$$

The summation is performed here over those n that are smaller than N. Under certain conditions, the widths of the magnetoimpurity levels are small in comparison with the magnitude of the splitting off Δ, and the latter is large in comparison with the impurity and thermal broadening of the neighboring Landau level, i. e., the magnetoimpurity states are well defined and stable with respect to impurity and thermal effects. The concept of magnetoimpurity states [57, 58] was used by A.M. Ermolaev and M.I. Kaganov to explain the beats in the de Haas-van Alphen effect, experimentally discovered by N.B. Brandt and L.G. Lyubutina (see Ref. [69]) in Bi with impurities Te and Se. Brandt and Lyubutina noted that if the oscillations are due to electrons, beats in the plot of the magnetic susceptibility versus field appear only when Bi is doped with donor impurities Te or Se. Replacement of donors with acceptors (Pb) leads to the disappearance of beats. If the oscillations are caused by holes, the situation is the opposite: beats exist when bismuth is doped with acceptors and are absent if the acceptors are replaced by donors. This feature of the effect is adequate to the concept of magnetoimpurity states of electrons on shallow donors and holes on acceptors [63]. As the magnetic field increases, the magnetoimpurity levels, like the Landau levels, move across the Fermi boundary with a different frequency, which leads to beats in the de Haas-van Alphen effect. The difference between these frequencies is

$$\delta W = \frac{\Delta}{\varepsilon_F}W_0,$$

where ε_F is the Fermi energy, W_0 is the frequency of the Lifshits-Kosevich oscillations [1, 2]. It was also established in Ref. [69] that the beat frequency depends on the type of impurity and is proportional to the oscillation frequency W_0. The noted here and other features of the beat effect were explained in Refs [57, 58].

The studies mentioned above, as well as the experimental detection of magnetoimpurity states in experiments on photoabsorption in semiconductors (for references to original papers, see the review paper [70] by S.P. Andreev) stimulated the appearance of many papers devoted to magnetoimpurity states arising at both Coulomb and neutral impurity centers in semiconductors [71 – 75]. Ref. [71] considers Raman scattering of light in conductors with magnetoimpurity states of electrons. They give rise to an additional contribution to the Faraday angle of rotation [72] and the specific features of the chemical potential of metals [73]. In Ref. [74], it was shown how these states affect the phonon spectrum in nonideal conductors. In Ref. [75], these states were studied by a functional method. They were also studied in papers [76 – 79]. In particular, in Refs. [78, 79], a model potential of an impurity atom (Gaussian separable potential) was used.

In papers [80 – 82], the contribution of magnetoimpurity states to the transverse magnetoresistance of metals in strong fields is calculated. It turned out that this contribution contains a part that is monotonic depending on the magnetic field:

$$\delta\rho_{\perp} = \rho_0 \, \pi \frac{a}{r_H},$$

where

$$\rho_0 = \frac{m}{ne^2\tau_0}$$

(n is the concentration of electrons; τ_0 is the free path time due to potential scattering of electrons by impurities in the absence of a magnetic field);

$$r_H = \frac{cp_F}{eH}$$

is the Larmor radius (p_F is the Fermi momentum). The expression $\delta\rho_{\perp}$ refers to single crystals with closed Fermi surfaces. It increases linearly with the field in the region $\omega_c\tau_0 \gg 1$, where saturation is predicted by

the classical theory. The oscillating contribution leads to beats in the Shubnikov-de Haas effect.

Back in 1929 P.L. Kapitsa found that the transverse magnetoresistance of polycrystalline samples increases linearly with field in strong magnetic fields. This law was explained in the paper by I.M. Lifshits and V.G. Peschansky [83]. In the papers by A.M. Ermolaev [80 – 82] considered another reason for the linear increase in the magnetoresistance of metals.

The main reason for the existence of various types of weakly damped electromagnetic waves in metals in an external magnetic field is the collective motions of conduction electrons [84, 85]. There are two types of such motions.

The first of them is the collective drift (Hall or polarization) of electrons in a plane perpendicular to the magnetic field. In uncompensated metals, the Hall drift leads to the existence of helicons (for references to original papers, see Refs. [84, 85]). In semimetals, the collective polarization drift due to temporal dispersion leads to the appearance of magnetoplasma waves. Longitudinal drift motion of electrons (along the magnetic field) in quantizing magnetic fields leads to the possibility of the existence of collective oscillations with a linear spectrum (quantum electromagnetic waves).

Another type of collective motions, leading to the existence of weakly damped electromagnetic excitations, is associated with various resonances in the electron-hole system. Thus, near the frequencies of cyclotron resonance, cyclotron waves exist in metals. Doppler-shifted cyclotron resonance results in a variety of dopplerons. In the vicinity of resonance frequencies corresponding to transitions between magnetic surface levels the specific surface waves appear. It can be argued that, in general, corresponding electromagnetic waves should exist near any resonance, and the resonance frequency coincides with the limiting frequency in the spectrum of such collective excitations.

In Refs. [59 – 62, 86 – 90], the new classes of weakly damped waves in metals were predicted, which arise due to resonances in the scattering of electrons by attractive impurities in a magnetic field.

It was noted above that impurity atoms in the metal play a double role. On the one hand, they limit the mean free path of conduction electrons and determine the collisional damping of electromagnetic waves, which is usually described by introducing a phenomenological constant ν – collision frequency. On the other hand, impurity atoms can radically change the structure of the electron energy spectrum of a metal, leading to the appearance of the magnetoimpurity states described above. Such states

correspond to resonances in the scattering of electrons by an attractive impurity in a magnetic field. They appear as Breit-Wigner peaks in the energy dependence of the scattering probability and collision frequency of electrons. These peaks exist against the background of a smooth dependence due to potential electron scattering.

Magnetoimpurity states open new channels for resonant absorption of electromagnetic waves in conductors. The frequencies of resonant transitions of electrons between filled magnetoimpurity levels and free Landau levels are equal $\omega_s = \Delta/\hbar + s\omega_c$, where $s = 0,1,...$ is the resonance number. If we neglect the widths of the levels involved in the transitions, the real part of the conductivity, which determines the damping of the waves, will have a root singularity, when $\omega \to \omega_s + 0$. This feature reproduces the feature of the density of states of electrons at the Landau level participating in the transition. It follows from the Kramers-Kronig dispersion relations that the root singularity will have the imaginary part of the conductivity at $\omega \to \omega_s - 0$. On the features of the imaginary part of the conductivity, new branches of the spectrum of electromagnetic excitations of the metal are formed. Waves appearing below frequencies ω_s, where the imaginary part of the conductivity is large in comparison with the real one, are called magnetoimpurity [59, 60]. The physical reason for the existence of these waves is that the magnetic localization of electrons on impurities leads to an even greater limitation of the freedom of motion of electrons than only the magnetic field. Due to this, the role of dissipative processes decreases, and transparency windows are opened for new waves, which do not propagate in conductors without magnetoimpurity states. Electromagnetic waves in conductors with magnetoimpurity states of electrons were also studied in [91 – 94].

In the vicinity of frequencies

$$\omega_s = \frac{1}{\hbar}\left(2\mu H + \Delta\right) + s\omega_c$$

(μ is the magnetic moment of the electron) in non-ferromagnetic metals there are weakly damped oscillations of the spin density of electrons, called magnetoimpurity spin waves [95 – 97]. They exist in narrow bands of transparency outside the Stoner sectors. Therefore, there is no collisionless damping of these waves. Their damping is determined by the width of the magnetoimpurity level and the electron collision frequency

due to potential scattering by impurities. Scattering of neutrons by magnetoimpurity spin waves was considered in Ref. [98].

In modern solid state physics, along with the traditional three-dimensional electron systems that have just been discussed, low-dimensional electron systems (two-dimensional – 2D, zero-dimensional – 0D, and one-dimensional – 1D) are increasingly being studied [99]. The motion of conduction electrons belonging to low-dimensional systems , is limited by narrow quantum wells, which significantly determines both the energy spectrum unperturbed by the impurity potential and the nature of the formation of bound states.

The purpose of this monograph is to present the theory of localization of conduction electrons in the aforementioned nanosystems within the framework of the method of local perturbations by I.M. Lifshits, as a general approach that is valid for any dimension and form of confinement potential in a magnetic field. The generality of the approach consists, in particular, in the fact that the derivation of the equation for the spectrum of impurity levels is carried out from “first principles”, which include the use of the equations of the quantum theory of scattering and functional methods as the initial ones.

The universality of the consideration proposed here also lies in the fact that the type of nanosystem and the nature of the confinement potential are not concretized from the very beginning: consideration of a definitely chosen nanosystem (quantum dot, quantum wire, nanotube, thin ring, mesoscopic ring, two-dimensional electron gas) occurs only at the stage intermediate equations for overlap integrals. Among all possible models of the scattering potential of an impurity atom, a potential of a special form was chosen – the operator of the first rank of the theory of degenerate perturbations by I.M. Lifshits. Thus, the above considered magnetic localization of electrons on impurity atoms, caused by the quasi-one-dimensional motion of electrons in a strong magnetic field, stimulated the study of magnetoimpurity states in low-dimensional systems (quantum dot, quantum wire, nanotube, thin ring, mesoscopic ring, two-dimensional electron gas) by the method of I.M. Lifshits local perturbations.

CHAPTER ONE

NANOSYSTEMS: CLASSIFICATION AND DEVELOPMENT OF RESEARCH

An introduction to the physics of low-dimensional systems is the excellent book by Davis [99]. This book examines a wide range of issues in solid state physics – from methods of creating low-dimensional structures to the theory of physical phenomena in them. The presentation of the material is available to the inexperienced reader, it is made at a high level. There are helpful exercises at the end of each chapter. The book focuses on the properties of quantum wells. Other structures are also being considered. Much attention is paid to electron-impurity scattering. Unfortunately, the author said little about quantum dots, wires and nanotubes. In the theory of electron scattering by impurity atoms, he limited himself to potential scattering, therefore the book does not contain a theory of local and resonance states of electrons in the field of isolated impurity atoms.

1.1. Quantum Dots

A quantum dot is a system of conductor electrons moving in a limited nanometer-sized volume. The problem of creating such a structure is to impose an additional potential (confinement potential) on the conduction electrons in the conductor, which limits their motion in all directions. This is achieved using a special molecular beam epitaxy technique. Using this technique, nanostructures with one-dimensional electron gas are created: quantum wires and zero-dimensional structures several nanometers in size, which are called quantum dots. It is important to emphasize that a quantum dot 10 nm in size contains about 100 atoms along its diameter, and although its inner part retains the crystalline symmetry of the bulk material, all the properties of this nanostructure strongly depend on the state of its surface.

It is well known [100] that the electron energy spectrum of an isolated quantum dot is a set of discrete dimensional quantization levels. Therefore, it can be considered as a giant artificial atom with controlled parameters such as the depth of the potential well, the nature of the confinement potential, the quantity of electrons, and the size of their localization region.

The interest in quantum dots and other nanostructures is due to a number of reasons [100]. The influence of microelectronics on society is constantly growing. The successes of computer technology, informatics, radio-electronics are based on the achievements of microelectronics. It forms the element base of all modern devices of receiving, transmitting and processing information, automated control systems. The desire to reduce the size of devices, to increase their speed has led to the fact that solid state physics has turned into nanophysics – the physics of nanosystems. These systems are interesting not only because they are functional elements of modern gadgets and devices, but also because they exhibit quantum effects. It is impossible to predict the properties of future technical devices without the use of quantum mechanics. The study of quantum effects in semiconductor nanostructures gave impetus to the emergence of new classes of devices: resonant tunnel diodes, transistors, which have a high speed of response and a wide range of capabilities. The problems of creating quantum integrated circuits and quantum computers, the main elements of which will be quantum dots, wires, wells, and nanotubes, are intensively discussed.

The properties of a nanosystem in a magnetic field are of particular interest. The presence of a magnetic field leads to the appearance of the hybridization effects of the dimensional and magnetic quantization of the motion of electrons. These effects include the quantum Hall effect in a two-dimensional electron gas.

Theorists are interested in quantum dots because they are convenient objects for testing new calculation methods. Of particular interest are the properties of quantum dots with impurity atoms in a magnetic field. A small number of electrons in a quantum dot leads to the fact that even one impurity atom has a strong effect on its properties. In a limited volume of a quantum dot in a magnetic field, interesting effects of hybridization of spatial and magnetic quantization of the motion of electrons are manifested. These effects include the considered here localization of electrons on individual impurity atoms.

The monograph [100] is devoted to the electron and optical properties of quantum dots and other low-dimensional semiconductor structures. This monograph discusses methods for producing quantum dots, ways to

implement their confinement, new types of optoelectronic devices containing quantum dots. Quantum dots in systems with two-dimensional electron gas (2DEG) at the GaAs-AlGaAs interface are considered. The Fock-Darwin parabolic confinement is used [101, 102]. The magnetic field is perpendicular to the 2DEG plane. The effect of one or several impurity atoms in quantum dots scattering electrons is taken into account. The author Ref. [100] limited himself to considering the impurity repulsive potential of the Gaussian type

$$V\left(\vec{r}\right)=V_0\exp\left[-\frac{\left(\vec{r}-\vec{R}\right)^2}{d^2}\right] \qquad \left(V_0>0\right),$$

where d is the potential radius, $\vec{R}$ is the position of the impurity atom. This potential does not localize electrons. The author of this monograph does not consider impurity states of electrons in a quantum dot. New methods (epitaxial growth, lithography method) and other methods for the synthesis of quantum dots are considered in monograph [103] and Petrov's papers № 4, 5 (4. P.M. Petroff "Epitaxial Growth and Electronic Structure of Self-assembled Quantum Dots"; 5. P.M. Petroff "Self-assembled Quantum Dots Devices") in the collection of articles [104]. These papers noted the properties of quantum dots that are important for practical applications: confinement of carriers and their pairs, which localizes particles; features of the density of states of electrons that bring a quantum dot closer to an isolated atom; multiparticle effects. It is noted that semiconductor quantum dots based on InAs, GaAs, InAs-GaAs can be used in devices and gadgets: detectors, batteries, lasers, transistors, quantum computers, generators of individual photons.

The theory of quantum transport of electrons in systems with quantum dots, taking into account the electron-electron interaction, was considered in a review paper [105]. The authors of this paper examined low-temperature electrical transport through a quantum dot, which is associated with two conducting contacts. The role of electron-electron interaction in this phenomenon is decisive. The conductance of the system is calculated in a wide temperature range. It is found out how the Coulomb blockade and Kondo scattering of electrons affect the temperature dependence of conductance. The influence of electron-electron interaction on weak localization and Aharonov-Bohm oscillations in quantum dots is considered in paper [106].

1.2. Quantum wires

The methods of quantum field theory [107, 108] are actively used in the study of quantum wires and other one-dimensional systems [109 – 112]. In this case, the following methods of theoretical physics are used: supersymmetry, bosonization, Luttinger fluid model, renormalization group, conformal symmetry.

Monograph [111] considers strongly correlated electrons in low-dimensional systems. The bosonization method is used, based on the idea of converting fermions into bosons, proposed by Jordan and Wigner [113]. Part III of this monograph is devoted to disordered one-dimensional systems. The potential scattering of electrons by an isolated impurity center in a three-dimensional sample is considered by the bosonization method. The phases of the scattered electron waves are obtained. They are compared with the results of the standard scattering theory. Local and quasilocal states of electrons in the field of isolated impurity atoms are not considered in this monograph.

In monograph [112], in the theory of wire, the method used earlier in the study of the Luttinger fluid was used. A quantum wire is considered as an experimental realization of a Luttinger fluid. The electron-electron interaction is taken into account. The problem of electron confinement in the GaAs-AlGaAs system in a narrow channel is discussed. Quantization of the motion of electrons perpendicular to the channel is taken into account, which leads to the formation of minibands in the energy spectrum of electrons. The conditions are found under which one can restrict oneself to taking into account the influence of the lower miniband in the process of calculating the conductivity. The conductivity and conductance of wires are calculated in cases of isolated impurities and extended disorder. The properties of carbon nanotubes with metallic conductivity are considered on the basis of the Luttinger liquid method.

1.3. Nanotubes

Carbon nanotubes were discovered by Iijima [114] in 1991. Together with fullerenes and mesoporous carbon structures, they form a new class of nanomaterials, the properties of which differ significantly from the properties of other forms of carbon: graphite and diamond. Interest in these systems is due to their unique properties – high strength and conductivity, magnetic, optical, capillary properties. The technology for preparing nanotubes is being continuously improved [115]. They are usually obtained by rolling a graphene sheet into a tube. Depending on the

rolling method, the tube has metallic, semiconducting, or dielectric properties [116, 117]. Soon, semiconductor nanotubes were synthesized [118 – 120], as well as nanotubes with superlattices [121]. A new era in the physics of carbon nanotubes began in 2004 after obtaining a building material for them – graphene layers [122]. It turned out that the energy spectrum of electrons in graphene is Dirac [123, 124]. This affects, in particular, the spectrum of impurity states of electrons in graphene layers [125]. The method of controlled adsorption of atoms, as well as the lithographic method, make it possible to create superlattices on graphene layers and on carbon, as well as semiconducting nanotubes [126].

The prospects of using nanotubes in field effect transistors, highly sensitive sensors, liquid crystal displays, solar cells, and spin transistors are tempting in Ref. [127]. The possibility of using nanotubes as an element base for spintronics is discussed in Ref. [128]. They can be used for storing and transmitting information, as additives to polymers, as energy converters, in supercapacitors and quantum computers [129]. The question of the possibility of using nanotubes in medicine is being discussed in Ref. [130].

The review papers [131 – 133] discuss the results of studies by the authors of this monograph devoted to the study of thermodynamic characteristics and spectra of collective excitations (plasma and spin Landau-Silin waves) of an electron gas on the surface of a semiconductor nanotube with a superlattice in a magnetic field.

1.4. Mesoscopic rings

A good introduction to the physics of mesoscopic systems is Imri's book [134]. In a small volume of the book (234 pp.), the author acquaints the reader with the basic concepts and phenomena of mesoscopic physics: Anderson localization, dephasing of wave functions by electron-electron interaction, quantum interference effects, quantum Hall effect, mesoscopic superconductivity, noise in mesoscopic systems, etc. The presentation of the book begins with a description of methods for synthesizing mesoscopic systems. The results of calculations of physical quantities are presented. More cumbersome mathematical transformations have been moved to the Appendix.

1.5. 2D electron gas

The increased interest of researchers in the physics of two-dimensional conductors is due to a number of circumstances. One has to deal with them

when studying a wide range of physical systems: two-dimensional metals and semiconductors [2, 135 – 138], semiconductor surfaces [139], inversion and accumulation layers at the semiconductor-dielectric interface [140], heterostructures with selective doping [141], thin films of metals [135, 141], layered compounds [141], electrons on the surface of liquid helium [141], high-temperature superconductors [142]. A number of phenomena have been discovered in two-dimensional conductors, the interest in which is still unabated. These include the quantum Hall effect [143 – 146], mesoscopic effects [2], and an unusual spectrum of two-dimensional plasmons [140]. Two-dimensional conductors serve as the basis for many gadgets and devices of microelectronic technology. Suffice it to recall the field-effect transistor. Research over the past few decades has breathed new life into this important element of modern electronics. The advances in the technology of growing thin films with thicknesses up to atomic, along with the development of theoretical methods for studying condensed matter, continue to stimulate the development of this topical area of physics – the physics of two-dimensional electronic systems.

Plasmons with an unusual non-activation spectrum in two-dimensional electron systems were predicted long ago [140, 147 – 149]. In 1977, they were discovered experimentally [150] in an inversion layer at the interface between silicon and silicon dioxide. Soon, two-dimensional magnetoplasmons were also discovered in an inversion layer placed in a magnetic field perpendicular to the layer [151]. Intensive study of magnetoplasma waves in two-dimensional conductors has been going on for several decades. In Ref. [152] considered a strictly two-dimensional electron gas embedded in a dielectric placed in a magnetic field perpendicular to the electron layer. A dispersion equation for magnetoplasmons propagating in the plane of the layer is obtained, and its solution is analyzed. In Ref. [153], the Fermi-liquid theory of magnetoplasma and spin waves in two-dimensional systems is constructed. Collisions of electrons with impurity atoms and the associated damping of magnetoplasmons are not considered in Refs [152, 153]. The connection between two-dimensional magnetoplasmons and phonons was studied in [154].

Physics of another type of collective excitations in two-dimensional non-ferromagnetic conductors (spin waves) is also intensively developing. A large number of papers are devoted to the calculation of the dynamic spin susceptibility of two-dimensional electrons, which appears in the dispersion equation for spin waves. The results of calculations of the static susceptibility of a free electron gas in a magnetic field perpendicular to the plane of electron motion are contained in Ref. [155]. The exact expression

for the dynamic spin susceptibility, as well as the density-density reaction function of a free degenerate electron gas, is given in Refs. [140, 156, 157]. The quantizing magnetic field was taken into account in Ref. [158]. The high-frequency asymptotics of the spin susceptibility of a two-dimensional Fermi liquid was obtained in Refs. [159, 160]. The influence of impurity atoms, potentially scattering conduction electrons, on the susceptibility was considered in Ref. [161]. The relaxation of the spin magnetization of two-dimensional electrons in a magnetic field, as well as the localization of magnetoplasma and spin excitations in such systems in the field of impurity atoms were studied in Refs. [162, 163]. The scattering of light by these excitations was also considered in Ref. [164, 165].

Despite the large number of papers devoted to magnetoplasma and spin excitations in two-dimensional disordered systems in a quantizing magnetic field, the problem of collective excitations in such systems cannot be considered finally solved. Usually, collisions of electrons with impurity atoms and crystal lattice defects of the sample are taken into account by introducing the collision frequency into the conductivity tensor and spin susceptibility, which is assumed to be constant. As a result, collisional damping of collective excitations appears, which dominates in those regions of frequencies and wave vectors where Landau damping is absent. This is permissible when electrons experience only potential scattering by impurity atoms. Meanwhile, the role of impurity atoms in conductors is more complex. It has long been known [8, 18] that impurities shift and broaden the energy levels of electrons and cause different types of localization in the system. In the energy spectrum of electrons, there are local and quasi-local states [8, 18], which must be taken into account when calculating the kinetic characteristics of conductors. Taking into account the localized states of quasiparticles is one of the important problems of quantum kinetics [8, 166, 167]. This problem is especially relevant in two-dimensional conductors. It is known [64] that, in the two-dimensional case, an arbitrarily weak attraction impurity is capable of forming a bound state of a particle. The corresponding local level is located at the edge of the conduction band. This state corresponds to the pole of the electron scattering amplitude by an isolated impurity atom located on the physical sheet of the Riemann surface of the amplitude as a function of the electron energy [168, 169]. The binding energy in the local state is exponentially small compared to the depth of the impurity potential well [64]. As noted recently, bound states have a significant effect on the low-temperature properties of two-dimensional systems. The existence of such states means that it is

impossible to calculate the kinetic characteristics of a two-dimensional electron-impurity system on the basis of perturbation theory from the scattering impurity potential. Thus, it is necessary to use an exact equation for the scattering amplitude.

As noticed earlier, the amplitude of scattering of quasiparticles by impurity centers in a solid is usually calculated using the method of local Lifshits perturbations [8, 18, 170] or the method of zero-radius potentials [68, 171]. The exact equation obtained by these methods for the amplitude of electron scattering by short-range impurity atoms was used to calculate the static conductivity of a two-dimensional electron gas [172]. In this paper, the features of the static conductivity of two-dimensional metals, inversion layers and heterojunctions, which cannot be obtained on the basis of perturbation theory, are predicted.

In a quantizing magnetic field perpendicular to the plane of electron motion, there is a system of local levels alternating with Landau levels. Their positions in the case of short-range impurity potentials of various types were found by the method of local perturbations in [170, 173, 174]. Such a structure of the spectrum of an electron-impurity system manifests itself in optical experiments with a two-dimensional electron gas in a magnetic field [140].

Local impurity states have a significant effect not only on static kinetic coefficients, but also on high-frequency ones. High-frequency conductivity and spin susceptibility are included in the dispersion equations for magnetoplasma and spin waves. Therefore, one should expect a rearrangement of the spectrum of collective excitations in two-dimensional conductors taking into account the impurity states of electrons in the presence of a magnetic field. Such a rearrangement takes place in the three-dimensional case [59 – 63].

CHAPTER TWO

THE METHOD OF LOCAL LIFSHITS PERTURBATIONS

2.1. Local perturbations in metals and semiconductors

In this subsection, following the book [26], the application demonstrates of the method of local perturbations by I.M. Lifshits for a simple example of a three-dimensional conducting crystal. In the case of an ideal crystal described by the Hamiltonian $\hat{H}_0$, it is convenient to use the quasimomentum projections $\vec{k}$ as quantum numbers:

$$\hat{H}_0 \left|\vec{k}\right\rangle = \varepsilon_{\vec{k}} \left|\vec{k}\right\rangle, \tag{2.1.1}$$

where $\varepsilon_{\vec{k}}$ is the energy of the electron in the state $\left|\vec{k}\right\rangle$. However, it is convenient to take into account the presence of a local perturbation V in the crystal using the site representation. The basic functions $\left|n\right\rangle$ of this representation are localized at the lattice sites and have the form [3]

$$\left|n\right\rangle = \frac{1}{\sqrt{N}} \sum_{\vec{k}} \mathrm{e}^{-\mathrm{i}\vec{k}\vec{R}_n} \left|\vec{k}\right\rangle, \tag{2.1.2}$$

where $\vec{R}_n$ is the lattice vector, N is the number of sites in the crystal. The orthogonality of the basis $\vec{k}$ implies the orthogonality of the basis (2.1.2). The case of one energy band of an ideal crystal is considered. It should be emphasized that it is inconvenient to describe the eigenfunctions of the perturbed Hamiltonian $\hat{H} = \hat{H}_0 + \hat{V}$ using the quasimomentum.

Consider the operator $\left(E - \hat{H} - \mathrm{i}\delta\right)^{-1}$. In the site representation, it has the form of a matrix with elements

$$G_{nm}(E) = \langle n | \left(E - \hat{H} - \mathrm{i}\delta \right)^{-1} | m \rangle . \qquad (2.1.3)$$

This expression coincides with the Green's function of the Schrödinger equation. The operator in Eq. (2.1.3) can be represented as a sum of an unperturbed term and a correction describing the perturbation by an impurity:

$$\frac{1}{E - \hat{H} - \mathrm{i}\delta} = \frac{1}{E - \hat{H}_0 - \mathrm{i}\delta} + \frac{1}{E - \hat{H}_0 - \mathrm{i}\delta} \hat{V} \frac{1}{E - \hat{H} - \mathrm{i}\delta} .$$

If we take from this identity the matrix element $G_{nm}(E)$ between the states $\langle n|$ and $|m\rangle$, then we can make sure that it satisfies the Dyson-type equation:

$$G_{nm}(E) = G^0_{nm}(E) + \sum_{pl} G^0_{np}(E) V_{pl} G_{lm}(E) . \qquad (2.1.4)$$

Here $V_{pl} = \langle p | \hat{V} | l \rangle$ is the matrix element of the operator $\hat{V}$ in the site representation, G^0_{nm} is the Green's function for an ideal crystal:

$$G^0_{nm}(E) = \frac{1}{N} \sum_{\vec{k}} \frac{\mathrm{e}^{\mathrm{i}\vec{k}\left(\vec{R}_n - \vec{R}_m\right)}}{E - \varepsilon_{\vec{k}} - \mathrm{i}\delta} . \qquad (2.1.5)$$

Equation (2.1.4) can be rewritten symbolically:

$$\hat{G} = \hat{G}^0 + \hat{G}^0 \hat{V} \hat{G} . \qquad (2.1.6)$$

Its symbolic solution is:

$$G = \frac{1}{1 - G^0 V} G^0 = G^0 + G^0 V \frac{1}{1 - G^0 V} G^0 . \qquad (2.1.7)$$

It is known [3, 108] that the poles of the Green's function give the spectrum of elementary excitations of the crystal. In Eq. (2.1.7), in addition to the poles of the G^0-function, there are also poles $\left(1 - G^0 V\right)^{-1}$. These poles contribute, due to the presence of impurities, to the excitation spectrum of the crystal [8, 13, 14, 16]. The equation whose solution these poles are is called the Lifshits equation [8, 18]:

$$\det\left[1 - G^0(E) V \right] = 0 . \qquad (2.1.8)$$

Generally speaking, to solve this equation it is necessary to calculate the determinant of the matrix $N \times N$. This is a difficult task due to its excessive cumbersomeness. However, the perturbations V is quasi-local. Its intensity decreases rapidly with distance from the impurity. This means that we can consider the impurity perturbation only in the region of the first few coordination spheres. In other words, instead of a matrix $N \times N$, we consider a matrix $n_0 \times n_0$ (here n_0 is the number of atoms falling within the sphere of the perturbation).

To begin with, consider the perturbation localized within the impurity atom. Let it be in a site $n = 0$. Then the perturbation matrix has the form

$$V_{nm} = u_0 \delta_{n0} \delta_{m0}, \tag{2.1.9}$$

where u_0 characterizes the intensity of the impurity perturbation. Examples of such perturbations can be an impurity atom in a metal, with a potential screened by conduction electrons; or an atom that differs in mass from the environment (in this case, changes in the force constants of the lattice are neglected).

Within the framework of perturbation (2.1.9), equation (2.1.4) takes the form:

$$G_{nm} = G^0_{nm} + \frac{u_0 G^0_{n0} G^0_{0m}}{1 - u_0 G_0}. \tag{2.1.10}$$

Here

$$G_0 \equiv G^0_{00} = \frac{1}{N} \sum_{\vec{k}} \frac{1}{E - \varepsilon_{\vec{k}} - \mathrm{i}\delta}. \tag{2.1.11}$$

From Eq. (2.1.10) it is easy to find the density of states $\nu(\varepsilon)$ in the spectrum of single-particle excitations:

$$\nu(\varepsilon) = \frac{1}{\pi N} \operatorname{Im} \operatorname{Sp} \hat{G}(E). \tag{2.1.12}$$

The factor $1/N$ arises due to the normalization of the density of states per one atom of the matrix. The Spur included in Eq. (2.1.12) is easily calculated:

$$\operatorname{Sp} \hat{G} = \sum_n G_{nn} = \operatorname{Sp} \hat{G}^0 + \frac{u_0}{1 - u_0 G_0} \sum_n G^0_{n0} G^0_{0n}. \tag{2.1.13}$$