Spacecraft Manoeuvring in the Vicinity of a Near-Circular Orbit

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PREFACE

In this book the author shares his more than 45-years’ experience in the field of spacecraft manoeuvre determination in near-circular orbits.
In the second half of the 70s, the ballistics center of the Keldysh Institute of Applied Mathematics in the USSR Academy of Sciences was assigned a task to solve the problem of manned and unmanned spacecraft ballistics and navigation support. The greatest challenge was to solve the multiple-impulse rendezvous problem for these types of spacecraft and space stations.

The author studied the literature available at the time concerning the rendezvous problem in order to find an analytical or numerical analytical solution but there was no answer to that question. As a rule, some cumbersome numerical solutions were used.

Analytical and numerical analytical solutions presented in the works of J.-P. Marec and J.E. Prussing were only related to the nondegenerate rendezvous problem (Prussing studied the coplanar rendezvous problem on circular orbits). In addition, constraints on the impulse application moments and their orientation were not considered.

Initially a numerical solution method was designed at the Keldysh Institute of Applied Mathematics in the USSR Academy of Sciences. This method was suitable for solving the problem but did not give an explanation as to why this or that solution had been chosen, which made it difficult to use in emergency situations. A numerical analytical method was designed in the early 80s. It was intended to solve degenerate coplanar and noncoplanar rendezvous problems in the classical statement with and without constraints. A geometric interpretation of impulses in the space of eccentricity vector components provided an opportunity to explain the nature of the found solution. A graphical user interface made it easy to select a new manoeuvring scheme in case of emergency situations. The method’s simplicity and reliability allowed it to be used on board spacecraft. Later, analytical solutions for coplanar and noncoplanar nondegenerate rendezvous were found. In the late 90s, a solution for rendezvous with significant initial right ascension of the ascending node (RAAN) deviation and long flight duration was found. Thus, the theory for optimal multiple-impulse rendezvous was fully developed.

The next step in the evolution of manoeuvring theory in the vicinity of circular orbits was the development of solution methods for coplanar and noncoplanar rendezvous and transfers using low-thrust engines in the 2000s. The types of optimal solutions and their existence domains were defined. The parameters of these solutions were determined using simple analytical and numerical analytical algorithms.

The almost fully developed theory of optimal manoeuvring on near-circular orbits allowed the author to find numerical analytical solutions for all of the practical problems he faced during more than 45 years of work with real spacecraft. The solution time for these problems is significantly smaller in comparison with the time needed to solve the same problems using classical numerical methods. In addition, it became possible to explain the physical nature of the found solution. This was the answer to the question that the author had been searching for since the beginning of his work on the problem, which was a long time ago.

The problems of coplanar and noncoplanar rendezvous of different duration, as well as the problems of deployment and maintaining the given configuration of satellite constellations, were solved. Lately, the primary focus has been shifted to space debris manoeuvring problems.

In addition to the fact that this book contains the accumulated knowledge of solving complex practical problems, it will also be useful for young specialists making their first steps in the field of spacecraft manoeuvring. Each theoretical section ends with an example using the specified algorithms. The author’s lecturing experience in Bauman Moscow State Technical University (BMSTU) and the Academy of Engineering Faculty of the Peoples’ Friendship University of Russia (RUDN) shows that it will help students to better understand the material and to learn how to solve real nonsimplified problems.

Those readers who have extensive experience in solving complex practical problems can be helped by learning new approaches to problem solutions and explanations of their nature.

INTRODUCTION

Manoeuvres play an important role in the spacecraft flight process. A manoeuvre is a target driven alteration of spacecraft orbit parameters with the help of a propulsion system. Generally, manoeuvres help to change the orbit with the necessary precision, which allows the spacecraft to fulfill its mission. The important role of manoeuvres, as well as their variety and presence practically in each space mission, determines the attention that is paid to them in the literature on space flight mechanics.

Released in 1969, F.W. Gobetz and J.R. Doll's report (Doll and Gobetz 1969, 801–834) already encompassed more than 300 articles dedicated to the optimal spacecraft manoeuvring. At the present time, there are thousands of articles related to the topic, as well as dozens of monographs including notable works by V.V. Ivashkin (Ivashkin, 1975), K.B. Alexeev, G.G. Bebenin, V.A. Yaroshevsky (Alexeev, Bebenin, and Yaroshevsky, 1970), V.A. Egorov (Egorov, 1965), T.V. Solovyev, and E.V. Tarasov (Soloviev and Tarasov, 1973). D.F. Lawden's work is considered to be the foundational one (Lawden, 1966).

The problem of optimal manoeuvring parameter determination for spacecraft in near-circular orbits holds a special place in the theory of optimal manoeuvring. Firstly, these problems are of big practical interest as most of the real spacecraft operate in these orbits. Secondly, these problems are less complicated than those in the classical statement and they can sometimes be solved using analytical and numerical analytical methods. Naturally, lots of articles are dedicated to problems of this type. Their references will be given in successive chapters. The most noticeable are the monographs by J.-P. Marec (Marec, 1979); V.A. Ilyin and G.E. Kuzmak (Il'in and Kuzmak, 1976); and M.F. Reshetnev, A.A. Lebedev, V.A. Bartenev, M.N. Krasilshikov, V.A. Malyshev, and V.V. Malyshev (Bartenev, Krasilshikov, Lebedev, Malyshev et al., 1988).

The necessary conditions of the strict local optimality of impulsive flybys in arbitrary and Newton's gravity fields were obtained on the basis of the variation approach (Il'in and Kuzmak, 1976). The solution for the transfer problem (the flyby without time constraint) between the near-circular orbits in the linearized statement and the solution for the rendezvous problem are given. A brief and sufficiently complete solution of the transfer problem in the noncoplanar near-circular orbits is given in

Edelbaum (1967, 66–73). In the work of Bartenev, Krasilshikov, Lebedev, Malyshev, et al. (1988), the problems of orbit determination and manoeuvring with high and low-thrust engines (mostly transfer problems) are considered. In J.-P. Marec's monograph (Marec, 1979) major attention is paid to the nondegenerate rendezvous problem in near-circular orbits. A fundamental up-to-date review of articles on rendezvous problems can be found in Guojin, Jin, and Yazhong (2013, 1–11).

The rendezvous problem consideration started in the mid-60s. Even now, the articles by J.E. Prussing are still cited (Prussing 1969, 928–935; Prussing 1970, 1221–1228). Prussing considered the two-spacecraft rendezvous in coplanar orbits with 1–3 revolutions duration. In the book by J.-P. Marec (Marec, 1979), research was conducted into the classic average duration noncoplanar rendezvous in near-circular orbits. It was shown which diapasons of orbit element values could be achieved while using different types of optimal nondegenerate solutions (reversed problem solutions). The difficulty in understanding the book's material was one of the reasons why its ideas were not widely used. The algorithm for the rendezvous manoeuvre parameter determination corresponding to the nondegenerate hodograph of primer vector was introduced in Jones (1976, 55–90).

In the 60s, the first spacecraft dockings were conducted. It turned out that additional constraints should be taken into account while solving practical problems: velocity impulse application time, its orientation and value, transfer orbit parameters, etc. The problem got far more complex in comparison with the classic one from the first theoretical articles. It became impossible to use the solutions from those articles. New effective numerical methods for solving practical problems were developed.

Nowadays, three major approaches for solving complex multiple-impulse spacecraft manoeuvring problems are mainly used. In the first case, the problem is divided into several simple problems. For example, the problem of manoeuvring within an orbit plane and the problem of orbit plane rotation are solved separately. The orbit plane rotation in this scheme is carried out by the single velocity impulse, which is applied on the orbit plane intersection line. Such a scheme[1] was used for the rendezvous of Shuttle and the ISS in Fehse (2003, 441–449). A similar approach is used for guiding geostationary satellites (Bulynin 2008, 73–74), and satellites in a constellation (Rylov 1985, 691–714), (Bobronnikov, Fedorov, Krasilshikov, Malyshev et al. 2001, 43–45), etc. The advantage of the scheme is its simplicity and reliability, clarity of the physical nature of each of the

[1] "Shuttle Press Kit: STS-92". Accessed March 25, 2007. http://www.shuttlepresskit.com/STS-92/

manoeuvres, and the usage of simple spacecraft orientation systems. The disadvantage of the scheme is the excessive total delta-v expenditure on manoeuvres.

In the second case, numerical methods are applied, allowing finding the optimal solution of the most complex multiple-impulse problems with different constraints. Numerical methods have been successfully used for decades for the rendezvous manoeuvre determination of “Soyuz” and “Progress” spacecraft with long-term orbital stations (Bazhinov and Yastrebov, 1978; Petrov 1985; Baranov 1986, 324–327). The articles by V.P. Gavrilov and E.V. Obuhov had a great influence on development of these methods (Gavrilov and Obukhov 1980, 163–172). Numerical methods allow finding the solutions with minimum total delta-v, which is sometimes crucial for the accomplishment of the mission. For example, the orbital module “Kvant” only docked on the “Mir” orbital station after the third attempt with almost depleted fuel tanks. If only one of three attempts had not been optimal, there would not have been a sufficient amount of fuel for docking. The simplex method is mostly used for manoeuvre parameter determination (Lidov 1971, 687–706; Lidov and Teslenko 1978, 112–141; Gavrilov and Obukhov 1980, 163–172; Gavrilov 1995; Kolegov, 2007; Bakshiyan, El'yasberg, and Nazirov, 1980). The method was first used for the abovementioned purposes by M.L. Lidov.

The disadvantage of all numerical methods is the absence of information on why we arrived at this or that type of solution and how the solution would alter with an initial condition change. It is especially critical in the case of emergency situations occurring during the flight when a new manoeuvring scheme with additional constraints is needed. Similar problems arise at the stage of development when the future manoeuvring scheme is being selected. Generally, it is easier to determine the manoeuvre parameters than the manoeuvring scheme. When choosing the scheme, we should determine the number of velocity impulses needed for an optimal solution, the intervals of manoeuvring and rendezvous duration. After that we can evaluate the manoeuvre parameters. We can name the cumbersomeness of the numerical methods as their disadvantage. This creates additional difficulties when using them on board a spacecraft. For another thing, numerical methods need a significant amount of computing time to solve problems. This factor appears to be a significant disadvantage when solving complex combined problems. Sometimes the solution of lots of conventional transfer and rendezvous problems are needed to obtain a complex optimal solution. The same problem appears when we use the space debris catalogue and need to calculate the parameters of thousands of manoeuvres.

The third approach to solving the rendezvous problem can often be found in articles from universities. This approach was first implemented in articles by P.M. Lion and M. Handelsman (Handelsman and Lion 1968, 127–132) and D.J. Jezewski and H.L. Rozendaal (Jezewski and Rozendaal 1968, 2160–2165). At the first stage of this method, the parameters of the two-impulse solution using Lambert's problem are obtained. Then the primer vector hodograph for the found solution is analyzed. If necessary, additional velocity impulses are added for the optimal solution. With this technique and gradient optimization algorithms, multiple-impulse solutions for a two-body problem were obtained (Gross and Prussing 1974, 885–889; Chiu and Prussing 1986, 17-22; Guzman, Hughes, and Mailhe 2003, 85–104). The most common and developed method of solving rendezvous problem is the use of Lambert's problem (Ohotsimsky and Sikharulidze 1990; Battin 1966; Battin 1977, 707–713; Ermilov, Ivanova, and Panushin 1977; Dashkov and Kubasov 1979; Escobal 1970; Herrick 1978). We can distinguish as a separate group the solution of multiple-revolution for Lambert's problem (Prussing 2000, 31–148; Shen and Tsiotras 2003, 50–61; Han and Xie 2004, 9–13) and the universal algorithms of its solution (Pitkin 1968, 270–27; Kriz 1976, 509–513; Sukhanov 1988, 483–491). Nowadays, the method for searching for an optimal multiple-impulse solution for the rendezvous problem based on Lambert's problem is also widespread (Guo-Jin, Hai-Yang, Ya-Zhong, and Yong -Jun 2007, 946–952).

Problems of orbital station manoeuvre parameter determination can be put in a special group, although, despite their peculiarity, they might be considered to be rendezvous problems. They are rather specific due to the long-time interval between the initial and terminal manoeuvres and multiple, often controversial, constraints on the parameters of the interim orbit. The basic research in this field was conducted by E.K. Melnikov (Melnikov 2004, 176–186; Melnikov 2009; Kolegov and Melnikov 1990, 158–165).

The purpose of this book is to introduce numerical analytical methods for solving multiple-impulse rendezvous problems in near-circular orbits to the reader. These methods combine the advantages of the first two methods. The methods are simple enough, as well as illustrative and reliable. By using them, we can obtain close-to-optimal solutions. This allows us to use the results obtained in the articles by G.E. Kuzmak, T.N. Edelbaum, and J.-P. Marec when solving actual practical problems. The suggested numerical analytical methods are several times quicker than the numerical methods and they also give an explanation as to why this or that optimal solution was obtained. They allow us to determine the existence domains of optimal solutions of different types. It is hard to determine these domains using numerical methods. It is especially important when we choose the manoeuvring scheme: how many velocity impulses should we use and on which manoeuvring intervals can they be applied? The found solutions give us a simple geometric interpretation which simplifies their explanation and allows the use of a graphic dialogue while solving the problem.

Also, the basic problems of manoeuvring with low-thrust engines are considered. Special attention is paid to satellite constellation and satellite formation flying deployment and keeping. Problems of the determination of manoeuvres performed by an active space object, problems of space debris collision avoidance, problems of large space debris de-orbiting, and problems of the transfer of spacecraft to disposal orbit (DO) are considered.

The structure of the book is arranged in accordance with the adopted classification of manoeuvres in near-circular orbits (Doll and Gobetz 1969, 801–834).

In the first chapter a solution of linearized equations of motion system in a cylinder coordinate frame is given. The statement of the optimal manoeuvre parameter determination problem in near-circular orbits and necessary optimal conditions are presented. The iterative procedure is explained. It allows the realization of terminal conditions with the necessary accuracy and with due regard to the noncentral gravitational field, atmosphere, and spacecraft propulsion system etc. The geometric interpretation of the impulse components' influence on different orbit elements is given. The relative motion of two approaching spacecraft with different target vectors is depicted. It is shown that an impulse transversal component sum in every manoeuvring interval can be determined analytically with good precision during the spacecraft-orbital station rendezvous. The equations for the determination of optimal phase desynchronization diapason between the spacecraft and orbital station are given. The equations include options for shifting first interval manoeuvres to later revolutions and shifting the launch date. Further material is given in accordance with the adopted manoeuvre classification.

In the second chapter, the problems of transfer between coplanar (three types of two-impulse solutions) and noncoplanar orbits (three types of two-impulse solutions and one type of three-impulse solution) are considered. The equations for the determination of the manoeuvre parameters of these solutions are given. A comparison between problem solutions in linearized and accurate statements is made. Equations for the impulse components of the optimal coplanar two-impulse solution for fixed impulse application angles are given.

In the third chapter, the coplanar rendezvous is considered. Three types of possible solutions, which correspond to the point, ellipse, and cycloid primer vector hodographs, are analyzed. The existence domains for three types of solutions are determined. The algorithms for the parameter determination of two-, three-, and four-impulse solutions for each type of possible primer vector hodographs are considered. Example cases are presented. It is shown that the total delta-v of the optimal Lambert's problem solution might be two or even three times higher than the delta-v for an optimal three-impulse solution. The rendezvous problem with a constraint on transfer orbit altitude is considered. The impact of orbit determination errors and manoeuvre realization errors is analyzed. The procedure for the solution selection for the minimum influence of these errors is presented.

In the fourth chapter, a universal algorithm for a four-impulse multiple-revolution rendezvous in noncoplanar near-circular orbits manoeuvre parameter determination and a numerical analytical algorithm of long-range guidance for Soyuz- and Progress-type spacecraft manoeuvre parameter determination are shown. The six-impulse solutions which correspond to the spiral hodograph of primer vector are analyzed. The equations for the determination of optimal impulse application angles for these solutions are presented. The numerical analytical algorithm for several dozen rendezvous manoeuvres determination is shown.

There are lots of problems in practical work which are hard to solve without using numerical methods. This is why Chapter Five is dedicated to numerical methods. The numerical method, which was used over a number of years in the ballistics center of the Keldysh Institute of Applied Mathematics at the Russian Academy of Sciences for real spacecraft manoeuvre parameter determination, is also presented with selected examples.

The ballistics center of the Keldysh Institute of Applied Mathematics at the Russian Academy of Sciences, whose creator and long-term head, E.L. Akim, was the corresponding member of the Russian Academy of Sciences, participated in ballistics navigation flight support for all major scientific spacecraft. The ballistics center played an important role in the ballistics navigation support of manned spacecraft. The author of this book took part in the determination of manoeuvre parameters of "Soyuz 19" (1974), "Soyuz 20" (1975, Apollo-Soyuz Test Project), and "Soyuz 22" (1976). Over a number of years (since 1978), the ballistics center has participated in ensuring the functioning of the orbital stations "Salut-6", "Salut-7", and "Mir"; in the activities connected with the deployment of the International Space Station (ISS); and the ballistic support of the "Buran" space system. As part of these activities, the author has calculated the manoeuvre parameters of approximately 140 spacecraft of different types: “Soyuz”, “Soyuz-M”, “Soyuz-TM”, “Progress”, and “Progress-M”, as well as the orbital modules of “Kvant”, “Priroda”, “Spectre”, “Zvezda”, and the “Buran” space system, etc. Participation in the ballistics navigation support of these flights gave the opportunity to develop and test different methods of manoeuvre parameter determination, and to choose the simplest and most reliable ones with maximum adjustment to the peculiarities of the flight of real spacecraft. Since the methods presented in this book were designed for solving practical problems, they take into account the constraints on moments of impulse application, their magnitudes and orientation, and interim orbit altitude limits, as well as meet the strongest demands in terms of performance and reliability.

Due to their advantages, the methods were chosen by CNES as a basis for their own method of calculating the manoeuvre parameters of rendezvous problem for the ATV approach to ISS (Carbonne, Chemama, Julien, Kudo et al. 2009, 1091–1106).

Emergency situations occurring in orbit proved the necessity of the graphic dialogue, which rapidly reselects manoeuvring schemes with additional constraints caused by emergency situations.

The first use of the graphical dialogue while solving the manoeuvring problem was described in the article by A.K. Platonov and R.K. Kazakova (Kazakova and Platonov 1976). Lambert’s solution was used for the manoeuvre determination. The book by Y.A. Zakharov gave the description of the graphic dialogue for the calculation of interorbital transfer with finite thrust. A transition to the solution of the problem with finite thrust from an impulsive solution was made by using nonlinear programming. Both graphical dialogues were developed at the stage of spacecraft flight design.

The graphical dialogue description, which can be used at the stage of ballistics design and during the flight in emergency situations, including space debris collision avoidance, is described in the fifth chapter. Unlike the previous two dialogues, the solution analysis and its alteration take place in the space of eccentricity vector projections, and not in the space orbits themselves.

It was assumed in the problems mentioned above that the burn duration is noticeably shorter than the orbit period. It allowed us to solve the problem in an impulsive statement and helped us to recalculate the engine regime parameter accurately with the help of an iterative procedure. Still, in a number of practical problems, the manoeuvre duration is comparable with the orbit period. For example, such a situation appears when orbital module big-sized manoeuvres are carried out with the use of docking and attitude thrusters. Another example is the usage of electric propulsion engines. In these cases, impulsive approximation is not enough and special manoeuvre determination methods are needed.

Problems of this type have a special place among optimal spacecraft manoeuvring problems. A number of articles are dedicated to them. Articles by the group of authors led by T.M Eneev and V.A. Egorov (Beletsky and Egorov 1964, 360–391; Akhmetshin, Efimov, Eneev, and Yegorov 2000, 279–305; Ermilov, Ivanova, and Panushin 1977; Akhmetshin, Beloglazov, Belousova, Efimov et al. 1985; Egorov, Grigoriev, and Ryzhov 2005), as well as articles by M.P. Zapletin and I.S. Grigoriev (Grigoriev, Zapletina, and Zapletin 2007, 758–762; Grigoriev and Zapletin 2009, 1499–1513) can be mentioned. Several interesting monographs have been published (Grodzovski, Ivanov, and Tokarev 1966; Lebedev, 1968; Bartenev, Krasilshikov, Lebedev, Malyshev et al., 1988; Zakharov, 1984). Articles by M.S. Konstantinov (Konstantinov 1997) and V.G. Petukhov (Petukhov 2004, 250–268; Petukhov 2008, 219–232; Petukhov 2012, 249–261) can also be noted. Due to the complexity of the problems in which manoeuvring is performed by low-thrust engines, they were traditionally solved by numerical methods with the use of Pontryagin's maximum principle or by dividing the problem into simpler problems which have trivial solutions, just like the Shuttle manoeuvre calculations. In recent years, Y.P. Ulybyshev has successfully been using the method of inner point for long-duration manoeuvre problems (Sokolov and Ulybyshev 1999, 95–100; Ulybyshev 2008, 135–147; Ulybyshev 2012, 403–418).

The aspects of manoeuvring with limited thrust engines are described in Chapter 6. The algorithms of the transfer problem manoeuvre parameter determination on coplanar orbits with fixed orientation of engines in orbital and inertial coordinate frames and the algorithm of optimal engine orientation change determination are depicted. In the space of deviations of semimajor axis and eccentricity between final and initial orbits the areas with optimal engine orientation mentioned above are determined. The numerical analytical algorithm for the manoeuvre parameter determination for a noncoplanar transfer is shown. A coplanar rendezvous with manoeuvring on each revolution and on two separate intervals is considered.

In the previous theoretical articles from the 60s and early 70s, no research was conducted in the field of spacecraft manoeuvring in satellite constellations, which play an important role in modern cosmonautics. Nowadays, lots of satellite constellations differ in terms of application, the number of satellites in a constellation, the types of orbits, and the relative satellites positions used. The most common are satellite constellations on near-circular orbits. The algorithms mentioned in the previous chapters might be used for the calculation of a spacecraft manoeuvre parameter in satellite constellations. At the same time, the manoeuvres of such satellites have their own peculiarities.

Dozens of publications have been dedicated to satellite constellations. At first, the satellite constellation configuration that can provide the necessary coverage of the Earth should be selected. In this field, the articles by G.V. Mozhaev (Mozhaev, 1968; Mozhaev 1972, 833–843; Mozhaev 1973, 59–68), J.G. Walker (Walker 1971, 369–384), B.P. Byrkov (Byrkov and Razoumny 1992, 62–68), Y.N. Razoumny (Razoumny 1993), E. Lansard (Frayssinhes, Lansard, and Palmade 1998, 555¬564), and V.K. Saulsky (Saulsky 2005, 34–51) can be noted. Recently, some articles by Y.N. Razoumny (Kozlov, Razoumny, and Razoumny 2015, 200–204; Kozlov, Razoumny, and Razoumny 2015, 196–199), and S.Y. Ylybyshev (Ulybyshev 2015, 311–¬322; Ulybyshev 2016, 1–11) concerning the multiple-tiered satellite constellations have been published. The satellites in these constellations move in circular orbits with different radii.

Despite the variety of satellite constellations, there are two basic types of optimal manoeuvre problems: satellite constellation deployment and keeping.

The first one is close to the classic rendezvous problem. Each satellite is considered separately. The satellite needs to be transferred to a specified point in the final orbit in a fixed period of time. In this case, the time of the satellite transfer to a specified point is not important and can be selected in a wide range unlike in the case of the manoeuvring problem, which is calculated when docking “Soyuz” or “Progress” to a long-term space station. It is connected with the search for a compromise solution between the transfer duration and the total delta-v expenditures. Total delta-v expenditures usually increase with the decrease of the transfer duration to a specific orbit point. Also, when dealing with low Earth orbits, we face a shift of RAAN even after the simplest satellite transfer along the orbit. And this needs to be corrected. This case is thoroughly examined in the first paragraph of Chapter 7. The algorithm of optimal impulse application angle selection and the algorithm of compromise rendezvous time selection are shown.

The problem gets more complicated when the satellite operational orbit RAAN differs from the initial orbit RAAN by dozens of degrees. For example, such a situation occurs when a single launch vehicle injects several satellites in orbit and some of them need to be transferred to different operational planes or when a spare satellite needs to be transferred to another orbit plane in order to replace the malfunctioning one. The optimal one in terms of total delta-v costs satellite transfer needs significant time (several hundreds of revolutions). The problem of satellite transfer from the circular orbit to a specific point of analogous circular orbit with a big RAAN difference is also shown in Chapter 7.

It was shown that, with the increase of RAAN deviation, the total delta-v expenditures do not grow proportionally as might be expected. They alter in a fashion that corresponds to the sine-shaped fading law over the line corresponding to the expected proportionality. The magnitudes of these oscillations are noticeable within several degrees of variations of RAAN. In this case, the total delta-v expenditures for the compensation of a substantially bigger deviation of RAAN can be tens of percent less than the cost of the compensation for relatively small deviations.

In general, it is necessary to have a universal manoeuvre parameter computing algorithm for a long-duration rendezvous when being transferred to the given point of the final orbit from an arbitrary injection orbit. This algorithm becomes more complicated when there is a big difference in RAAN between the initial and final orbits.

It is impossible to solve this problem using the existing methods for short and medium duration rendezvous problems. In these methods, the influence of the Earth's oblateness was omitted (Prussing 1969, 928–935; Prussing 1970, 1221–1228; Jones 1976, 55–90; Marec 1968; Marec, 1979) or accounted for using the iterative procedure (Petrov 1985; Bazhinov and Yastrebov, 1978; Gavrilov, Obukhov, Skoptsov, and Zaslavsky 1975; Baranov, Gundobin, Ivanov, Kapralov et al. 1992, 26–27) when obtaining the given tolerance of terminal conditions. The Earth's oblateness can be used for the reduction of total delta-v expenditures in the problem of the satellite transfer to the given plane, whose RAAN differs from the initial one by several tens of degrees.

In the available articles, the problems of the parameter determination of satellite transfer to another operational plane were hardly considered. The two problem solving methods for a rendezvous with big initial deviations of RAAN (Bollman, D'Amario, Lee, Roncoli et al. 1999; Breeden, Guinn, and Ocampo, 2001, 1–20) were presented at international conferences held by AIAA. The methods were demonstrated for the solution to rendezvous on Mars orbit. RAAN needed to be changed by 182 degrees. The first one (Bollman, D'Amario, Lee, Roncoli et al. 1999), the NASA project, is similar to the Shuttle-to-ISS docking method. The correction of orbital elements in the orbital plane and orbital plane rotation are carried out separately. The solution obtained is not optimal (the total delta-v expenditures are 60% higher than the necessary ones). Due to the complexity of the problem, ten velocity impulses were used instead of five for NASA's classical rendezvous problem solution. The second method (Breeden, Guinn, and Ocampo, 2001, 1–20) developed by JPL and the University of Texas is numerical. It obtains the optimal solution with three velocity impulses, but the time needed to arrive at the solution is extremely long and a good initial guess is needed. It is hard to perform multiple solutions for the problem using these methods.

In Chapter 7, a simple, reliable, and fast numerical analytical method for obtaining an optimal solution of such a type (Baranov and Baranov 2009, 256–262) is given. It allows the formulation of the dependence of the consumption of the total delta-v on the duration of the flight with a single solution to the problem. Besides, the computational process is well illustrated. It is always clear why the optimal solution has its specific look and how it alters due to the change of initial conditions. It is possible to obtain solutions which can decrease the influence of manoeuvre realization errors.

The given method was used for a number of situations including the solution to abovementioned rendezvous problem on Mars orbit, and the solution to the manoeuvre parameter problem while deploying satellite constellations (e.g., “Globalstar”) and satellite formation flying. The examples of manoeuvre parameter calculations for the initialization of a cluster for atmospheric tomography and for the satellite formation flying “Aqua Train” deployment are given. Due to the close positions of satellites in formation flying, significant attention must be paid to collision avoidance while calculating the manoeuvres for their deployment. This problem was considered in (Baranov, Boutonnet, Escudier, and Martinot 2005, 913–920; Baranov, Boutonnet, Escudier, Matinot et al. 2003, 83–96; Baranov, Boutonnet, Escudier, and Martinot 2003).

Special attention is also paid to the problem of keeping the given satellite constellation configuration, which has several significant differences from the satellite constellation deployment problem. Two strategies can be distinguished: absolute and relative keeping. In the absolute keeping regime, the motion of each satellite should comply with some given motion, which allows the manoeuvres for each satellite to be calculated separately from other satellites in the constellation (Chao and Schmitt 1991; Baranov and Wang 2015, 68–83). In the relative keeping regime, the cooperative motion of all satellites in a constellation is ensured. It is significantly cheaper than the absolute regime because there is no need to correct orbit elements which practically alter in the same fashion repeatedly for all satellites in a constellation. For example, the semimajor axis reduces in practically the same fashion for all satellites due to atmospheric drag and does not need to be corrected. The only thing to do is to control the relative angular distances between the satellites which determine the satellite constellation configuration. Relative keeping is a more complicated problem as it is necessary to take into consideration the location of other constellation elements while calculating manoeuvres of the current satellite. In the articles by G.V. Mozhaev (Mozhaev 2001, 634–647) and R.F. Murtasin (Murtazin 1998, 173–182), two keeping policies were considered and it was shown that relative keeping leads to a smaller number of manoeuvres needed and lesser total delta-v expenditures.

The problem of manoeuvre parameter calculation in the relative regime of constellation keeping has been examined by a number of authors. A simplex method (Bernussou, Brousse, Dufour, Foliard et al. 1997; Bernussou, Dufour, and Lasserre 1996, 169–174; Fedorov, Malyshev 2001, 45–46) or the numerical solution of the Riccati equation (Ulybyshev 1998, 109–115) are usually used for its solution. In these cases, the system conditions are controlled after fixed and equal time gaps. It may be needed to perform some optimal orbit element corrections before or after one of these fixed moments, which is a disadvantage of both methods. Analytical solutions were suggested in some articles; however, it was agreed that the manoeuvres were performed in the initial (Mozhaev 2001, 634-–47) or in the initial and final moments of time (Murtazin 1998, 173–182). The constellation was not controlled in the intermediate moments. The numerical analytical method allows the manoeuvre parameters to be evaluated analytically; this ensures the necessary configuration keeping on the whole interval is given in Chapter 7. Taking into consideration the physical peculiarities of problems helps to decrease the number of manoeuvres used. The suggested geometrical interpretation of the maintenance process gives a comprehensive explanation of the nature of the optimal solution. Numerical solutions do not allow this.

The development of methods for the deployment manoeuvre calculation and relative satellite constellation station keeping was carried out in the work initialized and supported by CNES. J.-P. Carrou, J.-P. Bertiasse, P. Legandre, J. Folliard, P. Brousse, J.P. Guster, and F. Dufour made great contributions in organizing this and other works. The development of the universal manoeuvre parameter determination method of satellite transfer to a specified point of orbit with several dozens of degrees RAAN deviation was made together with P. Labourdette (Baranov and Labourdette 2003, 130–142; Baranov and Labourdette 2002).

Absolute keeping is used far more often and is considered in the last paragraphs of Chapter 8. The example contains the case of the absolute keeping of a microsatellite in the sun-synchronous orbit which fulfills Earth remote sensing problems.

The satellite group absolute configuration initialization and its keeping during a considerable time interval were illustrated by the satellite formation flying "Tandem". Much attention is paid to the collision avoidance problem when maintaining orbit.

Nowadays, the spacecraft safety problem during flight draws more and more attention due to increasing space debris collision hazards. Manoeuvring problems have an important place in the space debris problem. Chapter 9 and 10 are dedicated to this issue.

The four trends in space debris manoeuvre calculating problems can be distinguished. The first and second trends are considered in Chapter 9, and the third and fourth in Chapter 10.

The first trend is the determination of manoeuvres which allow collisions with space debris to be avoided. Special collision avoidance manoeuvres are usually used. They are relatively simple. Their description is given in the beginning of Chapter 9. It would be more interesting to solve a more complicated problem like finding the solution to the rendezvous problem which will allow collisions on the phasing orbit (drift orbit) to be avoided. A safe orbit can be obtained by searching for magnitudes and times of velocity impulse applications on the first manoeuvring interval, and not by adding additional collision avoidance manoeuvres. A compromise solution can be obtained without a substantial increase of the fuel needed thanks to the abovementioned graphic dialogue with the problem.

The second trend is to determine manoeuvres performed by an active space object. The assessment and future forecasting of such manoeuvres will allow modeling an active space object movement with higher accuracy and, hence, to avoid collisions with them. Single- and two-impulse manoeuvres were assessed. The two-impulse manoeuvre parameter determination algorithm reduces the problem solution time by several orders in comparison with Lambert's solution, which is traditionally used for these purposes. The manoeuvre determination accuracy is also improved. Single- and two-impulse long duration manoeuvres were also assessed. The solution time was also decreased by several orders in comparison with traditional methods (Alfriend, Kamensky, Stepanyants, and Tuchin 2009, 3–22; Borovin, Stepanyantz, Tuchin, Tuchin et al. 2012). The assessment algorithms of short and long duration single-impulse manoeuvres with considerable errors in the determination of initial and, especially, final orbits were presented. In this case, the manoeuvre assessment helps to improve the final orbit accuracy (it is obtained by applying a calculated manoeuvre to the initial orbit) and thus helps us to increase the approach (of the protected spacecraft with the given object) determination accuracy. These algorithms can be used while assessing the impulses which occur during the setup of the given spacecraft orientation; this helps to increase the orbit forecast accuracy.

Manoeuvre assessment problem statements and approaches to their solutions have been discussed with V.M. Agapov several times.

The third trend is the transfer of the decommissioned spacecraft into orbits where they cannot be dangerous for active space objects. For geostationary orbit spacecraft, this means a transfer into orbits with altitudes 250–350 km higher than regular geostationary orbits, while, for low Earth orbits, it is the transfer to orbits where the existence time is limited to 25 years. An orbit parameter determination algorithm was given, and this type of elliptical and circular orbits evolution was considered. The dependences of elements of orbits with reduced existence time against the time of their size and orientation adjustment, ballistic coefficients etc. were considered.

The fourth trend is large space debris transfer (cross-section area is not less than 5 m2, final rocket stages, boosters) in orbits with a significantly reduced orbital lifetime which will not be dangerous to active space objects.

There are lots of projects for space debris mitigation measures. The most effective ones are the two schemes in which one of the servicer spacecraft (space vehicle collector [SV-collector]) can remove several large space objects (LSO).

The first scheme includes a successive flyby of several spacecraft with their collection or insertion in the exhaust sections of small spacecraft with their own autonomous control and enough fuel supply (thrust de-orbit kits [TDKs]) for braking and large space debris transfer to the disposal orbit (DO). The flyby is performed by an SV-collector and, when it is out of fuel and TDKs, it is refueled by a tanker spacecraft.

The second scheme suggests the usage of one spacecraft which manoeuvres between the objects and provides their transfer to the DO. This scheme is less effective, but it performs the flyby faster. Both schemes are considered and compared in Chapter 10.

Five groups of objects with close values of orbit inclinations were identified. The strategy of the flyby was chosen, and the compromise times of each flyby were found to meet the time constraints of the whole mission. The total delta-v expenditures on different flyby schemes were calculated and the most preferable schemes of each group flybys were chosen. It was determined which amount of fuel supply and the number of TDKs the SV-collector and tanker spacecraft should have. It was also estimated how many SV-collectors and tanker spacecraft were needed for the almost complete cleanup of all groups.

Practically all the algorithms described were supported by example cases. This will allow readers who intend to use the algorithms from the book to check their own realizations.

Solutions are given for the problems which can be met in practical work. In those solutions, terminal conditions were calculated with the setup precision using the iteration procedure described in the first chapter, which takes into consideration all the necessary perturbations.

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CHAPTER ONE

PROBLEM STATEMENT: GENERAL SOLUTION SCHEME

1.1. Spacecraft Equations of Motion

A particular interest in manoeuvring in near-circular orbits can be explained by the fact that most real spacecraft orbits are in this class of orbits. The circular orbital velocity in a central gravitational field can be determined as:

$$V_0 = \sqrt{\frac{\mu}{r_0}},$$

where r_0 = the circular orbital radius and $\mu =$ Earth's gravitational constant (the product of the gravitational constant by mass of the attracting body, for Earth $\mu = \gamma M \approx 3.986028 \cdot 10^5 \text{km}^3/\text{s}^2$ and for Earth's radius $R_e = 6{,}371 \text{ km}$).

In practice, there are always some perturbations in real and circular orbits.

The three main groups of perturbing factors can be distinguished as:

1. Deviations in the initial conditions (by velocity, radius or angle) from the conditions which provide motion along the circular orbit;
2. Additional external forces: the influence of the noncentrality of the gravitational field; atmospheric drag; attraction between bodies; solar radiation pressure; and the influence of the magnetic field, etc.
3. Forces caused by the spacecraft's propulsion system.

These perturbations can significantly change the circularity of an orbit. However, in most practically significant cases, deviations from circular orbits are relatively small and linearized equations of motion can be used (at least as a starting point) when studying them. Here, we assume that a circular orbit of radius r_0 is unperturbed, while the orbit under investigation is perturbed.

1.1.1. Equations of Motion in a Cylindrical Coordinate Frame

For the description of motion, we use a cylindrical coordinate frame r, u, z (Fig. 1-1). Its origin *O* is situated as the center of attraction. Here, r = the distance between the attraction center and the spacecraft's projection on the unperturbed orbital plane; u = the angle within the plane of the unperturbed orbit starting from an arbitrary initial Ox axis in the direction of the satellite's movement; and z = the distance between the unperturbed orbital plane and the satellite. The z axis is perpendicular to the unperturbed orbital plane. It is aligned in such a manner that if one were to follow the direction of increasing z, the satellite would move in a clockwise fashion. It is suggested that the time reference starting point (t = 0) is the moment when the satellite passes the point at which $u = 0$.

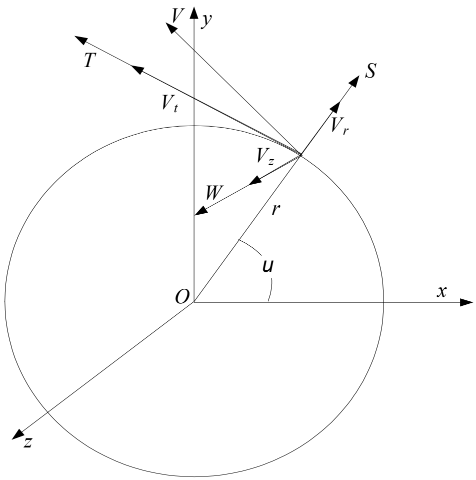

Fig. 1-1. The cylindrical coordinate frame connected to a satellite position in orbit

Here, we consider perturbing acceleration as summed acceleration caused by all forces except for the force produced by the central gravitational field ($g = \mu / r^2$). In addition, let the projection of the perturbing acceleration be in the direction of radius-vector r; the projection of the perturbing acceleration move along a normal line to the radius vector r in the unperturbed orbital plane; and the projection of the perturbing acceleration on the z axis be *S*, *T*, and *W* (Fig. 1-1), respectively. Keeping in mind that ratio z/r is small, and neglecting the effects of the second order infinitesimal, one can obtain the satellite equations of motion relative to the introduced coordinate frame in Suslov (1946):

$$S - g = \ddot{r} - r\dot{u}^2,$$
$$T = \frac{1}{r}\frac{d}{dt}(r^2\dot{u}), \qquad (1\text{-}1)$$
$$W = \ddot{z} + g\frac{z}{r}.$$

We will only investigate the case of the minor (with respect to the major acceleration *g*) perturbing accelerations *S*, *T*, and *W*, and minor (with respect to r_0 and V_0) deviations from circular motion caused by perturbing accelerations.

With accuracies of small quantities of the first order, the influence of deviations between the perturbed orbit and the unperturbed circular orbit in terms of the *S*, *T*, and *W* values can be omitted and these accelerations can be calculated as accelerations corresponding to the unperturbed orbit. This assumption allows us to solve the first two parts of Eq. 1-1 independently of the third equation, as the values S and T, with the accuracies of small quantities of the first order, are not affected by the lateral displacement z .

Let $V_r = \dot{r}$ and $V_t = r\dot{u}$ be the velocity vector projections on the radius vector and the normal line in the unperturbed orbital plane, respectively. By substituting these values in Eq. 1-1 we get:

$$\dot{V}_r = S - \frac{\mu}{r^2} + \frac{V_t^2}{r},$$
$$\dot{V}_t = T - \frac{V_t V_r}{r}, \qquad (1\text{-}2)$$
$$\dot{r} = V_r,$$
$$\dot{u} = \frac{V_t}{r}.$$

This system of three differential equations with the unknown values r, V_r, V_t cannot be evaluated in a closed form with arbitrary values of the perturbing accelerations *S* and *T*.

1.1.2. Equations in Deviations from a Basic Circular Orbit

For the approximate solution of Eq. 1-2, we will assume that the basic characteristics of the perturbed motion under consideration on the time interval of interest deviate little from the corresponding characteristics of the unperturbed circular motion. Let $\Delta r, \Delta V_r, \Delta V_t, \Delta u$ be the deviations between the respective values of the perturbed and unperturbed orbits:

$$\begin{aligned} r &= r_0 + \Delta r, \\ V_r &= \Delta V_r, \\ V_t &= V_0 + \Delta V_t. \end{aligned} \tag{1-3}$$

The first equation of Eq. 1-2 for the unperturbed orbit can be stated as:

$$0 = -\frac{\mu}{r_0^2} + \frac{V_0^2}{r_0}. \tag{1-4}$$

Let us substitute Eq. 1-3 in Eq. 1-2 and subtract Eq. 1-4 from the first equation of 1-2. In this case, by using Eq. 1-4 and assuming the values $\Delta r, \Delta V_r, \Delta V_t$ to be small (with accuracy of the first order of smallness), we get the following relationships (with accuracies of small quantities of the first order):

$$\begin{aligned} &\Delta \dot{V}_r - 2\lambda_0 \Delta V_t - \lambda_0^2 \Delta r = S, \\ &\Delta \dot{V}_t + \lambda_0 \Delta V_r = T, \\ &\Delta \dot{r} - \Delta V_r = 0, \\ &\Delta \dot{u} = \frac{1}{r_0}\left(\Delta V_t - \lambda_0 \Delta r\right), \end{aligned} \tag{1-5}$$

where λ_0 = the angular velocity of satellite motion along the unperturbed orbit, which can be found by:

$$\lambda_0 = \frac{V_0}{r_0}. \tag{1-6}$$

We get the equation system of four linear differential equations with constant coefficients and a set of unknown variables $\Delta r, \Delta V_r, \Delta V_t, \Delta u$.

In a similar manner, we get the equation for the deviations of lateral displacement:

$$\ddot{z} + \lambda_0^2 z = W. \tag{1-7}$$

1.1.3. Equations of the Motion System Solution

The equation systems 1-5 and 1-7 have a solution that can be stated as follows (El'yasberg 1965):

$$\Delta r = (2 - \cos\lambda_0 t)\Delta r_0 + \frac{\sin\lambda_0 t}{\lambda_0}\Delta V_{r_0} + \frac{2(1-\cos\lambda_0 t)}{\lambda_0}\Delta V_{t_0}$$

$$+ \frac{1}{\lambda_0}\int_0^t S(\xi)\sin\lambda_0(t-\xi)d\xi + \frac{2}{\lambda_0}\int_0^t T(\xi)[1-\cos\lambda_0(t-\xi)]d\xi,$$

$$\Delta V_r = \lambda_0 \sin\lambda_0 t\cdot\Delta r_0 + \cos\lambda_0 t\cdot\Delta V_{r_0} + 2\sin\lambda_0 t\cdot\Delta V_{t_0}$$

$$+ \int_0^t S(\xi)\cos\lambda_0(t-\xi)d\xi + 2\int_0^t T(\xi)\sin\lambda_0(t-\xi)d\xi,$$

$$\Delta V_t = -\lambda_0(1-\cos\lambda_0 t)\Delta r_0 - \sin\lambda_0 t\cdot\Delta V_{r_0} - (1-2\cos\lambda_0 t)\Delta V_{t_0}$$

$$- \int_0^t S(\xi)\sin\lambda_0(t-\xi)d\xi - \int_0^t T(\xi)[1-2\cos\lambda_0(t-\xi)d\xi,$$

$$\Delta u = \Delta u_0 - \frac{3\lambda_0 t - 2\sin\lambda_0 t}{r_0}\Delta r_0 - \frac{2(1-\cos\lambda_0 t)}{V_0}\Delta V_{r_0}$$

$$- \frac{3\lambda_0 t - 4\sin\lambda_0 t}{V_0}\Delta V_{t_0} - \frac{2}{V_0}\int_0^t S(\xi)[1-\cos\lambda_0(t-\xi)]d\xi$$

$$- \frac{1}{V_0}\int_0^t T(\xi)[3\lambda_0(t-\xi) - 4\sin\lambda_0(t-\xi)]d\xi, \tag{1-8}$$

$$z = \cos\lambda_0 t\cdot z_0 + \frac{\sin\lambda_0 t}{\lambda_0}V_{z_0} + \frac{1}{\lambda_0}\int_0^t W(\xi)\sin\lambda_0(t-\xi)d\xi,$$

$$V_z = -\lambda_0 \sin\lambda_0 t\cdot z_0 + \cos\lambda_0 t\cdot V_{z_0} + \int_0^t W(\xi)\cos\lambda_0(t-\xi)d\xi.$$

The members of the equations under the integrals determine the influence of the perturbing forces, whereas the other members of the equations determine the influence of the minor initial perturbations on the current deviations of the orbit from the unperturbed circular orbit. For now, we will consider only the influence of the minor initial perturbations. The equations for them in dimensionless form can be found below:

$$\begin{aligned}
&\frac{\Delta r}{r_0} = k_{11}\frac{\Delta r_0}{r_0} + k_{12}\frac{\Delta V_{r_0}}{V_0} + k_{13}\frac{\Delta V_{t_0}}{V_0},\\
&\frac{\Delta V_r}{V_0} = k_{21}\frac{\Delta r_0}{r_0} + k_{22}\frac{\Delta V_{r_0}}{V_0} + k_{23}\frac{\Delta V_{t_0}}{V_0},\\
&\frac{\Delta V_t}{V_0} = k_{31}\frac{\Delta r_0}{r_0} + k_{32}\frac{\Delta V_{r_0}}{V_0} + k_{33}\frac{\Delta V_{t_0}}{V_0},\\
&\Delta u = k_{41}\frac{\Delta r_0}{r_0} + k_{42}\frac{\Delta V_{r_0}}{V_0} + k_{43}\frac{\Delta V_{t_0}}{V_0} + k_{44}\Delta u_0,\\
&\frac{z}{r_0} = k_{55}\frac{z_0}{r_0} + k_{56}\frac{V_{z_0}}{V_0},\\
&\frac{V_z}{V_0} = k_{65}\frac{z_0}{r_0} + k_{66}\frac{V_{z_0}}{V_0}.
\end{aligned} \tag{1-9}$$

Here, $k_{ij}\ (i, j = 1,2,...,6)$ = dimensionless coefficients, which can be written as:

$$\begin{aligned}
&k_{11} = 2 - \cos\varphi, k_{12} = \sin\varphi, k_{13} = 2(1 - \cos\varphi),\\
&k_{21} = \sin\varphi, k_{22} = \cos\varphi, k_{23} = 2\sin\varphi,\\
&k_{31} = -(1 - \cos\varphi), k_{32} = -\sin\varphi, k_{33} = -(1 - 2\cos\varphi),
\end{aligned}$$

$$\begin{aligned}
&k_{41} = -(3\varphi - 2\sin\varphi), k_{42} = -2(1 - \cos\varphi), k_{43} = -(3\varphi - 4\sin\varphi),\\
&k_{55} = \cos\varphi, k_{56} = \sin\varphi, k_{44} = 1,\\
&k_{65} = -\sin\varphi, k_{66} = \cos\varphi,
\end{aligned}$$

where $\varphi = \lambda_0 t$ = the unperturbed value of the angle u.