Realistic
Interpretation
of Quantum Mechanics
Realistic Interpretation of Quantum Mechanics

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Quantum mechanics has been labeled magic because it combines an extremely good efficiency for the prediction of experimental results with a relatively simple formalism, but it does not provide a picture of the material world. Furthermore, when one attempts to get an intuitive picture logical contradictions appear. For instance, there are empirical facts suggesting that an electron is a point particle, or at least an object much smaller than an atom. However, interference effects and other phenomena seem to prove that it is an extended object (a wave). As a consequence, the interpretation of quantum mechanics has been the subject of continuous debate from the early period, almost one century ago, until today. The interpretations range from the pragmatic one supported by Bohr, with emphasis in the experiments, to those dazzled by the formalism, like the many-worlds that claims its universal validity, against common intuitions of human beings that do not believe to be simultaneously in several branches of a wave-function of the universe.

There is a dichotomy in current interpretations, namely they either reject a quantum picture of the world or offer a picture drastically departing from what we may derive from our everyday experience. In contrast, in this book I support ‘realism’, that is the view that science in general, and physics in particular, should explain how the world is, rather than just offering rules for the prediction of the results of the observations or experiments. That is, the book attempts a realistic interpretation that might provide a picture of the material world. Any picture must be free from contradictions and should be understood without sophisticated mathematical theories. In summary, realism includes the view that the world is made up of real stuff, existing in space and changing with time or, better stated, existing in a spacetime continuum.
The book may be seen as a kind of scientific memoir of the author. It is an organized survey of the attempts to understand quantum mechanics made along more than fifty years. Asides from revisiting relevant work of the author, the book contains many original contributions.

A fundamental hypothesis of the proposed interpretation of quantum theory is the reality of the vacuum fields. They appear as an unavoidable consequence of quantization although there is no agreement with respect to their nature. For some people they are an artifact of the formalism having observational consequences. The vacuum fields are labeled ‘virtual’, a word without a clear meaning that is used to avoid any commitment about their actual existence. This book strongly supports the reality of the vacuum fields as stochastic fields.

The interpretation of quantum theory resting on the reality of the vacuum fields opens a door for the solution of several problems in fundamental physics, some of them considered open and other allegedly closed but the common solution being still disputed by some people. I will mention six problems that will be discussed in this book: local realism, entanglement, quantum gravity, dark energy, dark matter and black holes. In the following I will comment briefly on each of them, stressing my personal opinion, that in most cases does not agree, or at least not fully, with the common view.

Local realism versus quantum mechanics has been the late stage of a debate that arose soon after the discovery of quantum mechanics in 1925. As is well known the main actors of the debate were Niels Bohr and Albert Einstein, who supported completeness and incompleteness of the theory, respectively. In 1935 Einstein, Podolsky and Rosen (EPR) introduced locality, or relativistic causality, as an argument for the incompleteness, but the debate did not end. In 1965, 10 years after Einstein’s death, John Bell derived his celebrated inequalities that have been interpreted by most authors as a vindication of Bohr. I do not agree and the subject is treated extensively in this book, namely in chapter 1, section 1.1, chapter 2 section 2.3, the whole chapter 3 and chapter 6 section 6.6, asides from comments in other parts of the book.

Entanglement is a concept introduced by EPR and discussed in more detail by Schrödinger the same year 1935. There is a clear mathematical definition in terms of vectors in a Hilbert space (or wave-functions) and their consequences are extremely relevant, it being a crucial concept in the increasingly important field of quantum
information and quantum computation. However there is no clear physical interpretation of entanglement. It is closely related to the Bell inequalities and it is particularly studied in chapter 2 section 2.2.3, chapter 3, section 3.2.5, chapter 5 section 5.4 and chapter 6 section 6.6.

The remaining 4 problems belong to astrophysics or cosmology, and the whole chapter 7 is devoted to them. In section 2 the idea of ‘quantizing the gravitational field’ is revisited taking into account that gravity is not a force. The so-called gravity effects derive from the curvature of spacetime. Therefore general relativity is not a theory of gravity, but a theory of (curved) spacetime. Quantizing gravity is actually quantizing the spacetime curvature and this is understood in this book as an epistemological rather than ontological question. In fact, as said above ‘quantization’ means the need of studying everything in the material world as stochastic.

The rest of chapter 7 deals with the consequences of the hypothesis that the quantum vacuum fields may produce spacetime curvature and that curvature modifies those fields. In particular, it is necessary to take into account that the vacuum fields fluctuate. In section 7.4 it is argued that, as these fluctuations are incompatible with Minkowski space, it is worth to study the minimal modification produced by them and the result is that in the absence of matter they give rise to a cosmological constant term or, in other words, they are plausibly the origin of the ‘dark energy’. On the other hand, in space containing baryonic matter the combination of that matter with the vacuum fluctuations may produce new effects or, in other words, modify the dark energy. It is also proposed that vacuum fluctuations might give rise to effects currently attributed to dark matter. This possibility is studied in section 7.5.

In astrophysical compact objects where the baryonic matter is able to produce a strong spacetime curvature, it is plausible that the effect on the quantum vacuum fields should be extremely big. In section 7.6 it is proposed that these changes might modify the evolution of such compact relativistic objects stopping collapse before a Schwarzschild singularity is produced.

Plan of the book. The book consists of seven chapters. In chapter 1 a number of nude observations, usually assumed specifically quantal, are analysed in order to show that they might be explained without departing from our proposed realistic view of nature. The chapter
starts with an epistemological introduction that supports the Einstein (realistic) against the Bohr-Heisenberg (positivistic and pragmatic) views of science. At the end of the chapter a sketch is presented of the view about the quantum world.

Chapter 2 is devoted to the standard, or canonical, Hilbert-space formalism of quantum theory. We start with the postulates of the theory followed by a critical analysis of the most popular interpretations of the formalism. A purpose of that analysis is to point out that we should not attempt to interpret the standard formalism, but rather the observations or experiments, that are independent of any theory. For the sake of completeness I also include a study of the proposed logical structure in terms of lattices of propositions and the comparison with classical logic, the Bell inequalities being a crucial test.

Chapter 3 deals in more detail with the Bell inequalities, that have been during half a century most relevant in discussions about the interpretation of quantum mechanics. The inequalities are assumed to be necessary conditions for a local realistic interpretation of nature. In fact, Bell’s work seems to prove that there is a conflict between quantum mechanics and local realism, understanding locality as relativistic causality. I shall discuss to which extent this is true. In addition, a short survey is presented of the experiments performed or proposed in order to test the inequalities against the quantum predictions.

In chapter 4 alternative formulations of quantum theory are presented, namely de Broglie-Bohm, stochastic mechanics, Weyl-Wigner in phase space, and Feynman path integrals. These formulations either contradict some predictions of the standard formalism, and also experiments, or seem incompatible with a realistic interpretation. For instance, the Weyl-Wigner function seems to imply ‘negative probabilities’ and Feynman path integrals ‘imaginary probabilities’. I shall argue that nevertheless both of them might be free from these shortcomings if correctly interpreted.

Chapter 5 is a survey of stochastic electrodynamics, a theory that studies within classical electrodynamics the motion of charged particles under the action of given forces but in the presence of a random electromagnetic radiation field. The theory agrees with quantum predictions in a limited domain, but disagrees in other cases. The relevance of stochastic electrodynamics is that it provides hints for a realistic interpretation of the whole quantum theory. In fact, it is a
theory that may be considered classical but makes predictions that fit in experiments allegedly quantal.

Chapter 6 provides a realistic analysis of several effects of the quantum vacuum radiation field that offers, for some experiments, a clear intuitive picture commonly claimed to be impossible. In particular, we discuss experiments showing wave-particle behaviour or violations of Bell inequalities.

Chapter 7 deals with quantum effects in astrophysics and cosmology. In the first part a personal view is presented of the meaning of general relativity, which in several respects differs from the current view. The rest of the chapter deals with the possibility of understanding dark energy and dark matter as effects of the quantum vacuum fluctuations. It is also suggested that the quantum vacuum fields might prevent collapse to black holes.

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1.1. The debate about the interpretation of quantum theory

1.1.1. Early interpretations: Schrödinger, Heisenberg and Bohr

Quantum theory began with Max Planck’s formula for the blackbody spectrum, presented in December 14, 1900, a date later named by Sommerfeld the birthday of quantum theory. After years of slow progress without a definite theory, quantum mechanics appeared almost simultaneously during 1925-26 in two different forms: ‘quantum mechanics’ of Heisenberg and ‘wave mechanics’ of Schrödinger.
Soon afterwards Schrödinger himself and Dirac proved that both theories are equivalent; that is, they make the same predictions for the experiments. For the history of the early period see Jammer [1].

Quantum mechanics had a rapid and deep impact. It was soon applied to atoms, molecules and solids with great success. However the interpretation of the theory was not straightforward. Schrödinger suggested that his wave-function describes a continuous electric charge distribution. That picture was abandoned after correct criticisms by Bohr and other people. In particular a detailed calculation of the ionization energy of the helium atom proved that the electron should be seen as a particle much smaller than the atom.

In contrast, Heisenberg introduced his quantum theory as a set of calculational rules involving arrays of numbers (matrices in mathematical language) devoided of any intuitive picture. Furthermore physics without images should be considered a superior form of science because the only condition for the validity of a theory is the agreement of its predictions with the empirical evidence. That view was reinforced with Dirac’s formulation in terms of an abstract vector space.

The approach of Heisenberg was supported by Bohr, who elaborated it introducing the ‘complementary principle’, with the aim of solving the particle (localized)-wave (extended) duality of quantum objects, and stressing the role of the Planck constant as an indivisible element of ‘action’. This led to the Copenhagen interpretation, which became dominant for many years. This interpretation may be labeled as pragmatic because the referent of the theory is not the physical world but the experiment. However, it produced discomfort in some people, e.g. Einstein, and a long debate arose that lasts until today [2], [3], [4], [5], [6], [7], [8], [9], [10], [11]. For a recent review with extensive bibliography, see Drummond [12].

The interpretations usually refer to quantum mechanics as formulated in Hilbert space, a formalism to be treated in chapter 2. Therefore I postpone to that chapter a discussion of the most popular interpretations. In the following I comment on the two main classes of quantum interpretations, namely pragmatic and realistic.

1.1.2. The pragmatic approach to quantum mechanics. None of the interpretations proposed till now offer a clear intuitive picture of the quantum world. Nevertheless, most physicists do not worry for the lack of a picture and embrace a pragmatic approach close to the
Copenhagen interpretation. They accept a minimal interpretational framework with the following key features [13]:

1. Quantum theory is viewed as a scheme for predicting the probabilistic distribution of outcomes of measurements made on suitably prepared copies of a system.

2. The probabilities are interpreted in a statistical way as referring to relative frequencies.

Behind the pragmatic approach there is usually a philosophical position about physics (or science in general) that may be summarized as follows. It is common wisdom that a physical theory has at least two components [14]: (1) the formalism, or mathematical apparatus, of the theory, and (2) the rules of correspondence that establish a link between the formalism and the results of measurements. As an example let us consider the formalism of quantum mechanics based on the mathematical theory of Hilbert spaces (to be discussed in more detail in chapter 2). The formalism involves two kinds of operators: density operators, \( \hat{\rho} \), that represent states, and self-adjoint operators, \( \hat{A} \), that represent observables. The link with the measurement results is given by the postulate that the expectation value, \( \text{Tr}(\hat{\rho} \hat{A}) \), corresponds to the statistical mean of the values obtained when one realizes several measurements on identically prepared systems (which determines \( \hat{\rho} \)) by means of an appropriate apparatus (that corresponds to \( \hat{A} \)).

If we assume that the formalism and the correspondence rules are the only objects required to define a physical theory, in the sense that the statistical regularities need not be further explained, then we get what has been called a minimal instrumentalistic interpretation of the theory [15], [13]. It may be identified with the purely pragmatic approach mentioned above.

Most people claiming to support that approach accept the following positions:

1. The notion of an individual physical system ‘having’ or ‘possessing’ values for all its physical quantities is inappropriate in the context of quantum theory.

2. The concept of ‘measurement’ is fundamental in the sense that the scope of quantum theory is intrinsically restricted to predicting the results of measurements.

3. The spread in the results of measurements on identically prepared systems must not be interpreted as reflecting a ‘lack of knowledge’ of some objectively existing state of affairs.
The *instrumentalistic* approach is quite different from, even opposite to, the *realistic* view traditional of classical physics. Between these two extremes there are a variety of approaches.

### 1.1.3. Realistic interpretations.

The main opponent to a purely pragmatic approach to quantum mechanics was Albert Einstein. Indeed, his discussions with Niels Bohr are the paradigm of a scientific debate, hard in the scientific arguments but hearty from the personal point of view. One of the most celebrated moments of the debate was a 1935 article by Einstein, Podolsky and Rosen [16] (EPR) soon followed by Bohr’s reply [17]. The former begins as follows: “Any serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concepts with which the theory operates. These concepts are intended to correspond with the *objective reality*, and by means of these concepts we picture this reality to ourselves” (my emphasis).

It is true that in the years elapsed since the EPR paper the concept of ‘objective reality’ has been questioned as not clear. Due to the difficulties with the interpretation of quantum mechanics, many people working on foundations dismiss the ‘realism’ of EPR as ‘naive’. Thus more sophisticated forms of realism have been proposed [7]. In any case, a deep discussion about the philosophical aspects of reality or realism is outside our scope.

In this book I strongly support Einstein’s view. That is, I believe that a realistic interpretation is possible. The starting point is the claim that any physical theory should offer a *physical model* in addition to the *formalism and rules for the connection with the experiments*. The latter are obviously essential because they are required for the comparison of the theory with empirical evidence, which is *the test* for the validity of the theory. But in my opinion physical models are also necessary in order to *reach a coherent picture* of the world. Many quantum physicists apparently support the uselessness of pictures, but it is the case that when they attempt popular explanations of quantum phenomena they frequently propose actual pictures, many of them rather bizarre. For instance it has been claimed that quantum mechanics *compel us* to believe that there are a multiplicity of ‘me’ in parallel universes, or that an atom may be present in two distant places at the same time. This is an indication that the need for a ‘picture the reality to ourselves’ [16] cannot be easily dismissed.
Furthermore the existence of physical models might open the possibility for new developments and applications of quantum theory and therefore it is not only an academic question.

The contrast between the two great theories of the 20th century, quantum mechanics and relativity is interesting. The latter provides a beautiful physical model: There is a four-dimensional manifold with intrinsic curvature and all material objects (e.g. particles or fields) are defined in that continuum and the basic quantities of physics, like mass, energy or momentum, become geometric properties. But the calculational tool of general relativity (derived from the Riemann geometry) is rather involved, the fundamental (Einstein) equation being \textit{nonlinear}. In quantum mechanics there is a relatively simple \textit{linear} formalism involving vectors and operators in a Hilbert space. Indeed, the fundamental (Schrödinger) equation is linear. However, there is no coherent physical model behind. I would say that general relativity has \textit{physical beauty}, the quantum formalism possesses \textit{mathematical elegance}.

Historically the renunciation to physical models in quantum mechanics was a consequence of frustration caused by the failure of the models proposed during the first quarter of the 20th century. This was specially the case after Bohr’s atom, consisting of point electrons moving in circular orbits around the nucleus. The model, generalized with the inclusion of elliptical orbits, certainly produced progress in the decade after 1913. However, it was obvious that the model mixed contradictory laws, namely classical electrodynamics and Bohr postulates. The success of quantum mechanics in the quantitative interpretation of experiments did not solve the problem, which became more acute. Thus the failure to find a good physical model of the microworld led to an almost universal acceptance of the current view that models may be unnecessary or even misleading.

I do not agree with that view, but this book is a defence of a realistic interpretation of the quantum phenomena. I am aware that the task is extremely difficult as is proved by the lack of such an interpretation after a century of quantum mechanics. However, I am convinced that many of the obstacles derive from some assumptions that are not necessary for the interpretation of the experiments. These assumptions have been introduced along the historical development of the theory and are now a part of the common view. Pointing out the main obstacles and how they might be removed is the purpose of this book. I do not pretend to provide a coherent and complete realistic
interpretation, but I hope that some of the ideas put forward might be useful in the progress towards a better understanding of quantum mechanics.

1.1.4. A note on the epistemology of physics. In order to practice science some previous philosophical questions should be answered. For instance, what is science? Or, what is the purpose of science? There are different philosophical positions about these questions that are closely connected with the different interpretations of quantum mechanics.

There is some agreement that the criterion to distinguish science from nonscientific knowledge is the proposal of Karl Popper [18], [19]: A claim is scientific if it may be refuted by observations or experiments. This definition is a consequence of a well known fact, namely the possible existence of several different theories all of them predicting correctly the results of experiments in a given domain. In other words the correctness of a theory is sufficient, but not necessary, for the appropriate prediction of empirical facts. For this reason a single experiment may refute a theory but a theory can never be fully confirmed empirically, and this is essentially the Popper thesis. As a consequence several different theories may exist that are able to predict correctly the empirical results, but suggesting quite different pictures of the microworld.

Popper’s criterion is good enough as a matter of principle, but it is not so good in practice. In fact, it is the case that rarely an established theory breaks down as a consequence of a single experiment contradicting it. As Lakatos [20] has pointed out, well tested theories are protected in the sense that the empirical refutation of a single prediction may be interpreted without rejecting the theory, for instance assuming that the analysis of the experiment was incorrect. Indeed, it is a historical fact that established theories are only abandoned, or better superseded, when there is a new theory in agreement with the former one in its domain of validity but possessing a wider domain or other virtues.

Quantum mechanics is today a fully established theory and therefore it is very well protected in the sense of Lakatos. I do not only mean protection in the domain where the theory has been tested. What I want to stress is that over the years people have introduced a number of assumptions, today widely accepted, that are additions without possibility of empirical test. See section 2 below for several
examples. These unnecessary additions are also protected and, in my opinion, they are the main cause of the strong difficulties in reaching a realistic physical model of the quantum world.

Most working quantum physicists adhere to the pragmatic approach as described above. The support has its roots in a ‘positivistic’ attitude. Positivism is the philosophical doctrine that, in a broad sense, states that all knowledge should be founded on empirical evidence. In this sense it is accepted by everybody. But in a more strict sense it is a tendency to give value to the crude empirical data in detriment of the theoretical elaborations. For instance this was the opinion of Ernst Mach, who rejected the concept of atom because at that time (around 1900) atoms had not been directly observed.

Positivism was also behind Heisenberg’s initial formulation of quantum mechanics resting upon the belief that only sets of numbers corresponding to the possible results of measurements should enter the theory. This led him to elaborate quantum mechanics as a calculational tool involving matrices (it was sometimes called ‘matrix mechanics’). The combination of mathematical formalism and empirical results almost without further theoretical elaboration permeates the interpretation of quantum mechanics till now. An illuminating confrontation between the positivistic and realistic epistemologies is the conversation of Heisenberg with Einstein that took place in Berlin in 1926, as remembered by Heisenberg himself [21]. The most relevant part is the following.

As soon as we were indoors, he [Einstein] opened the conversation with a question that bore on the philosophical background of my recent work. “What you have told us sounds extremely strange. You assume the existence of electrons inside the atom, and you are probably quite right to do so. But you refuse to consider their orbits, even though we can observe electron tracks in a cloud chamber. I should very much like to hear more about your reasons for making such strange assumptions.”

“We cannot observe electron orbits inside the atom,” I must have replied, “but the radiation which an atom emits during discharges enables us to deduce the frequencies and corresponding amplitudes of its electrons. After all, even in the older physics wave numbers and amplitudes could be considered substitutes for electron orbits. Now, since a good theory must be based
on directly observable magnitudes, I thought it more fitting to restrict myself to these, treating them, as it were, as representatives of the electron orbits.”

“But you don’t seriously believe,” Einstein protested, “that none but observable magnitudes must go into a physical theory?”

“Isn’t that precisely what you have done with relativity?” I asked in some surprise. “After all, you did stress the fact that it is impermissible to speak of absolute time, simply because absolute time cannot be observed; that only clock readings, be it in the moving reference system or the system at rest, are relevant to the determination of time.”

“Possibly I did use this kind of reasoning,” Einstein admitted, “but it is nonsense all the same. Perhaps I could put it more diplomatically by saying that it may be heuristically useful to keep in mind what one has actually observed. But on principle, it is quite wrong to try founding a theory on observable magnitudes alone. In reality the very opposite happens, it is the theory which decides what we can observe. You must appreciate that observation is a very complicated process. The phenomenon under observation produces certain events in our measuring apparatus. As a result, further processes take place in the apparatus, which eventually and by complicated paths produce sense impressions and help us to fix the effects in our consciousness. Along this whole path—from the phenomenon to its fixation in our consciousness—we must be able to tell how nature functions, must know the natural laws at least in practical terms, before we can claim to have observed anything at all. Only theory, that is, knowledge of natural laws, enables us to deduce the underlying phenomena from our sense impressions. When we claim that we can observe something new, we ought really to be saying that, although we are about to formulate new natural laws that do not agree with the old ones, we nevertheless assume that the existing laws—covering the whole path from the phenomenon to our consciousness—function in such a way that we can rely upon them and hence speak of observations” (my emphasis).
The conversation continued for a while and at the end Einstein warned: “You are moving on very thin ice. For you are suddenly speaking of what we know about nature and no longer about what nature really does. In science we ought to be concerned solely with what nature does.” Einstein’s arguments are a clear support to a realistic epistemology, and I fully agree with his views.

I believe that, as stated by Einstein “it is wrong to try founding a theory on observable magnitudes alone”. But I believe that the following statement more close to Heisenberg’s view is correct: We should try to interpret as physically real just the observable magnitudes alone, but not the intermediate (mathematical) ones that appear in a calculation. This will be our guide in this book for a realistic interpretation of quantum mechanics.

1.2. Specific features of quantum physics

I propose that the difficulties for a realistic interpretation of quantum phenomena do not derive from the empirical facts, or not only. Thus in the following I shall briefly revisit the most relevant of those phenomena in order to see whether the nude empirical facts do prevent any picture of the microworld. Actually, most textbooks of quantum mechanics emphasize the difficulty, or impossibility, to interpret typical quantum phenomena with a realistic view. The purpose of the following paragraphs is just the opposite. It will be shown that in fact those phenomena are compatible in most cases with a picture of the microworld. Of course, the picture is somewhat different from the one offered by classical physics, but not dramatically different.

Nevertheless, I shall confess that serious difficulties remain, so that neither this section nor the whole book will provide a systematic realistic interpretation free from difficulties.

1.2.1. The stability of atoms. Soon after Rutherford’s experiment of 1911, that lead to the nuclear atom, Bohr proposed in 1913 a model which involved postulates contradicting classical electrodynamics. The common wisdom was, and it still is, that the contradiction cannot be avoided. That it appears even for the most basic empirical fact, the stability of the atom. But this is not true [22].

In fact, if studied with classical electrodynamics a hydrogen atom, consisting of one proton and one electron, cannot be stable if isolated. The reason is that an electron moving around the proton would radiate, and therefore it will lose energy until the atom collapses. But
the argument is not valid if there are many atoms in the universe because if all atoms radiate the hypothesis of isolation is not appropriate. It is more plausible to assume that there is some amount of radiation filling space. Then every atom would sometimes radiate but other times it would absorb energy from the radiation, eventually arriving at a dynamical equilibrium. This may explain, at least qualitatively, the stability of the atom. The picture that emerges is that matter and radiation of the universe cannot be treated independently, and the complexity of the universe compel us to treat the radiation as a background stochastic field. Therefore the electron of a hydrogen atom would move in a random way around the nucleus. That motion should be so complex that we cannot follow it in detail but only the probability distribution of positions can be determined. That distribution is what the Schrödinger wave-function provides via Born’s rule. The assumption of background fields filling space fits in the quantum vacuum fields that appear in field quantization. They are assumed real stochastic fields throughout this book, and make up the basic hypothesis for the realistic interpretation of quantum theory as discussed in the following.

1.2.2. The connection between energy and frequency. A standard method to study the radiation field in free space is to expand it in plane waves (or in normal modes if it is enclosed in a cavity). In free space the number of modes, $N$, per unit volume and unit frequency interval is

$$N = \frac{\omega^2}{\pi^2 c^3},$$

and the radiation energy is

$$E = \frac{1}{2} \hbar \omega$$

per normal mode of the radiation. That energy eq.(2) is just $1/2$ the one postulated by Einstein in his 1905 article where he introduced the concept of quantum of radiation, later named photon. I will discuss the concept of photon in chapter 6. In the following I will derive some consequences via an heuristic approach. Firstly I propose to generalize eq.(2) for all possible vacuum fields, associated to the forces of nature.

If a hydrogen atom is in a dynamical equilibrium with radiation, it is plausible that the main interaction with the vacuum fields takes place with the normal modes of the field that have frequencies close to those of the electron motion. Also, in a dynamical equilibrium
it is plausible that the mean kinetic energy of the electron should be close to half the average energy of those normal modes having greater interaction with the atom (the other half would correspond to potential energy). Then if the electron moved around the nucleus in a circle with energy $E$ (i.e. without emission or absorption of radiation), we might write the following equalities

$$|E| = \frac{1}{2}mv^2 = \frac{e^2}{2r}, \quad v = r\omega, \quad |E| \sim \frac{1}{2}\hbar\omega,$$

the latter corresponding to the condition of dynamical equilibrium with radiation. Of course, the motion is perturbed by the action of the vacuum fields, whence the electron motion would be irregular, not circular, but it is plausible that eqs.(3) would be roughly fulfilled. Hence the energy and the size of the atom may be got by eliminating the quantities $v$ and $\omega$ amongst the 4 equalities, which leads to

$$E \sim -\frac{1}{2} \frac{me^4}{\hbar^2}, \quad r \sim \frac{\hbar^2}{me^2},$$

in rough agreement with the quantum prediction and with experiments.

1.2.3. Statistical character. The statistical character of measurements in the quantum domain is a consequence of existence of random vacuum fields as discussed above. However, it is appropriate to comment on it in more detail due to the great relevance attributed to it in books and articles about foundations of quantum physics.

In the classical domain typical experiments are affected by statistical errors. That is, the same experiment performed in similar conditions may give rise to (slightly) different results. For this reason it is a standard practice to report the results of measurements accompanied by an uncertainty interval. In the macroscopic domain the uncertainty is attributed to the difficulty of controlling a very large number of parameters (the environment), with the consequence that never (or rarely) an experiment may be repeated in exactly the same conditions. In any case, it is usual that the uncertainty is only a small fraction of the measured quantity. In contrast, in the microscopic domain it is frequent that the uncertainties are of the same order than the measured result. This is equivalent to saying that the same experiment may give rise to a number of different results, every one with some probability. However, in contrast with macroscopic (classical) physics, in quantum physics the probabilities are usually not attributed to lack of control in the experiment.
The current view is that quantum probabilities are radically different from the classical, ordinary life, probabilities. The latter are introduced when there is incomplete knowledge (‘ignorance’), maybe unavoidable, about the truth of some assertion. For instance we may attach a probability 1/2 to the appearance of head when throwing a coin, because we cannot control all relevant variables in the experiment. In contrast, it is a common assumption that quantum probabilities are quite different, that they derive from a lack of strict causality of the natural laws, i.e. the fact that different effects may follow to the same cause. This is usually called the fundamental or essential probabilistic character of the physical laws. This is an example of a practical difficulty that has been (incorrectly in my opinion) raised to the rank of an ontological statement: “Natural laws are not strictly causal”.

Einstein disliked that assumption and strongly criticized it, as expressed in his celebrated sentence “God does not play dice”. I understand very well Einstein’s opinion. For him the rational understanding of nature was a kind of religion. The more loose (strict) the natural laws are, the smaller (greater) could be our rational understanding of nature. Accepting a weak causality is like accepting poor science. Nevertheless, some people are happy with the absence of determinism implied by the nonexistence of strict causality. For instance some claims have been made that the quantum lack of determinism may explain human free will. This question lies outside the scope of this book and shall not be further commented on.

But I do not support determinism in the mechanistic view of Laplace. As said above quantum mechanics is a stochastic theory. I believe that strictly causal laws might perhaps exist, but there is also a universal noise which permeates everything and prevents any practical determinism. Strict causality combined with stochasticity (randomness) is in practice indistinguishable from essential probability, and the former is more plausible. In order to clarify this matter let us think about Brownian motion. Under macroscopic observations the random motion of a Brownian particle may appear as lacking causality; but we assume that, taking into account the molecules of the liquid where the particle is immersed, the whole motion is governed by Newtonian dynamics, which is causal.

It may be argued that the uncertainties in the measurements in the quantum domain do not look like typical uncertainties derived
from noise. Indeed, the latter may be usually approximated by continuous probability distributions. In the quantum domain there are instances, e.g. the Stern-Gerlach experiment, where there is uncertainty between just two values (for more about that experiment see the subsection about discrete states, below). I shall not discuss further this difficulty here, but we will discuss it in more detail in later chapters.

1.2.4. Heisenberg uncertainty relations. The Heisenberg ‘uncertainty principle’ is the most frequently quoted evidence for the dramatic splitting between classical and quantum physics. In fact, the principle appears in popular writings like a kind of mysterious property of our world. However, the arguments given below strongly suggest that the Heisenberg inequalities are consequences of the stochasticity inherent to the microworld rather than a fundamental principle. I shall not discuss here the general relation dealing with conjugate dynamical variables, but restrict attention to the experimentally proved impossibility of determining simultaneously the position and the velocity (or momentum) of a particle. This implies that it is not possible to prepare a particle with both position and velocity sharply defined, and also that no measurement may provide the values of both these quantities at the same time. Hence it is impossible to determine the path of a particle.

In any motion under the action of a random force some constraints may appear for the simultaneous determination of position and velocity, that might be stated in the form of inequalities. As an illustrative example this is shown to be the case in Brownian motion. A Brownian particle possesses a highly irregular path whose instantaneous velocity cannot be measured (with ordinary, macroscopic set-ups). Only the mean velocity, $\bar{v}$, during some time interval may be measured, that is,

$$\bar{v} = \frac{|\Delta r|}{\Delta t},$$

where $|\Delta r|$ is the distance between the initial and final positions in the time interval $\Delta t$. On the other hand there is a relation, derived by Einstein in 1905, between the expected value of the square of the distance, $|\Delta r|^2$, and the time interval, $\Delta t$. Namely,

$$\langle |\Delta r|^2 \rangle = D\Delta t,$$
where $D$ is called the diffusion constant and $\langle \rangle$ means ensemble average, that is, the average over many measurements involving the same time interval. If we eliminate $\Delta t$ amongst the two equalities we get

$$\langle |\Delta r|^2 \rangle = \langle \bar{v}^2 \rangle \Delta t^2 \Rightarrow \langle |\Delta r|^2 \rangle \langle \bar{v}^2 \rangle \simeq D^2,$$

a relation having some similarity with the Heisenberg uncertainty relation. We conclude that a plausible interpretation of the Heisenberg principle is that the quantum motion possesses a random component having some similarity (not identity!) with Brownian motion. This similarity has been the basis for the development of stochastic mechanics, which provides an intuitive picture of some typically quantum phenomena. However, this theory presents difficulties as will be discussed in chapter 4. Actually, the Brownian motion inequality derived above is different from the Heisenberg inequalities because $\bar{v}^2$ is the mean squared velocity during a time interval, rather than the uncertainty of the velocity. In chapter 5 I will show that Heisenberg inequalities may be derived as a consequence of the radiation spectrum.

The Heisenberg inequalities become an obstacle for a realistic interpretation of quantum mechanics when the practical difficulty (or impossibility) of simultaneous knowledge of position and velocity is elevated to the category of an ontological statement: “Trajectories of quantum particles do not exist”. Of course, the Heisenberg inequalities are reinforced by the fact that they are predicted by the quantum formalism, but the analogy with the Brownian motion inequality suggests that the quantum formalism may be a disguised form of specifying a stochastic theory.

1.2.5. Discrete energy states. As is well known, the first quantum hypothesis, introduced by Planck in 1900, was that material systems may possess only energies belonging to a discrete set. The assumption was extended by the Einstein 1905 proposal that light consists of discrete pieces of energy (photons) and the successful application of this principle to the photoelectric effect. In 1913 Bohr incorporated the idea to his atomic model postulating that atoms can only exist in states having energies within a discrete set, $E_0, E_1, E_2, \ldots$. The model also assumed that the absorption and emission of light takes place with transitions between these states, the frequency, $\omega_{jk}$, of the light being related to the difference of atomic energies by

(5) \[ \hbar \omega_{jk} = E_j - E_k. \]