

The Fundamental Principles of Physics

The Fundamental Principles of Physics:

From Atom to Molecule

By

Paul Blaise and Olivier Henri-Rousseau

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PREFACE

The book by Profs. Paul Blaise and Olivier Henri-Rousseau entitled "The Fundamental Principles of Physics: From Atom to Molecule" presents in an illustrative, didactic and attractive way current knowledge in the area of molecular physics at the basic and advanced level for undergraduate science students.

The book is composed of eight chapters describing constituents of matter, the wave and corpuscular nature of light and matter, quantization of energy in the hydrogen atom, semi-classical and quantum models of the hydrogen atom, Lewis and MO models of chemical bond and chemical reactivity and molecular orbitals in electrocyclic reactions.

The material is presented in an attractive and illustrative way. The book contains photos of eminent scientists and well designed figures. Most of the chapters end with well prepared tutorials containing problems to be solved and answers.

This book is a valuable source of knowledge in molecular physics at the undergraduate level and is recommended to students of physics, chemistry and molecular science.

Prof. Dr. hab. Marek J. Wójcik
Jagiellonian University
Krakow, Poland

FOREWORD

This book corresponds to an introductory course on the structure of matter for students in the first years of their physical sciences degrees. Starting from the fundamental constituents of matter and highlighting their mutual interactions, it gradually leads to a more precise idea of what an atom or molecule is by using theories that seek to reproduce the experimental behaviors of matter while remaining within the commonly accepted scientific framework. The aim of this course is therefore to awaken in physical science students an acknowledgement of the complexity of reality, as well as the methods used to achieve some understanding of the world around us. To this end, since the beginning of the 20th century physics has possessed a powerful theoretical tool to understand certain aspects and behaviors of matter: quantum theory, which uses quantum mechanics. However, being aware of the difficulties in understanding quantum theory that can arise in the minds of students, we have often insisted on the importance of the physical principles underlying this theory, such as the Coulomb relation, the particle-wave duality, the notions of determinism and indeterminism, estimation of the probability of presence, and the virial theorem.

This book is composed of eight chapters. At the end of each chapter, we have added tutorials containing several exercises along with the answers.

Chapter I provides a directory of atomic constituents.

Chapter II is dedicated to the study of light, experimentally highlighting the wave-like behavior and the corpuscular character of electromagnetic vibrations in general.

In **Chapter III**, we are led to consider material particles using an identical approach and consider the behavior of both corpuscular and wave-like matter generally.

In **Chapter IV**, remaining in the microscopic domain, we discuss the various experiments that show the energy of electrons in atoms to be quantized.

Then, in **Chapter V**, we discuss the various semi-classical theories that explain the quantization of energy in the atom based on fundamental principles, as well as discussing their limitations and the need for more rigorous theory.

Chapter VI addresses the full quantum theory of the atom through the resolution of the Schrödinger equation; its resolution allows us to clarify the details of energy quantization with the model of stationary states. This model, extended to atoms with several electrons, is able to describe each atom by its electronic configuration and account for its physicochemical properties by explaining the periodic classification of the elements.

Chapter VII, being the largest chapter of this book, is the culmination of previous chapters. It is devoted to an analysis of chemical bonding both from the electronic point of view with Lewis's elementary theory, which allowed the principles defining a chemical bond between two atoms to be set down, and with the theory of molecular orbitals based on the linear combination of atomic orbitals. Part of this chapter is devoted to showing how the basic principles of physics are at work even in rudimentary models of atoms and molecules. In relation to polyatomic molecules, the notion of geometry intervenes with the shape of molecules in space. The predictive nature of a theory, such as the VSEPR method, combined with the theory of hybridization of atomic orbitals and the theory of molecular orbitals can provide the most relevant picture possible of what constitutes a molecule.

Finally, in **Chapter VIII**, which goes beyond the study of the structure of matter, we examine how this knowledge of molecules is at work in attempting to predict their reactivity. Thus, the use of the theory of molecular orbitals of polyenes is able to account for the experimental results of electrocyclic reactions.

CHAPTER I

THE CONSTITUENTS OF MATTER

I-I. The Macroscopic and Microscopic Domains

The world can be roughly divided into two domains: the macroscopic and the microscopic.

The macroscopic domain concerns that which is on the human scale or above.

The microscopic domain concerns that which is at the scale of the atom or below.

Thus, a comparison can be made between what is at the microscopic scale and what is at the macroscopic scale.

Table I-1: Microscopic and macroscopic domains

Microscopic domain	Macroscopic domain
Atom=microscopic unity of matter	Element=macroscopic unity of matter
Homoatomic molecule	Pure element
Heteroatomic molecule	Pure compound

1. The macroscopic and microscopic ratio: the mole N



A. Avogadro

This ratio is given by the Avogadro¹ number, denoted by N . This overall classification must be clarified using a more quantitative measure of the microscopic character. A mole corresponds to a collection of N “individuals”. In particular, we talk about the mole of atoms (N atoms), the mole of molecules (N molecules), or the mole of electrons (N electrons).

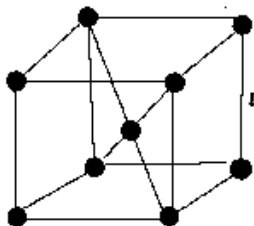
2. The value of N

As early as 1875, J. van der Waals estimated this value by interpreting the laws of real gases (based on the ideal gas law). Let us take the example of a variety of iron, α -iron, consisting of a regular assemblage of atoms, the basic pattern of which (conventional unit cell) is described as “centered cubic”.



J. van der Waals²

In this model, the edge of the cube is l . The ball-and-stick form is shown on the left side of **Fig. I-1**. In fact, since the atoms are in contact with each other, the conventional unit cell should be represented as the *space-filling* form located on the right side of the same figure.



Ball-and-stick model.



Space-filling model.

Figure I-1a: Iron α conventional unit cell.

¹ Amadeo Avogadro (1776-1856). Italian physicist and chemist.

² Johannes Diderik van der Waals (1837-1856). Dutch physicist and mathematician (1837-1923). He won the Nobel Prize in Physics in 1910.

Let us now detail the various steps leading to the calculation of the Avogadro number in the case of the α -iron crystal. Consider the space-filling form. It can be seen that there is $1/8$ of an iron atom at each corner and thus each corner corresponds to a volume of $(1/8)v_{At}$, where v_{At} is the total volume of an atom. All 8 vertices correspond to a volume of approximately

$$v = 8(1/8) v_{At}$$

Let us add that there is a whole atom of iron in the center of the cell. If the residual gaps are neglected, this gives the cell the following total volume

$$v_{cell} \simeq \left(8 \frac{1}{8} + 1\right) v_{At} = 2v_{At}$$

then

$$v_{At} \simeq \frac{v_{cell}}{2}$$

In addition, there are N atoms in a mole of atoms and thus the volume of a mole of atoms is

$$V_{At} = N v_{At} = N \frac{v_{cell}}{2}$$

However, the volume of the cell is, of course, that of a cube of edge l

$$v_{cell} = l^3$$

From this, we can then obtain the expression of the volume of the mole of an atom of iron

$$V_{At} = N v_{At} = N \frac{l^3}{2} \tag{I-1}$$

However, the density of iron is, by definition, the ratio between the mass of a mole of iron atoms M_{Fe} and its volume V_{Fe}

$$\rho_{Fe} = \frac{M_{Fe}}{V_{Fe}} \tag{I-2}$$

then

$$V_{At} = \frac{M_{Fe}}{\rho_{Fe}}$$

Let us identify the two expressions (I-1) and (I-2) for the volume of a mole of iron atoms

$$\mathcal{N} \frac{l^3}{2} = \frac{M_{Fe}}{\rho_{Fe}}$$

As a result, the expression of the Avogadro number can be determined experimentally by the following expression

$$\mathcal{N} = 2 \frac{M_{Fe}}{\rho_{Fe} l^3} \quad (\text{I-3})$$

Now, we have to measure the distance l . To do this, an X-ray machine called the Debye-Scherrer chamber is used.



P. Debye³



P. Scherrer⁴



W. Bragg⁵

Here, the X-ray beam is focused on the iron crystal. If this beam is punctual, the crystal diffracts it. There is a symmetry of revolution around the axis of the beam. The diffracted rays form cones, the axis of which is the incident beam. On a photographic plate, we can then observe Debye-Scherrer rings, which are the traces of these cones. **Figs. I-1b** and **Ic** illustrate these rings.

³ Peter Debye (1884-1968). Dutch physicist known for his contributions to the study of dielectric materials. Winner of the Nobel Prize in Chemistry in 1936.

⁴ Paul Hermann Scherrer (1890-1969). Swiss physicist known for his work on crystallography and, in particular, his contribution to the Debye-Scherrer method.

⁵ William Bragg (1890-1971). British mathematician and crystallographic physicist. Winner of the Nobel Prize in Physics in 1915.

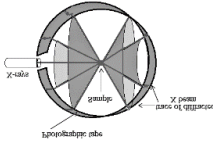


Figure I-1b:
Debye-Scherrer chamber.

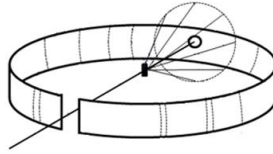


Figure I-1c:
Details of diffraction rings.

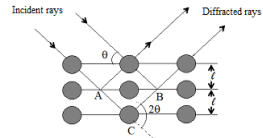


Figure I-1d:
X-ray diffraction.

As schematized in **Fig. I-1d**, in a crystal there is a three-dimensional periodic repetition of elements (atoms or molecules) called nodes; these appear as dark discs on the figure representing the reticular planes passing through the centers of these elements, spaced at a distance of l . The angle θ , called the *Bragg angle*, determines the impact of a parallel X-ray beam on these reticular planes. Note that this is the complement to the usual angle of incidence in optics. The difference in the path between the two light rays depicted has a value of $AC+CB = 2l \sin \theta$. When this path difference is equal to an integer number n of wavelengths, they constructively interfere.

If λ is the wavelength of the radiation, θ is the angle of diffraction, and l is the inter-reticular distance of the diffracting crystal plane, then the 2θ directions of the space in which we have peaks of intensity that check Bragg's law are

$$2l \sin \theta = n\lambda$$

As such, the distance l can be deduced as

$$l = \frac{n\lambda}{2 \sin \theta} \tag{I-4}$$

where $n = 1$. For the iron crystal, we can find experimentally that

$$l = 2.864 \times 10^{-10} m.$$

This last result, introduced in Eq. (I-3), and using the atomic mass of iron $M_{Fe} = 55.8 \cdot 10^{-3} \text{ kg}$ and the iron density $\rho_{Fe} = 7.89 \cdot 10^3 \text{ kg}\cdot\text{m}^{-3}$, allows us to propose a more precise value for the Avogadro number

$$\mathcal{N} = \frac{2 \times 55.8 \times 10^{-3}}{(2.864 \times 10^{-10})^3 \times 7.89 \times 10^3} = 6.03 \times 10^{23}$$

Note that the most accurate current value⁶ of the Avogadro number is

$$\mathcal{N} = 6.02214076 \times 10^{23}$$

I-II. Highlighting the Complexity of Atoms

1. Experiment

The electron is an elementary or fundamental particle⁷. This can be highlighted by studying the radioactive decomposition of a piece of radium, which spontaneously emits several types of radiation:

α -radiation, which is an emission of material particles deflected in the direction of the field and which generates helium gas.

γ -radiation, which is a very high energy electromagnetic emission. This radiation is not sensitive to the action of the electric field and so the rays propagate in a straight line.

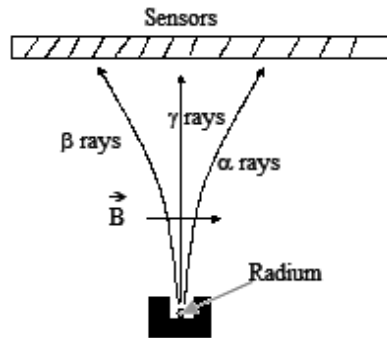


Figure I-2: Radioactive decomposition of radium.

β -radiation, which is made up of particles that are deflected in the opposite direction of the electric field and therefore carry an electrical charge inverse to that of the α -particles. Later, we will describe how this radiation is the result of the electrons. These three types of radiation are shown in **Fig. I-2**. Matter therefore appears to be made up of positive particles and negative particles.

⁶ Peter J. Mohr, David B. Newell, & Barry N. Taylor, "CODATA Recommended Values of the Fundamental Physical Constants: 2014" [archive], July 30, 2015.

⁷ An elementary or fundamental particle is a particle the composition of which is unknown or is not made up of other smaller particles.

2. Measuring the electron charge

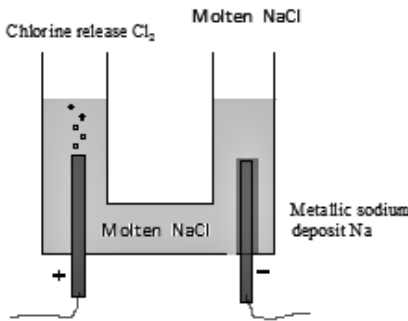
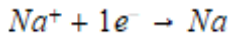


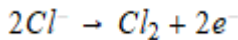
Figure I-3: Voltmeter for performing electrolysis of molten sodium chloride.

A voltmeter can be used to measure the charge of the electron, (See **Fig. I-3**):

Sodium chloride, under the action of the electric current, splits into Na^+ and Cl^- ions. Each ion moves to the electrode of the opposite sign. As such, metallic sodium is deposited at the anode (negative)

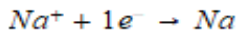


while at the cathode (positive) we have the reaction

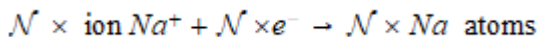


and Cl_2 molecular chlorine is released.

Let us consider the reaction at the anode. To deposit one Na atom, you have to circulate one electron



For one mole, we have



The amount of electricity needed to deposit one mole of sodium atoms has been measured as $Q = 96485 \text{ Coulomb}^8$. This is the Faraday constant⁹.

⁸ Charles-Augustin Coulomb (1736-1806). French officer, engineer, and physicist. He clarified the laws of solid friction and formulated the law of attraction between electrified solids.

⁹ Michael Faraday (1791-1867). British physicist and chemist.



C.-A. Coulomb



M. Faraday

This amount must correspond to N times the elementary charge, i.e.

$$\mathcal{N}q = 96485 \text{ Coulombs}$$

so that

$$q = \frac{96485}{6.02214076 \times 10^{23}} = 1.602 \times 10^{-19} \text{ C}$$

$$q_e = -q = -1.602 \times 10^{-19} \text{ C}$$

3. Measuring the electron mass: the J. J. Thomson¹⁰ experiment

In a famous experiment, pictured in **Fig. I-4** and called the J. J. Thomson experiment, the ratio q_e/m_e is determined.

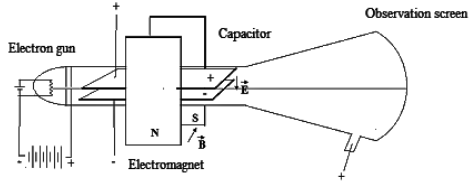


Figure I-4: Thomson experimental device.

This determination is made by studying the trajectory of an electron in an electric field perpendicular to its movement.

The electrons, emitted by the thermoionic effect (resistive heating) in an electron gun, are accelerated by the potential difference with the aid of a positive electrode before entering between the plates of a capacitor and the hole of an electromagnet. If the electric field is acting alone, the electrons of charge q_e undergo the action of an electrostatic force \mathbf{f}_{el}

$$\vec{f}_{el} = q_e \vec{E}$$

which is in the direction of the electric field with the modulus

¹⁰ Sir J. J. Thomson (1856-1940). British physicist. Nobel Prize in Physics in 1906 for his studies on electrical conductivity in gas and discoverer of the electron.

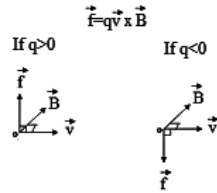
$$\left\| \vec{f}_{el} \right\| = q_e E$$

If induction \mathbf{B} is acting alone, an electron of charge q_e and velocity \mathbf{v} that enters a magnetic induction \mathbf{B} perpendicular to the velocity vector \mathbf{v} undergoes the action of a magnetic force \mathbf{f}_{mgt} , the direction of which is given by the right-hand rule (see the figure on the right)

$$\vec{f}_{mgt} = q_e \vec{v} \times \vec{B}$$

with the modulus

$$\left\| \vec{f}_{mgt} \right\| = q_e v B$$



If the electric field and the magnetic field act simultaneously, so that the electrostatic force is equal and opposite to the electromagnetic force, then we can write

$$\left\| \vec{f}_{el} \right\| = \left\| \vec{f}_{mgt} \right\|$$

i.e.

$$q_e E = q_e v B$$

Simplifying the expression by q_e , we get

$$E = v B$$

Hence the expression for the speed with which the electron will continue its path in a straight line without being deflected is

$$v = \frac{E}{B} \tag{I-5}$$

and, as such, we can know the speed modulus, which will be useful to us in the following.

4. Determining the electron q/m ratio

Now, let us consider an electron with mass m_e and charge q_e arriving at point O with uniform velocity v in the space between the capacitor plates. One can schematically represent the trajectory of the electron as shown in Fig. I-5.

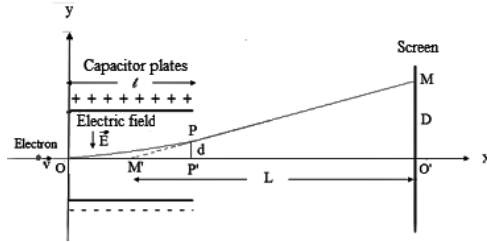


Figure I-5: Electron trajectory.

The electron is subject solely to the action of the electrostatic force. It follows from point O and, as long as it remains between the capacitor plates, describes a trajectory that will become parabolic. Once it has passed point P, marking the end of the space between the capacitor plates, its trajectory will show a straight and uniform movement and it will reach the observation screen at point M.

Consider the entry point O. The electron is subjected to an electrostatic force and its acceleration γ can be broken down in both directions of the plane, either x or y. Given the fundamental relationship of the dynamics $\mathbf{f} = m\mathbf{\gamma}$, for each component we can write

$$\gamma_x = 0; \quad \gamma_y = \frac{f}{m_e} = \frac{q_e E}{m_e}$$

By integrating each expression with respect to time, according to the x and y axes, respectively, we obtain

$$v_x = \gamma_x t + v_{0x}; \quad v_y = \gamma_y t + v_{0y}$$

where v_{0x} is the initial velocity of the electron along the x axis.

We can identify that the velocity v of the electron calculated in v_{0y} is the initial velocity along the y axis. Before the application of the electrostatic force, this is zero. The velocity v_y along the y axis is the integral in relation to the acceleration γ_y , so that

$$v_x = v \quad ; \quad v_y = \frac{qE}{m}t$$

Let us integrate these speeds with respect to time in order to get the coordinates for x and y

$$x = vt \quad ; \quad y = \frac{1}{2} \frac{q_e E}{m_e} t^2 \tag{I-6}$$

By reversing the left equation of Eq. (I-6), for time t we get

$$t = \frac{x}{v}$$

This allows us to rewrite y, as given by the right equation of Eq. (I-6), in the form of a dependence on the initial velocity v of the electron, its mass m_e , and the intensity of the electric field \mathbf{E}

$$y(x) = \frac{1}{2} \frac{q_e E}{m_e} \left(\frac{x}{v} \right)^2$$

According to Eq. (I-5), this leads to

$$y(x) = \frac{1}{2} \frac{q_e E}{m_e} \left(\frac{Bx}{E} \right)^2 \tag{I-7}$$

By rearranging the following expression, which is that of a parabola (the left-hand part of Eq. (I-8), the derivative of which is given by the right-hand part of the same equation), we obtain

$$y(x) = \frac{1}{2} \frac{q_e B^2}{m_e E} x^2 \quad ; \quad y'(x) = \left[\frac{dy(x)}{dx} \right] = \frac{q_e B^2}{m_e E} x \tag{I-8}$$

The slope a at point P' of the abscissa $x = l$ is the value of this derivative in $x = l$

$$a = \left[\frac{dy(x)}{dx} \right]_l = \frac{q_e B^2}{m_e E} l$$

The equation of the tangent $y'(x)$ to the parabola $y(x)$ is

$$y'(x) = ax + b \text{ i.e. } y'(x) = \left(\frac{q_e B^2}{m_e E} l \right) x + b$$

Let us now consider point M' , the intersection between the tangent and the x -axis for which its ordinate $y'(0)$ is obviously zero. Let us proceed, only for the tangent, to a change in the origin of the coordinate x for which the abscissa x becomes \bar{x} . The tangent equation thus becomes

$$y'(\bar{x}) = \left(\frac{q_e B^2}{m_e E} l \right) \bar{x} + b \quad (\text{I-9})$$

Let us set $\bar{x}_{M'} = 0$ in M' . At this point, the ordinate of the tangent is zero so that in the new system of the abscissa \bar{x} we have

$$0 = \left(\frac{q_e B^2}{m_e E} l \right) 0 + b$$

i.e. in the new coordinate system for \bar{x} , we have $b = 0$.

Eq. (I-9) can thus be simplified as

$$y'(\bar{x}) = \left(\frac{q_e B^2}{m_e E} l \right) \bar{x}$$

We now move on to consider point P' . The abscissa x_P for the parabola (I-8) is measured with respect to point O , i.e. $x = l$, while for the tangent in P , the abscissa x is measured with respect to the origin M' , i.e. $x = 0$. An examination of **Fig. I-5** shows that the ordinate $y'(x = x_P)$ of the tangent $y(x)$ in $x = x_P$ is equal to the ordinate d of the parabola $y(x = l)$

$$y(x = l) = d = \frac{1}{2} \frac{q_e B^2}{m_e E} l^2 \quad (\text{I-10})$$

and

$$y'(\bar{x} = \bar{x}_{P'}) = d = \left(\frac{q_e B^2}{m_e E} l \right) \bar{x}_{P'}$$

By equalizing the two expressions, we get

$$\left(\frac{q_e B^2}{m_e E} l \right) \bar{x}_{P'} = \frac{1}{2} \frac{q_e B^2}{m_e E} l^2$$

and by simplification we obtain

$$\bar{x}_{P'} = \frac{l}{2}$$

Let us return to **Fig. I-5**. Consider the similar triangles $M'MO$ and $M'PP'$. Given the properties of similar triangles, one has

$$\frac{M'P'}{M'O} = \frac{d}{D} \quad (\text{I-11})$$

However, the length $M'P'$ is none other than the coordinate $\bar{x}_{P'}$, which, as we have just seen, is equal to $l/2$. The tangent therefore cuts the x -axis at the mid-point of distance l , i.e.

$$M'P' = \bar{x}_{P'} = \frac{l}{2}$$

In addition, by definition we have

$$M'O' = L$$

Using Eq. (I-11), we can write

$$\frac{l}{2L} = \frac{d}{D}$$

and the value of d is then deduced as

$$d = \frac{Dl}{2L}$$

Recall that d represents the ordinate of the parabola at point P' when the abscissa is l , given by the equation of the parabola Eq. (I-10). Then, we have

$$\frac{1}{2} \left(\frac{q_e B^2}{m_e E} l \right) l = \frac{Dl}{2L}$$

so that, after simplification and rearrangement, we have

$$\boxed{\left(\frac{q_e}{m_e}\right) = \frac{D}{L} \frac{E}{B^2 l}} \quad (\text{I-12})$$

The geometric parameters D, L, and l are known, geometrically, from experimentation, while E and B are determined by the experimenter so that the electron crosses the space between the capacitor plates without being deflected with simultaneous application.

Various experiments have given the following result for the charge-to-mass ratio of the electron

$$\boxed{\frac{q_e}{m_e} = -1,759 \times 10^{11} \text{ C. Kg}^{-1}}$$

Given the value of the charge q_e determined by other procedures such as electrolysis, which has been described above, or by Millikan's oil drop Experiment¹¹, i.e.

$$q_e = -1,602 \times 10^{-19} \text{ C}$$

we obtain the following value for the mass of the electron

$$\boxed{m_e = 9,1 \times 10^{-31} \text{ Kg}}$$

5. The nucleus

a. Description of the Rutherford¹² experiment

We have seen above that some natural radioactive elements, such as radium, emit α -radiation made up of He^{2+} cations carrying a positive charge equal

¹¹ R. A. Millikan, Phys. Rev. (Series I) 32, 349 (1911).

¹² Ernest Rutherford (1871-1937). New Zealand and British physicist. He discovered that radioactivity was accompanied by the disintegration of chemical elements and won the Nobel Prize in Physics in 1908.

to twice the absolute value of the electron charge. These α -particles are emitted with a very high initial speed.

One can produce a narrow beam of these particles by placing two successive diaphragms, D_1 and D_2 , in their path, as shown in **Fig. I-6**.



E. Rutherford

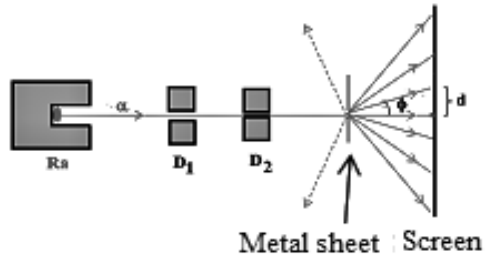


Figure I-6a: The Rutherford experiment

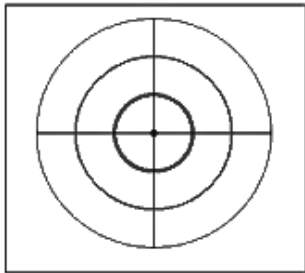


Figure I-6-b: Impact distribution.

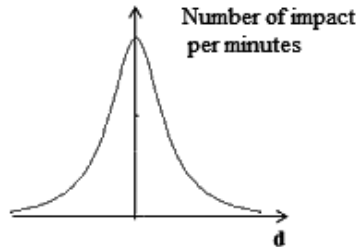


Figure I-6-c: Number of impacts per mn.

The α -particles can then be detected by the scintillation they produce in a screen of zinc sulfide. The apparatus is placed in a high vacuum environment. When a thin sheet of gold is inserted between the second diaphragm and the screen, we observe that:

- (i) the gold leaf is not damaged;
- (ii) the majority of the α -particles cross the gold leaf without deviating from the initial trajectory;
- (iii) some particles are deflected by angle ϕ and meet the screen at distance d from the central spot;

- (iv) the number of deviating particles decreases as angle ϕ increases;
- (v) and the number of deviating particles increases with the atomic number Z of the metallic element that constitutes the sheet.

b. Interpretation of the Rutherford experiment

The Rutherford experiment shows that:

- (i) Atoms are made up almost entirely of empty space, i.e. vacuum. In fact, if the atoms formed a compact whole, the particles would bounce off the metal sheet.
- (ii) The force producing the observed deviations is the Coulomb force, which is exerted between the positively charged α -particles and centers of repulsion. These carry a positive charge.
- (iii) These positive centers, which are called nuclei, constitute almost the entire mass of the atom since the mass of the negative particle, the electron, is about 2,000 times smaller than the mass of the lightest atom.
- (iv) The force exerted on an α -particle, which carries the charge $q_\alpha = +2e$, is

$$f = k \frac{Qq}{r^2}$$

where Q is the charge of the positive center and k is a constant equal to $1/(4\pi\epsilon_0)$.

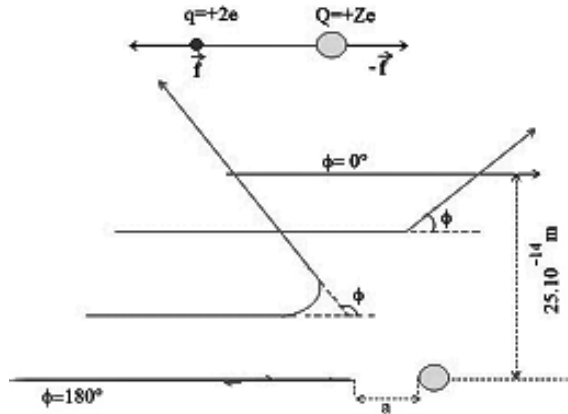


Figure I-6d: Repulsion and deviation between α particles and the positive center.

It can be seen that the (Q/e) ratio is an integer characteristic of the metal of the sheet and is the atomic number, denoted Z , of the metal. As we shall see later, it corresponds to the number of the box in which the metal is found in the periodic table.

Now, let us see what happens when an α -particle approaches the metal sheet. It will be subjected to the force of repulsion that we saw above and will describe a planar hyperbolic trajectory. The angle of deviation ϕ becomes all the more important as the initial direction of the particle passes closer to the nucleus. The results show that if the particle passes at a distance greater than 25×10^{-14} m from the positive center, it will not be deflected. The particles that are sent backwards ($\phi = 180^\circ$) are those going in the direction that passes through the positive center. Having approached the distance from the positive center, they turn back. At this stage, the initial kinetic energy is fully converted into potential electrostatic repulsion energy. The distance a of the particles decreases as their initial kinetic energy increases. It tends towards a limit, the distance a_0 , which we assimilate to the radius of the positive center. We find that a_0 is about 10^{-14} m.

This positive center corresponds to the nucleus of an atom of the metal constituting the sheet. This nucleus contains particles, known as subnuclear particles, the nature of which we shall now discover.

c. The proton

The name ‘proton’ comes from the Greek and means ‘first’ or ‘in the first place’. This name was coined by Rutherford, whose experience we have just recounted highlighting positive centers in matter called the nuclei.

In 1919, Rutherford discovered that hydrogen nuclei, known to be the lightest nuclei, could be produced as a result of collisions with nitrogen atoms. He came to the conclusion that the proton could be a fundamental particle making up the nuclei.

Today, the proton is no longer considered a fundamental particle.

The new theory of matter, known as the standard model, considers it to be a



Figure I-7: Quarks inside a proton.

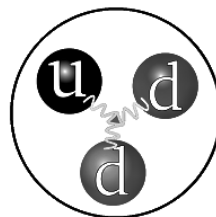


Figure 1-8: Quarks inside a neutron.

composite particle made up of three sub-particles known as quarks¹³. Quarks have a mass and an electrical charge that is fractional. These features are described in **Table I-2**.

¹³ A quark is an elementary particle and a constituent of observable matter. Quarks combine to form hadrons, composite particles, of which protons and neutrons, among others, are known examples. The two up quarks and the proton’s down quark are linked by a strong interaction, transmitted by gluons. These gluons are exchanged between quarks and, by the binding energy they represent, constitute about 99 % of the mass of the proton. In addition to these three valence quarks (which determine the quantum numbers of the particle) and gluons, the proton, like other hadrons, consists of a ‘sea’ of pairs of virtual quarks-antiquarks that appear and disappear permanently.