Experimental and Computational Methods for Strength Analyses of Gears
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By

Srečko Glodež
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Author
Gears are machine elements used in most engineering applications, mainly in the automotive and aerospace industries. When designing and dimensioning gear drives, the available standardised procedures are usually used for that purpose. The strength analysis (i.e. load carrying capacity) of gears involves the study of the static and fatigue strength (tooth bending strength and surface durability) as well as the scuffing and wear strength, with the purpose of ensuring the predetermined lifetime under the expected operating conditions. Generally, the international standard ISO 6336 is the most frequently used procedure for calculating the load capacity of spur and helical cylindrical gears made of different metallic materials. However, standardised procedures to analyse the load capacity of gears are based on a number of different coefficients that allow for proper consideration of real working conditions (additional internal and external dynamic forces, a contact area of mating gears, gear material, surface roughness, etc.). Furthermore, the standardised procedures are based exclusively on the experimental testing of reference gears, and they consider only the final stage of the fatigue process, i.e. the occurrence of final failure (tooth breakage in a gear tooth root or occurrence of pits on a gear flank). On the other hand, the complete fatigue process leading to fatigue failure may be divided into the following stages: (i) microcrack nucleation, (ii) short crack growth, (iii) long crack growth and (iv) occurrence of final failure. In engineering applications, the first two stages are usually referred to as the “Crack initiation period”, while long crack growth is referred to as the “Crack propagation period”. The complete service life of a gear can then be determined from the number of stress cycles $N_i$ required for fatigue crack initiation and the number of stress cycles $N_p$ required for a crack to propagate from the initial to the critical crack length ($N = N_i + N_p$).

When analysing the fatigue behaviour of mating gears, the appropriate computational tool (i.e. Finite Element Method) is often applied to obtain the comprehensive stress/strain field in a gear tooth root or on gear flanks. The computational results are combined with the appropriate fatigue design approach to obtain the fatigue life. The following approaches may be used for that purpose:
• Stress-life approach,
• Strain-life approach,
• Fatigue crack growth approach.

In recent years, sintered gears have been a cost-efficient alternative for machined gears in larger series in the capital goods industry (the automotive industry, electrical appliances, hand-tool industries and other high-volume industrial segments). Due to low prices, low waste, tight tolerances, and evermore improving mechanical properties, Powder Metallurgy (PM) is becoming an interesting alternative mass production process for the future. Significantly, the automotive industry has been using this technology to produce non-vital parts. In general, the international standard ISO 6336 is not suitable for calculating the load capacity of sintered gears. For that reason, alternative computational methods are often used for that purpose, considering all three fatigue design approaches (stress-life approach, strain-life approach, and fatigue crack growth approach). Besides sintered gears, polymer gears are also used widely in many industries and applications, such as office appliances, service and mechatronic devices, household facilities, computer and laboratory equipment, medical instruments, etc. Polymer gears can be produced by classical cutting processes or, for large series production, by injection moulding. Some of the main benefits of polymer gears are high specific mechanical properties, good tribological performance, high resistance against impact loading, ability to absorb and damp vibration, reduced noise, etc. However, polymer gears also have some disadvantages, such as less load-carrying capacity and lower operating temperatures if compared to metal gears, difficulties in achieving high tolerances, relatively high dimensional variations due to temperature and humidity conditions, etc.

This book consists of five chapters. The first chapter introduces the reader to the fundamental magnitudes of gear drives in relation to the cylindrical gear pairs, bevel gear pairs, and worm gear pairs. The second chapter explains the theoretical background of the load capacity of gears, where surface pitting load capacity and tooth root load capacity are described in detail. The third chapter focuses on the strength analyses of metal gears, while chapters four and five describe the strength analyses of sintered gears and polymer gears, including some typical practical examples.

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Author
CHAPTER 1
INTRODUCTION

Gear drives correspond to mechanical drives with the uniform transmission of rotational moving from the driving machine to the driven machine (see Fig. 1.1). Their primary function is to adjust the torque $T_1$ and rotational speed $n_1$ of the driving machine (i.e. electromotor, combustion engine, etc.) to the appropriate values $T_2$ and $n_2$, which should appear on the driven machine [1.1, 1.2]. Here, the power on the driven side, $P_2$, is smaller if compared to the power on the driving side, $P_1$ ($P_2 < P_1$). This is due to losses inside the drive system, which can be expressed by the efficiency of the gear drive $\eta = P_2 / P_1 < 1$.

![Figure 1.1: The basic principle of the use of gear drive](image)

Gear drives consist of one (one-stage drives) or more (multi-stage drives) gear pairs, which may be differently designed. In general, the following basic gear pairs are most often used in the praxis (see Fig. 1.2):

- **Cylindrical gear pair**: a pair of mating cylindrical gears with parallel axes (Fig. 1.2a).
- **Bevel gear pair**: a pair of mating bevel gears with intersecting axes (Fig. 1.2b).
- **Worm gear pair**: a pair of mating worm and worm wheel with crossed axes (Fig. 1.2c).
Chapter 1

Figure 1.2: The basic designs of gear pairs
a) Cylindrical gear pair, b) Bevel gear pair, c) Worm gear pair

The gear pair consists of two gears which are rotating in opposite directions (one clockwise and another counterclockwise). By cylindrical and bevel gear pairs, the smaller gear is called a *pinion*, while the bigger gear is called a *gear*. By worm gear pair, the smaller gear is called a *worm*, while the bigger gear is called a *worm wheel*. The number of teeth is designated as $z_1$ (for smaller gear) and $z_2$ (for bigger gear). Table 1.1 shows some typical characteristics of basic designs of gear pairs.

Table 1.1: Typical characteristics of basic designs of gear pairs [1.1]

<table>
<thead>
<tr>
<th></th>
<th>Power $P_{max}$ [kW]</th>
<th>Rotational speed $n_{max}$ [min$^{-1}$]</th>
<th>Circular velocity $v_{max}$ [m/s]</th>
<th>Transmission ratio $i_{max}$</th>
<th>Efficiency $\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylindrical gear pairs 1)</td>
<td>3000</td>
<td>100,000</td>
<td>40 … 100</td>
<td>6 … 8</td>
<td>0.97 … 0.99</td>
</tr>
<tr>
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<td>25 … 70</td>
<td>15 … 70</td>
<td>0.40 … 0.96</td>
</tr>
</tbody>
</table>

1) The listed maximum values are informative and are valid for one-stage gear drives.

Cylindrical gears consist of a pair of cylindrical toothed gear wheels and can be designed as spur gears, helical gears, double-helical gears and arrow-shaped gears (Fig 1.3). Furthermore, cylindrical gear pair may be external gear pair, internal gear pair (Fig. 1.4a) and rack gear pair (Fig. 1.4b).
Besides the basic designs of gear pairs, as shown in Fig. 1.2, some special designs of gear pairs are also known in the engineering praxis. The crossed helical gear pair (Fig. 1.5a) consists of two cylindrical helical gears mounted on the crossed (or skew) shafts. These crossed helical gears are also called screw gears. Unlike the helical gears, which give a line contact between the teeth, a point teeth contact appears in the crossed helical gears. Hypoid gears (Fig. 1.5b) are spiral bevel gears, the axes of which, however, do not intersect. Hypoid gears are usually used in motor vehicles (cars and trucks), with the axis of the pinion disposed below the crown wheel. Planetary gear drive (Fig. 1.5c) is a mechanism with gears where one axis (axis of planet gear) is movable around the main axis (axis of sun gear).
1.1 Basic kinematic magnitudes of gear pairs

In this Section, the basic kinematic magnitudes of gear pairs are presented in the case of cylindrical gear pairs. However, the same is also valid for all other gear pairs, as already presented above. In all equations, index "1" is related to the pinion (smaller gear), while index "2" is related to the gear (bigger gear). Fig. 1.6a shows the cylindrical gear pair where the pinion $z_1$ is mating with the gear $z_2$. Because such a gear pair should transmit the rotational moving from the driving shaft to the driven shaft uniformly, it could be replaced with the rolling of two cylinders with the pitch diameters $d_{w1}$ and $d_{w2}$. In contact point C (pitch point), the circular velocity of both cylinders should be the same: $v_1 = v_2$. 

Figure 1.6: Mating of two gears (a) and rolling of two cylinders (b)
1.1.1 Transmission ratio and gear ratio

The transmission ratio "i" of a gear pair is the ratio between the angular velocity \( \omega \) or rotational speed \( n \) of the driving shaft and driven shaft:

\[
i = \frac{\omega_{\text{driving}}}{\omega_{\text{driven}}} = \frac{n_{\text{driving}}}{n_{\text{driven}}}
\]  

(1.1)

The gear ratio "u" of a gear pair is the ratio between the number of teeth of the gear and that of the pinion:

\[
u = \frac{z_2}{z_1}
\]  

(1.2)

It is clear from equations (1.1) and (1.2) that, in the case of the driving pinion, we have \( u = i \), while in the case of the driving gear, we have \( u = 1/i \).

1.1.2 Main rule of toothing

Figure 1.7 shows the gear pair where the pinion and gear are mating in an arbitrary mating point Y, while the pitch cylinders with diameters \( d_{w1} \) in \( d_{w2} \) are contacting in pitch point C. The pinion rotates around the axis \( O_1 \) with angular velocity \( \omega_1 \) while the gear rotates around the axis \( O_2 \) with angular velocity \( \omega_2 \). The distances of mating point Y from axis \( O_1 \) and \( O_2 \) can be designated as \( O_1Y = r_{y1} \) for the pinion and \( O_2Y = r_{y2} \) for the gear, respectively. The circular velocities in an arbitrary mating point Y (\( v_{y1} = r_{y1} \omega_1 \) and \( v_{y2} = r_{y2} \omega_2 \)) can be distributed in the directions of common normal (\( v_{ny1} \) and \( v_{ny2} \)) and common tangent (\( v_{ty1} \) and \( v_{ty2} \)). Since the mating gear flanks must be in contact without backlash and no penetration may appear, the normal velocity components must be equal, i.e. \( v_{ny1} = v_{ny2} \). It follows from triangles \( O_1T_1Y \) and \( O_2T_2Y \):

\[
\frac{v_{ny1}}{v_{y1}} = \frac{O_1T_1}{r_{y1}} \Rightarrow v_{ny1} = \frac{v_{y1}}{r_{y1}} \cdot O_1T_1 = \omega_1 \cdot O_1T_1
\]  

\[
\frac{v_{ny2}}{v_{y2}} = \frac{O_2T_2}{r_{y2}} \Rightarrow v_{ny2} = \frac{v_{y2}}{r_{y2}} \cdot O_2T_2 = \omega_2 \cdot O_2T_2
\]  

(1.3)
Figure 1.7: Mating of pinion and gear in an arbitrary mating point Y
Considering the condition \( v_{ny1} = v_{ny2} \) and the assumption that pinion is a driving part, it follows:

\[
\frac{\omega_1}{\omega_2} = \frac{O_2T_2}{O_1T_1} = i
\]

(1.4)

It is evident from Fig. 1.7 that the common normal intersects the line \( O_1O_2 \) in the pitch point \( C \) and, therefore, builds the triangles \( O_1T_1C \) and \( O_2T_2C \). Considering the relations \( O_1C = \frac{d_{w1}}{2} \) and \( O_2C = \frac{d_{w2}}{2} \) we get:

\[
\frac{O_2T_2}{O_2C} = \frac{O_1T_1}{O_1C} \Rightarrow \frac{O_2T_2}{O_1T_1} = \frac{O_2C}{O_1C} = \frac{d_{w2}}{d_{w1}} = i
\]

(1.5)

Equations (1.4) and (1.5) define the transmission ratio \( i \), which should be a constant value. This requirement is satisfied if two mating profiles (gear flanks) perform a conjugated action. It means that during the rotation of the gear pair, the common normal to the gear flanks at an arbitrary mating point \( Y \) must always intersect the pitch point \( C \) (see Fig. 1.7). This rule is known as the \textit{main rule of toothing}. Therefore, the gear teeth must be designed so as to get a constant transmission ratio during mating and then to have a conjugate action. Among the possible profiles, involute and cycloid profiles are most often used in the praxis \[1.4, 1.6\]; see details in Section 1.1.6.

1.1.3 Construction of tooth profile and path of contact

Based on the main rule of toothing, it is possible to obtain the shape of the tooth profile analytically or graphically from the given tooth profile shape of the mating gear, as well as to obtain the path of contact (the line over which the tooth profiles contact during the rolling). Fig. 1.8 shows the construction of the tooth profile of a gear for a given tooth profile of pinion and pitch diameters \( d_{w1} \) and \( d_{w2} \). For given points \( X_1, Y_1 \ldots Z_1 \) on the tooth flank of a pinion, the appropriate points \( X_2, Y_2 \ldots Z_2 \) on the tooth flank of gear can be obtained graphically considering the procedure described below for point \( X_2 \). The normal on the tooth flank of the pinion intersects the pitch circle of the pinion at a point \( X_i' \). If the pinion rotates counterclockwise so that point \( X_i' \) coincides with pitch point \( C \), point \( X_i \) moves (rotates) to point \( X \). The location of point \( X \) can be determined with the intersection of the arc through point \( X_1 \) around the axes \( O_1 \) and the arc with radius \( X_1X_i' \) around pitch point \( C \). Because the normal on the tooth flank of the pinion in point \( X \) also goes through pitch point \( C \), point \( X \) is, according to the main rule of toothing, the current mating point. At point \( X \), point \( X_1 \) on the tooth flank of the pinion comes in contact with point \( X_2 \) on the tooth flank of the gear.
Point $X_2$ in its basic position is obtained with the rotating of the gear back for arc length $X_2'C$. Considering the assumption that pitch circles roll without sliding, the arc lengths $X_1'C$ and $X_2'C$ should be the same. The position of point $X_2$ is then defined with the intersection of the arc through point $X$ around the axes $O_2$ and the arc with radius $CX$ around point $X_2'$. Considering the procedure described above, the other points on the tooth flank of the gear can be obtained. Points $X$, $Y$ … $Z$ are the current mating points and represent the locations where the tooth flank of the pinion comes in contact with the tooth flank of the gear. The connection line through all mating points $X$, $Y$ … $Z$ is called the \textit{path of contact}. 

![Figure 1.8: Construction of tooth profile and path of contact](image)

\begin{align*}
X_1'C &= X_2'C \\
X_1'X &= X_2'X_2 = CX \\
Y_1'C &= Y_2'C \\
Y_1'Y &= Y_2'Y_2 = CY \\
Z_1'C &= Z_2'C \\
Z_1'Z &= Z_2'Z_2 = CZ
\end{align*}
Fig. 1.9 shows the mating between pinion and gear in three typical mating points: starting mating point A, pitch point C and ending mating point E. The starting mating point A can be obtained with the intersection of the tip circle of the gear and the path of contact. On the other hand, the ending mating point E can be obtained with the intersection of the tip circle of the pinion and the path of contact. The line \( AE = g_{p} \) represents the length of the path of contact between the pinion and gear during their rotation. It is clear that only active parts of tooth flanks \((A_1E_1 \text{ for the pinion and } A_2E_2 \text{ for the gear})\) mating with each other.

Figure 1.9: Principle determination of the length of the path of contact

1.1.4 Transverse contact ratio

Fig. 1.10a shows the mating between pinion and gear in starting mating point A and ending mating point E. The pitch circles of pinion and gear \( d_{w1} \) and \( d_{w2} \) intersect the belonging gear flanks in points \( W_1 \) and \( W_2 \) (in starting mating point A) and points \( W'_1 \) and \( W'_2 \) (in ending mating point E). Because pitch circles roll on each other without sliding, the arc lengths \( g_{p1} = W_1W'_1 \) and \( g_{p2} = W_2W'_2 \) must be the same \( (g_{p1} = g_{p2} = g_{p}) \). The mating arcs \( g_{p1} \) and \( g_{p2} \) represent those parts of pitch circles \( d_{w1} \) and \( d_{w2} \) at which the pinion mates with the gear.
Chapter 1

Figure 1.10: Magnitudes for determination of the transverse contact ratio

a) Mating arc $g_p$, b) Pitch $p_w$

Fig. 1.10b shows the pitch $p_w$, which is defined as the arc length between two neighbouring tooth flanks on the pitch circle. For the uniform operation of the gear pair, at least two tooth pairs must always mate, which means that the mating arc $g_p$ must always be greater than pitch $p_w$ ($g_p > p_w$). This can also be expressed with the transverse contact ratio $\varepsilon_\alpha$ as follows:

$$\varepsilon_\alpha = \frac{g_p}{p_w} > 1$$  \hspace{1cm} (1.6)

1.1.5 Sliding conditions between mating gear flanks

Fig. 1.11a shows the sliding conditions in an arbitrary mating point Y before pitch point C. The circular velocities in an arbitrary mating point Y ($v_{y1} = r_{y1} \omega_1$ and $v_{y2} = r_{y2} \omega_2$), which are acting perpendicular to the lines $O_1Y$ and $O_2Y$, can be distributed in the directions of common normal ($v_{ny1}$ and $v_{ny2}$) and common tangent ($v_{ty1}$ and $v_{ty2}$). As already described in Section 1.1.2, the normal velocity components must be the same, i.e. $v_{ny1} = v_{ny2}$. Furthermore, the tangential components $v_{ty1}$ and $v_{ty2}$ can be obtained from the triangles $O_1T_1Y$ and $O_2T_2Y$, and belonging hatched triangles as follows:

$$v_{ty1} = \frac{T_1Y}{r_{y1}} \Rightarrow v_{ty1} = \frac{v_{y1}}{r_{y1}} \cdot T_1Y = \omega_1 \cdot T_1Y$$

$$v_{ty2} = \frac{T_2Y}{r_{y2}} \Rightarrow v_{ty2} = \frac{v_{y2}}{r_{y2}} \cdot T_2Y = \omega_2 \cdot T_2Y$$

(1.7)
The sliding velocities of gear flanks in an arbitrary mating point Y (v_{gy1} for a pinion and v_{gy2} for a gear) are defined as a difference between the tangential velocity components v_{ty1} and v_{ty2}:

\[ v_{gy1} = v_{ty1} - v_{ty2} = \omega_1 \cdot T_{1Y} - \omega_2 \cdot T_{2Y} \]

\[ v_{gy2} = v_{ty2} - v_{ty1} = \omega_2 \cdot T_{2Y} - \omega_1 \cdot T_{1Y} = -v_{gy1} \]  

From triangles O_1T_1C and O_2T_2C and with consideration O_1C = d_{w1}/2 and O_2C = d_{w2}/2, it follows from Fig. 1.11a:

\[ \frac{T_{1C}}{O_1C} = \frac{T_{2C}}{O_2C} \Rightarrow \frac{T_{1C}}{O_1C} = \frac{T_{2C}}{O_2C} = \frac{d_{w1}}{d_{w2}} = \frac{\omega_2}{\omega_1} \]  

With consideration T_1C=(T_1Y + YC) and T_2C=(T_2Y − YC), it follows from Eqs. (1.8) and (1.9):

\[ v_{gy1} = -(\omega_1 + \omega_2) \cdot YC \]

\[ v_{gy2} = +(\omega_2 + \omega_2) \cdot YC \]  

Because the sum (\omega_1 + \omega_2) is a constant value, it is clear from Eq. (1.10) that the sliding velocity v_{gy} only depends on the distance of point Y from the pitch point C. When an arbitrary mating point Y coincides with pitch point C, the sliding velocity equals zero (see also Fig. 1.12).
1.1.6 Involute and cycloid profiles of gear flanks

As explained in Section 1.1.2, involute and cycloid profiles are often used in the praxis when designing gear pairs. Both of these profiles correspond to the main rule of toothing and provide uniform rotational moving from the driving to the driven shaft.

Involute gears

*Involute of the circle* (shorter *involute*), also known as *evolvent of the circle*, is the most often used curve when designing gear flanks. An involute is a path described by any point on a straight line that rolls without sliding on the base circle. The graphical construction of the involute is shown schematically in Fig. 1.13.

**Figure 1.13:** Schematic construction of involute of the circle
Fig. 1.14a shows the formation of involute tooth flanks schematically. The presented gear pair consists of pinion and gear with fixed centres of rotation O₁ and O₂ and having base circles whose respective diameters are d₁b and d₂b. T₁ and T₂ are the points of tangency of the straight-line with these base circles. According to the main rule of toothing, the common normal (straight-line) on gear flanks in an arbitrary mating point Y goes through the pitch point C. It is clear that the path of contact is a part of the presented straight-line, which forms the involute profile of both gear flanks. When dₚ₁ and dₚ₂ represent the diameters of the pitch circles, α is the pressure angle of the toothing with respect to these pitch circles. If such tooth flanks of pinion and gear (involute 1 and 2 in Fig. 1.14a) are limited upwards with tip diameters dₐ₁ and dₐ₂ and down with root diameters dₕ₁ and dₕ₂, the real gear teeth can be obtained (see Fig. 1.14b).

Gears with an involute tooth profile are called involute gears and cover approximately 90% of all globally manufactured gears [1.2]. This is due to the following advantages [1.3]: the conjugate action is independent of the variation of centre distance; it allows to obtain a high accuracy grade of the gears since the standard basic rack teeth have straight-sided profiles (as well as those of the cutting tools derived from them), so they can be made as accurately as possible; a single cutting tool can generate gear wheels of a given module, with any number of teeth.
Cycloid gears

Cycloid of the circle (shorter cycloid) is a path described by any point on a rolling circle that rolls without sliding on the base circle (Fig. 1.15). Based on the position of the rolling circle regarding the base circle, three typical cycloids are known: Orthocycloid (Fig. 1.15a), Epicycloid (Fig. 1.15b) and Hypocycloid (Fig. 1.15c).

Figure 1.15: Schematic construction of cycloid of the circle
a) Orthocycloid, b) Epicycloid, c) Hypocycloid

In cycloid toothing, the profile of the tooth flank consists of an epicycloid addendum and a hypocycloid dedendum of the tooth. Fig. 1.16 shows the cycloid gear pair where the pitch circles \(d_{w1}\) and \(d_{w2}\) coincide with the base circles \(d_{b1}\) and \(d_{b2}\) (\(d_{b1} = d_{w1}\), \(d_{b2} = d_{w2}\)). According to the main rule of toothing, the common normal in an arbitrary mating point must go through pitch point C, which means that mating gear flanks should have the same rolling circle. Because of the dedendum of pinion mats with the addendum of gear, their flank profiles can be obtained with the rolling of the rolling circle 1 on the pitch circle 1 (hypocycloid 1) and on the pitch circle 2 (epicycloid 2). Similarly, the addendum profile of pinion and dedendum profile of gear can be obtained with the rolling of the rolling circle 2 on the pitch circle 1 (epicycloid 1) and the pitch circle 2 (hypocycloid 2). It is evident from Fig. 16 that the path of contact consists of rolling circle arcs which contact at pitch point C. Under the assumption that the pitch diameters \(d_{w1}\) and \(d_{w2}\) are known, the shape of tooth flanks of pinion and gear is dependent only on the radii of rolling circles \(\rho_1\) and \(\rho_2\). The best mating conditions (low sliding between mating flanks, satisfy transverse contact ratio) can be obtained considering the following guidelines [1.1]:

\[
2\rho_1 \approx 0,3\cdot d_{w1} \quad \text{or} \quad 2\rho_2 \approx 0,3\cdot d_{w2}.
\]  
(1.11)
Cycloide gears have an advantage over involute gears due to lower friction loss and are used primarily on clockwork and similar fine mechanical mechanisms. Furthermore, cycloid gears could be manufactured with a small number of teeth (see examples in Fig. 1.17).

Figure 1.17: Practical examples of cycloid gear pairs [1.4, 1.7]

a) Transformation of rotation to the translation, b) Rotors of pump
1.2 Basic magnitudes of involute gears

When describing the basic magnitudes of involute gears [1.8], they are usually related to cylindrical spur gears. However, the cognitions can also be extended to the bevel and worm gears.

Fig. 1.18 shows the basic geometrical magnitudes of the cylindrical spur gear with the number of teeth $z$. Considering the assumption that the number of teeth $z$ is the whole number and the teeth are distributed uniformly around the reference circle (pitch $p$ is a constant value), the circumference of the reference circle of diameter $d$ can be obtained as follows:

\[
\text{circumference} = z \cdot p = \pi \cdot d \quad \Rightarrow \quad \frac{p}{\pi} = \frac{d}{z} = m \quad (1.12)
\]

Module $m$ [mm] is the basic magnitude of a gear, which determines the size of the basic rack tooth profile and, thus, the size of the associated gear teeth. Modules are standardised by ISO 54 [1.9]; see Table 1.2. Preferably, the modules from Series I should be considered.

\[d_b, \text{ base diameter} \quad \quad d_c, \text{ root diameter} \quad \quad d, \text{ reference diameter} \quad \quad d_h, \text{ tip diameter} \]
\[b, \text{ facewidth} \quad \quad h, \text{ tooth depth} \quad \quad e, \text{ space width on the reference circle} \]
\[s, \text{ tooth thickness on the reference circle} \quad \quad h_a, \text{ addendum} \quad \quad h_r, \text{ dedendum} \]
\[p, \text{ pitch on the reference circle} \quad \quad p_r, \text{ root fillet radius} \]

Figure 1.18: Basic magnitudes of cylindrical spur gear
Table 1.2: Standardised modules according to ISO 54 Standard

<table>
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<tr>
<th>m [mm]</th>
<th>I</th>
<th>II</th>
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<td>3,5</td>
<td>6,5</td>
<td>12</td>
<td>25</td>
<td>50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1) Preferably, the modules from Serie I should be considered.
2) This value can be used only exceptionally.

1.2.1 Involute function

As explained in Section 1.1.6, the involute of the circle is a path described by any point on a straight-line that rolls without sliding on the base circle with radii \( r_b \) and centre O (see Fig. 1.19). On the involute, the initial point U and an arbitrary point Y are marked. The rolling straight-line through point Y touches the base circle in point TY. The angle between the lines OU and OTY is called the \( \text{rolling angle of involute} \) \( \xi_y \), while the angle between the lines OY and OTY is the \( \text{pressure angle} \) \( \alpha_y \). The difference between \( \xi_y \) and \( \alpha_y \) in radians is the \( \text{involute function} \): \( \text{inv} \ \alpha_y = \xi_y - \alpha_y \).

Figure 1.19: Magnitudes for determination of the involute function
It is evident from Fig. 1.19 that the arc length between U and TY corresponds to the part of the rolling straight-line between Y and TY. Because the arc length between U and TY also equals to \( r_b \xi_y \) (\( \xi_y \) is given in rad), while the part of the rolling straight-line between Y and TY equals to \( r_b \tan \alpha_y \), it follows (the pressure angle \( \alpha_y \) should be given in [rad]):

\[
\text{inv } \alpha_y = \xi_y - \alpha_y = \tan \alpha_y - \alpha_y \tag{1.13}
\]

The function inv\( \alpha \) can be determined by direct computation, considering the pressure angle \( \alpha \) as given. Alternatively, a special table available in the scientific literature can be used [1.10]. Determination of the pressure angle \( \alpha \) considering inv\( \alpha \), as given, is the inverse operation which needs the solution of the non-linear equation: \( \tan \alpha - \alpha - \text{inv} \alpha = 0 \).

The part of the rolling straight-line between Y and TY also represents the curvature radii of involute \( \rho_y \) in point Y. It follows from the triangle OYTY:

\[
\rho_y = r_b \cdot \tan \alpha_y = \sqrt{r_y^2 - r_b^2} \tag{1.14}
\]

### 1.2.2 Mating of the involute gear pair

Fig. 1.20 shows the mating of the involute cylindrical gear pair without the profile shift (\( x_1 = x_2 = 0 \)) in pitch point C. In this case, the reference diameters \( d_1 \) and \( d_2 \) correspond to the pitch diameters \( d_{a1} \) and \( d_{a2} \) \( (d_1 = d_{a1} \text{ and } d_2 = d_{a2}) \). Here, the centre distance can be expressed as \( a = (d_{a1} + d_{a2})/2 = (d_1 + d_2)/2 \). According to the base definition of the involute function (see Section 1.2.1), the common normale in pitch point C should touch the base circles of pinion and gear \( d_{b1} \) and \( d_{b2} \) in points \( T_1 \) and \( T_2 \). Considering the main rule of toothing (see Section 1.1.2), the common normale in an arbitrary mating point should go through pitch point C. Based on these definitions, it can be concluded that, in the case of the involute gear pairs, the path of contact is part of a straight line between the points \( T_1 \) and \( T_2 \). The length of the path of contact is defined as the length between the starting mating point A and the ending mating point E. As shown in Fig. 1.20, the starting mating point A lies at the intersection between the common normale and gear tip diameter \( d_{a2} \), while the ending mating point E lies at the intersection between the common normale and pinion tip diameter \( d_{a1} \). The angle between the common normale in tangent on the pitch circles is the pressure angle \( \alpha \).
Figure 1.20: Mating of the involute cylindrical gear pair

If the pinion rotates counterclockwise with $\omega_1$, the gear rotates clockwise with $\omega_2$ in accordance with the following equation:

$$\frac{\omega_1}{\omega_2} = \frac{d_{w2}}{d_{w1}} = \frac{d_{b2}}{d_{b1}}$$

(1.15)
1.2.3 Standard basic tooth rack

The standard basic tooth rack (Fig. 1.21) defines the characteristics common to all cylindrical gears with tooth profiles having involute geometry. Therefore, each standardised gear wheel may be considered geometrically generated by the standard basic tooth rack with the straight-line profile. This basic rack is a fictitious rack which has, in the section normal to the flanks, the standard basic rack tooth profile, which corresponds to an external gear wheel with a number of teeth \( z = \infty \) and reference diameter \( d = \infty \).

![Figure 1.21: Standard basic tooth rack according to ISO 53 [1.11]](image)

The standard basic tooth rack profile refers to a theoretical toothing without backlash. The tooth depth \( h_P \) equals \( 2.25 \, m \) (\( m \) is the module), while the working portion \( h_{WP} \) equals \( 2 \, m \). Addendum \( h_aP \) and dedendum \( h_fP \) of this profile equal \( m \) and \( 1.25 \, m \), respectively. The working straight portion of the basic rack tooth profile is interconnected with the root line by the fillet, which has the shape of a circular arc with a radius equal to \( \rho_P \). The straight lines of the standard basic tooth rack profile are inclined at the pressure angle \( \alpha_P = 20^\circ \) with respect to the axis of symmetry of the tooth. On the datum line, the tooth thickness \( s_P \) is equal to the space width \( e_P \), and both are equal to one-half of the pitch \( p \), i.e.; \( s_P = e_P = p/2 = \pi \, m/2 \). The mating rack profile is the rack tooth profile symmetrical to the standard basic rack tooth profile with respect to the datum line P-P, and displaced by half a pitch relative to it. It is evident that the higher the bottom clearance \( c_P \) (and then the greater the ratio \( c_P/m \)), the larger the fillet radius of the basic rack \( \rho_P \), with the corresponding improvement of the bending strength of the tooth. However, the actual root fillet, which is outside the active profile, can vary depending on various influences such as the profile shift, number of teeth,