Long-Term Changes in the Earth's Climate

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By

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PREFACE

Factors causing the alternation of Ice Ages on the Earth constitute one of the most intriguing mysteries in the climate change problem (Zharov, 2019). The present monograph shows that this phenomenon is caused by the oscillation of parameters of the orbital and rotational motion of the Earth. The role of this factor was consistently and with increasing certainty clarified by J.J. Smulsky in three monographs (Melnikov and Smulsky, 2009; Smulsky, 2016a; Smulsky, 2018a). The problem of the evolution of the Earth's orbital motion was posed and investigated in (Melnikov and Smulsky, 2009). The monograph (Smulsky, 2016a) presents results on the evolution of Earth's rotational motion, on the variation of Earth's insolation, and on the correlation with paleoclimate data over a period of 200 thousand years.

The monograph (Smulsky, 2018a) and the present monograph analyze the whole problem for a period of 20 million years. The matter of interest is presented in a form that will be understandable to a wide range of readers. The main results pertaining to the solution of the differential equations of Earth's orbital and rotational motions are given at the end of the book, in the Appendices.

The book begins with a visual representation of the orbital and rotational movements of the Earth (Figure 1), from which daily and annual variations in the amount of solar heat coming to the Earth's surface can be deduced. This representation also illustrates the main aspects in the evolution of those movements, namely, the precession of the Earth's orbital axis $\vec{S}$ and the precession of the Earth's rotational axis $\vec{N}$ around different directions in space represented respectively by vectors $\vec{M}$ and $\vec{M}_2$. This picture will help the reader in comprehending the various important features in the evolution of Earth's movements, which are further considered in celestial-mechanical coordinates, and which are only known to a narrow circle of specialists.

Further, the monograph analyzes the evolution of the Earth's orbital motion over different time intervals. Similarly, it discusses the evolution of Earth's rotational motion over the time intervals from 0.1 year to ten thousand years. The author compares his results with available observational data and with numerical results obtained earlier by Sharaf and Budnikova (1969), Laskar et al. (2004a), and other researchers. All the compared results proved consistent. The differences between the author's results for the inclination of the Earth's orbital plane to the Earth's equator, i.e., for obliquity, and the analogous results obtained by other researchers were became noticeable after three thousand years. The reason for those differences lies in different degrees of taking into account the Earth's rotational motion in the Astronomical Theory of Climate Change. In the previous theory, also known as the Milankovitch theory, a simplified treatment of the phenomenon of the Earth axis' precession on the basis of Poisson equations was used.

Those equations were derived from the differential equations of Earth's rotational motion with omitted second-order derivatives and omitted products of first-order derivatives. J.J. Smulsky solves the latter equations numerically without simplification. As a result, he discovered that the oscillations of the Earth's rotation axis in the new theory are seven to eight times larger in amplitude than the oscillations of this axis in the earlier theories. At the end of the book, in the Appendices, this issue is analyzed in more detail. In the following three years, the author worked hard to independently check his solution to the problem of the Earth's rotation; this problem was additionally solved using another three methods. The results were successfully confirmed. In the present monograph (see Figures 65 and 66), J.J. Smulsky explains the physical cause of the large amplitude of Earth axis' oscillations. It turned out that the normal $\vec{S}$ to the Earth's orbit and the Earth axis $\vec{N}$ (Figure 1) both precess about different directions in space. The vector $\vec{S}$ precesses around the vector $\vec{M}$ of the total angular momentum of the Solar system. In celestial mechanics, the plane normal to this vector is called the Laplace plane. In turn, the Earth's rotation axis $\vec{N}$ precesses around another vector $\vec{M}_2$ declined from the momentum vector $\vec{M}$ by an angle of 3.2°.

This difference between the precessional motion of the Earth's orbit and that of the Earth's equatorial plane could only be established based on the solution of the differential equations of Earth's rotational motion with high accuracy and over large time intervals. This is a new and important result in celestial mechanics and astronomy. It will help researchers to gain a deeper insight into the structure of the Solar system and its evolution.

After the performed analysis of the orbital and rotational movements of the Earth, graphs follow which illustrate the variation of the Earth's obliquity and insolation over a period of 200 thousand years. The insolation extrema are enumerated, the insolation periods of climate change are introduced. It is shown that the data obtained for the last 50 thousand years are consistent with known changes in the paleoclimate. It follows from here that, indeed, insolation fluctuations present the main cause of long-period climate changes.

Further, the variations of the Earth's obliquity and insolation over the intervals of 1, 5, and 20 million years are considered in succession. For comparison, graphs show the changes in these quantities as they were revealed in the previous Astronomical theory of climate change. Following the consideration of data for five-million-year periods, summary plots are given for both the Earth's insolation and its orbital and rotational elements on which the magnitude of insolation depends. Following the consideration of the data for 20 million years, the statistics for obliquity and insolation changes are analyzed.

Changes in obliquity and insolation are also presented for the contemporary epoch and for a period of one million years into the future. The last chapter deals with some additional related issues, including a comparison of insolation periods with marine isotopic stages. The author arrives at a conclusion that marine isotopic stages do not adequately reflect changes in insolation and the paleoclimate.

The basis of this book is formed by graphs arranged in 67 illustrations and with text explaining these graphs. The graphs contain much more information than one can fit in a book of such a small volume. Those graphs will be the subject of analysis for researchers working in various fields of science.

The book presents an important milestone in understanding our world and the processes occurring in it. It is recommended for students and graduate students taking courses in physics and astronomy, and also to those specializing in Earth sciences.

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INTRODUCTION

Over the past 100 years, there have been several warm and cold spells in the Earth's climate. One of these was a warm period observed in recent decades. Over larger time intervals, lasting for tens of thousands of years, more significant climate changes occurred. During those changes, thick ice sheets formed in northern areas of Europe, Asia, and America. The causes and mechanisms of such long-term climate changes are discussed in the present book.

Almost one hundred years ago, Milutin Milankovitch (Milankovitch, 1920; 1939) developed his Astronomical Theory of Climate Change, also known now as the Astronomical Theory of Ice Ages, or the Orbital Theory of Paleoclimate, or simply the Milankovitch Theory. In this theory, on the basis of just three parameters—the eccentricity e of the Earth's orbit, the angular position of perihelion $\varphi_{p\gamma}$, and the angle ε of the inclination of the orbital plane of the Earth to its equatorial plane—Earth's insolation was calculated at various latitudes.

The solutions by M. Milankovitch were consistently reproduced by several generations of his followers (Brouwer & Van Woerkom, 1950; Sharaf & Budnikova, 1969; Berger & Loutre, 1991; Laskar *et al.*, 2004a; Edvardsson *et al.*, 2002; and others). However, those authors all followed the path that has been paved over centuries in celestial-mechanics studies. We took another way (Smulsky, 2016b). Namely, we did not use the equations proposed by our predecessors; instead, we derived such equations from starting fundamental principles. Second, while deriving our equations, we tried to only employ the minimum simplifications possible. Third, the problems were solved by numerical methods, using their most accurate versions or some new specially developed techniques.

The problems to be solved while developing the Astronomical Theory of Climate Change were the problem of the orbital motion of Solar-system bodies, the problem of the Earth's rotational motion, and the problem of the Earth's insolation as dependent on the Earth's orbital and rotational elements. In solving the first and third problem, our independent research has confirmed the results obtained by our predecessors. Yet, our results concerning the Earth's rotational motion proved to be different, with the oscillation amplitude of obliquity ε being seven times greater. Such variations yield insolation changes that clearly explain the climate changes that have occurred in the past.

This book examines the actual side of the problem: how our world functions, why the observed phenomena occur, and how they evolve. The history of the problem, the various aspects of its understanding, and the contributions made by the various authors in its development are not considered.

The problems of the evolution of the Earth's orbital and rotational motions, which form the basis of the Astronomical Theory, are among the most serious challenges in contemporary science. Those problems were successfully solved, and this book presents results that can be comprehended by a wide circle of readers. This is achieved with the help of 67 graphs and drawings. This abundance of illustrations has made the book more concise and informative. Data in the figures greatly exceed the text in volume, and they will serve as the basis for further research in various fields of science.

For those who want to understand the origins of our results, to check and verify them and, maybe, even obtain them for themselves, this book provides all the necessary information in the Appendices.

In this book, there are seven chapters and six appendices. The first chapter deals with the evolution of the orbital and rotational movements of the Earth. First, we present figures and graphs that introduce the main parameters of the Earth's movements in ordinary space and in a spherical coordinate system. Then, changes in the parameters of the Earth's orbital motion over the periods of six thousand and one million years, and changes in the Earth's rotational motion over a period of one million years, are considered.

Processes proceeding on the Earth depend on the relative change of the Earth's orbit with respect to its equator. Over different time intervals from 0.1 to ten thousand years, the evolution of the parameters of the relative motion is shown. In the same chapter, solutions obtained are compared with available observational data and with the numerical results that were obtained by other authors.

The second chapter begins with a consideration of the geometric characteristics of the Earth's insolation. The program for calculating insolation components is given in Appendix D. Next, we consider the evolution of the inclination between the orbital plane of the Earth and its equatorial plane, i.e., obliquity, as well as the evolution of Earth's insolation over the last 200 thousand years. The variation of insolation over the Earth's latitude in the contemporary epoch, and in the warmest and coldest epochs, is demonstrated. Over a time interval of the last 200 thousand years, 13 insolation periods of climate change have been identified. Over the interval of the last 50 thousand years, those periods are compared with paleoclimate data, and it is shown that the main events, that is, the Holocene optimum, the penultimate and last Ice Ages, and the warm period between the mentioned glacial periods, are consistent with the identified insolation periods.

The third chapter considers the evolution of the Earth's obliquity and insolation over a period of the last one million years. Climate changes due to insolation are analyzed, and a climate gradation is introduced in the form of six climatic levels. For instance, the grades for a cold climate are as follows: a moderately cold climate, a cold climate, and an extremely cold climate. Further, the evolution of obliquity and insolation during the second, third, fourth, and fifth millions of years is considered.

Those results are summarized in graphs for a period of the last five million years. Also discussed is the evolution of the parameters of the orbital and rotational motion of the Earth over the last five million years. The evolution of the characteristics of the Earth's orbital motion over the last 100 million years is presented in Appendix B, and the evolution of the characteristics of the Earth's rotational motion over the last 20 million years can be found in Appendix G.

In Chapter Four, changes in the obliquity and insolation are presented over million-year time intervals, starting from the sixth millions of years till the 20th millions of years. In all those graphs, as well as in the previous ones, changes in the mentioned quantities according to the previous Astronomical Theory of Climate Change are given for comparison.

Here, distributions of insolation over the Earth's latitude at its most extreme epochs during the last 20 million years are compared.

In the fifth chapter, types of climates due to insolation are analyzed over the last 20 million years. Periods with climate extremes, that is, with extreme cold and extreme warm climate levels, proved to be distributed rather unevenly. Climate characteristics over million-year time intervals, that is, a calm climate, an ordinary climate, and a not calm climate, are introduced. Their statistics are given.

In the sixth chapter, changes in insolation occurring during the span of five thousand years ago till 2050 at different latitudes in the Northern Hemisphere are considered at time steps of one year. Short period oscillations of insolation with a period of 18.6 years and less are demonstrated, and reasons for the occurrence of those fluctuations are identified.

The evolution of obliquity and insolation in future epochs is considered over the periods of 200 thousand and one million years. The basic properties of the climate proved to be the same as those in the past epochs. From a comparison of the data on insolation over the period of one million years, it follows that the span of the subsequent million years will see a milder climate than that in the previous epoch.

Chapter Seven, titled “Comparisons, Supplements, and Verification”, is devoted to the consideration of some additional issues widening the application of the new Astronomical Theory of Climate Change. In the first Section, the insolation periods of paleoclimate are compared with marine isotopic stages (MIS) over a period of the last five million years. MIS periods were identified by analyzing the change in the content of heavy oxygen isotope ^{18}O in the bottom sediments of the World Ocean. As a result of this analysis, it is found that MISs do not adequately reflect paleoclimate changes.

In the second Section, the movement of the Sun in the sky at different latitudes, and at different times of the day and in different years, is considered. On this basis, the length of daylight is analyzed, including the length of the polar day and the time of its onset. The results are provided in the form of graphs and illustrations, both for the present epoch and for others. The program for calculating solar phenomena is given in Appendix E.

Section Three discusses the evidence of the results obtained. The formulation of problems in the new Astronomical theory and in previous theories is considered. In the new statement of this theory, four problems are posed and evidence is presented for each of them. The physical reason that distinguishes the new Astronomical theory from its previous version is also explained here.

The Appendices present the differential equations of Earth's orbital and rotational motions, the evolution of the Earth's orbital motion over the last 100 million years, and the evolution of the Earth's rotational motion over the last 20 million years. The precession of the Earth's axis of rotation and its oscillations in different coordinate systems are analyzed. They also consider the statistics of obliquity oscillations over a period of the last 20 million years.

The present book is intended for a wide circle of readers. It will be of interest to the researchers working in the fields of earth sciences, astronomy, mechanics, and physics. The book can also be used by students and graduate students during the preparation of their term papers and dissertations.

The results of the new Astronomical theory of climate change reported in this book are based on the solutions of the problems of the Earth's orbital and rotational motion which were obtained on the supercomputers of the Joint-Use Center "Siberian Supercomputer Center of ICM & MG, SB RAS".

This book is a result of my 30-year activities at Institute of Earth's Cryosphere, Tyumen Scientific Center of SB RAS, Federal Research Center. While completing this research, I enjoyed the friendly help of my younger colleagues, many of whom were my students. At many important stages, my sons, Leonid and Yaroslav, also strived to help me. I express my sincere gratitude to all my disciples for their kind help.

Please send all comments and suggestions concerning this book to the following address: 625026, Tyumen, Malygina Street, 86, Institute of the Earth's Cryosphere.

CHAPTER ONE

EVOLUTION OF THE EARTH'S ORBITAL AND ROTATIONAL MOVEMENTS

The Earth's movements and their changes

The Earth moves in an elliptical orbit around the Sun, which is located at the focus of the ellipse (Figure 1). The smallest distance between the Sun and the Earth in perihelion is designated as R_p, and the largest one in aphelion, as R_a. The period of the Earth's motion relative to the fixed space attached to the Solar system is P_{sd} = 365.25636042 days. The period P_{sd} is called the sidereal period of the Earth's revolution around the Sun. The orbital motion of Earth proceeds in a counterclockwise direction if the Earth's orbit is observed from the North Pole of the Earth *N*. Perpendicular to the Earth's orbital plane is the orbital axis, which is designated as $\vec{S}$.

The Earth rotates around its axis $\vec{N}$ relative to fixed space with an angular velocity $\omega_E = 7.292115 \cdot 10^{-5}$ 1/sec in counter-clockwise direction, like it does when moving in orbit. The value of ω_E is such that the Earth executes a complete rotation in 0.99726968 day. The Earth's rotation axis $\vec{N}$ is inclined to its orbital axis $\vec{S}$ at an angle equal in the contemporary epoch to $\varepsilon = 23.43°$. During the orbital motion, the Earth's rotation axis $\vec{N}$ remains fixed in space (Figure 1). That is why at two points in the orbit, on the dates of 03.20 and 09.22, the axis $\vec{N}$ becomes perpendicular to the line that connects the Earth and the Sun; in other words, in such situations, the Sun occupies a position with respect to the Earth in the equatorial plane. Therefore, in the latter case, the Southern and Northern Hemispheres become equally illuminated by the Sun, with the duration of the day being equal to the night. These points (Figure 1) are called the day of the spring (*20.03*) and autumn (*22.09*) equinoxes. At the moment of 06.21, the axis $\vec{N}$ is least inclined to the line connecting the Earth and the Sun; hence, the Northern Hemisphere is illuminated by the Sun to the greatest extent here. On the date of 12.21, the axis $\vec{N}$ turns out to be most inclined to the above-mentioned line; therefore, the Earth’s Southern Hemisphere becomes better illuminated by the Sun and, at high latitudes, a polar night sets in in Northern Hemisphere. In the above two situations of extreme angles, with the approach to these angles and departure from them taken into account, a few days pass, and so these points are called respectively the day of the summer (*21.06*) and winter (*21.12*) solstices.

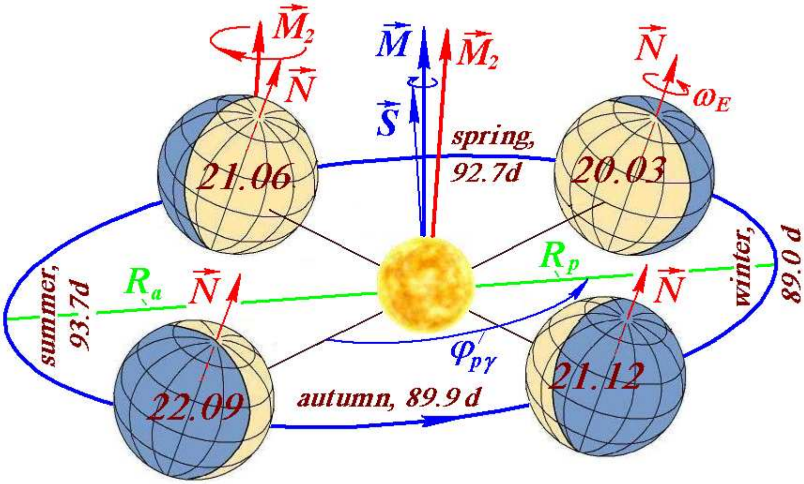

Figure 1. Position of the Earth in its orbit in 2025 by the day of spring equinox (*20.03*), summer solstice (*21.06*), autumn equinox (*22.09*), and winter solstice (*21.12*). The duration (in days) of the Earth’s motion in spring (*92.7 d*), in summer (*93.7 d*), in autumn (*89.9 d*), and winter (*89.0 d*): $\vec{N}$ is the Earth’s rotation axis and $\vec{M}_2$ is the vector around which this axis precesses with a period of 25.74 thousand years; $\vec{S}$ is the Earth’s orbital axis and $\vec{M}$ is the vector around which this axis precesses with a period of 68.7 thousand years; the value of the angular velocity ω_E corresponds to one complete revolution of the Earth around its axis performed in 0.99726968 day; $e = (R_a - R_p)/(R_a + R_p)$ is the Earth orbit's eccentricity; ε is the inclination of the rotation axis of the Earth $\vec{N}$ to its orbital axis $\vec{S}$; and $\varphi_{p\gamma}$ is the perihelion angle.

The inclination of the Earth's axis $\vec{N}$ to its orbital axis $\vec{S}$ leads to a change in light-day length both during a year and during one and the same day at different latitudes. On the day of summer solstice (*21.06*), there is a polar day on the territory from the North Pole to the Arctic Circle. Then, with decreasing latitude, the light day becomes shorter, reaching 12 hours at the equator while a polar night stands on the Earth's part below the Southern Polar Circle. On the contrary, on the day of winter solstice (*21.12*), there is a polar night on the territory from the North Pole to the Arctic Circle and then the day starts increasing. At the equator, the day length is 12 hours, and a polar day sets in on the territory below the Southern Polar Circle. When approaching the equinoxes of March 20 and September 22, the difference between the days at different latitudes diminishes, and the day length at those points at all latitudes becomes identical, and it is equal to 12 hours.

As the Earth moves in its orbit, seasons gave way to one another. The duration of the seasons is determined by the time during which the Earth moves in its orbit along certain segments of the latter. Over the section from the day of spring equinox (March 20) till the day of summer solstice (June 21), the duration of spring is 92.7 days; in the *summer* section, the summer duration is 93.7 days; in the *autumn* section, the autumn duration is 89.9 days; and in the *winter* section, the winter duration is 89.0 days.

The orbital and rotational movements of the Earth considered above determine its climate in the present epoch. However, those movements change as time passes, and the Earth's climate undergoes changes. The position of the Earth's orbit changes in space. The orbital axis $\vec{S}$ (see Figure 1) rotates or, in other words, precesses around a fixed direction in space $\vec{M}$. The precession proceeds in clockwise direction with a period of 68.7 thousand years. The Earth axis $\vec{N}$ also precesses clockwise around another direction $\vec{M}_2$, which is fixed in space. The period of the latter precession is 25.74 thousand years. Besides, the axes $\vec{S}$ and $\vec{N}$ oscillate about their own precession axis: $\vec{M}$ and $\vec{M}_2$ respectively. In addition to those movements, the shape of the orbit changes, i.e., its eccentricity varies in the range from 0 to 0.064 (the current value is 0.016). The position of the perihelion also undergoes changes. Today, the perihelion occupies a position in the *winter* section of the Earth's trajectory (Figure 1), with the winter having come to the Northern Hemisphere. Since the perihelion of the Earth's orbit rotates in counterclockwise direction with a mean period of 147 thousand years, its position in other epochs can be anywhere in the orbit.

These changes to the Earth’s orbital and rotational movements lead to a change in the Earth’s climate. Below, we will consider these changes in more detail.

Evolution of the Earth’s orbital motion

The evolution of the Earth's orbital motion is considered in a fixed coordinate system *xyz* with the origin located at the center *O* of celestial sphere *1* (Figure 2).

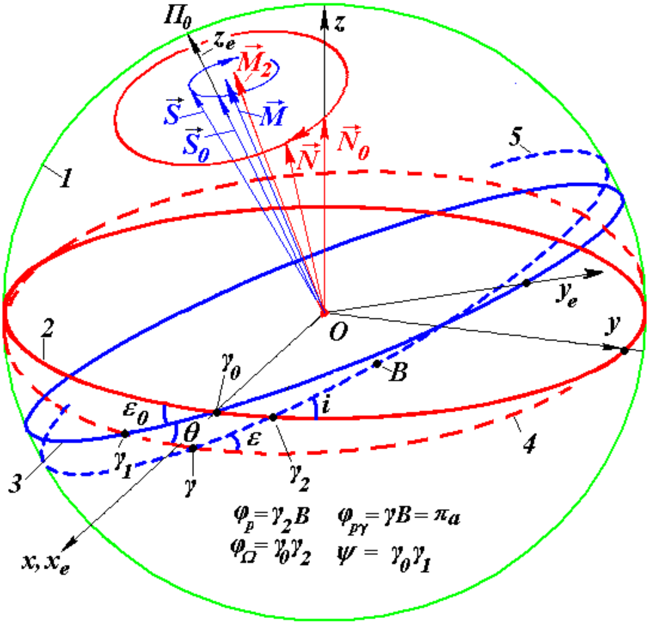

Figure 2. Parameters of the Earth’s orbit and axis in the stationary equatorial and stationary ecliptic coordinate systems (*xyz* and $x_e y_e z_e$, respectively): *1* is celestial sphere. The planes by epoch T_s: *2* the Earth’s equatorial plane and *3* the Earth’s orbital plane. The planes are shown by epoch *T*: *4* the Earth’s equatorial plane and *5* the Earth’s orbital plane. The unit vectors are as follows: $\vec{N}$ is the Earth axis’ unit vector; $\vec{S}$ is the Earth orbit’s unit vector; $\vec{M}$ is the unit vector of the Solar-system’s angular momentum; γ_0 is the vernal equinoctial point by epoch T_s; *B* is the position of perihelion on the celestial sphere; $\varphi_\Omega = \gamma_0\gamma_2$ is the angle of the ascending node of the orbit; φ_p.= $\gamma_2 B$ is the perihelion angle; and *i* is the angle of the orbital inclination.

Note that, depending on the particular problem under consideration, the point O can be located either at the center of mass of the Solar system, at the center of the Sun, or at the center of the Earth.

As a result of the interaction of Solar-system bodies, the equatorial plane of the Earth *2* and its orbital plane *3* change their positions: *2* and *3* in epoch T_s, and *4* and *5* in epoch T. Epoch T_s is considered to be the epoch of 2000.0 with the Julian-day number $JD_s = 2451545$. Since the annual motion of the Sun on the celestial sphere *1* relative to the Earth proceeds along circles *5* or *3*, the planes of those circles are also called the planes of moving and motionless ecliptics, respectively. The coordinate system xyz is related with the plane of motionless equator *2*. The moving plane of the Earth's orbit *5* is characterized by the angle $\varphi_\Omega = \gamma_0\gamma_2$, which defines the angular position of the ascending node γ_2, and by the inclination angle i.

The Earth moves around the Sun in an open trajectory similar in shape to an ellipse. At one point of the trajectory, at the perihelion, the Earth approaches the Sun to the closest distance R_p. At the opposite point, the aphelion, the Earth moves away from the Sun to the largest distance R_a. In Figure 2, the projection of the perihelion onto the celestial sphere *1* is marked with the character B, with its position being coordinated by angle $\varphi_p = \gamma_2 B$. The shape of the orbit is defined by the eccentricity

$$e = (R_a - R_p)/(R_a + R_p) \qquad (1)$$

and the orbit's semi-major axis is $a = (R_a + R_p)/2$.

The evolution of the Earth's orbital motion can be determined as a result of the solution (Smulsky, 1999; Smulsky, 2003; Grebenikov & Smulsky, 2007; Melnikov & Smulsky, 2009; Smulsky, 2012a; Smulsky, 2012b) of the differential equations of motion of Solar-system bodies (see Equations (7) in Appendix A). The points in Figure 3 represent the dynamics of the Earth's orbital elements: eccentricity e and angles φ_Ω, i, and φ_p. Over a span of six millennia (from -3 to 3 thousand years), the eccentricity e and the orbital inclination i both decrease in magnitude while the perihelion angle φ_p increases. Simultaneously, the angle of the ascending node φ_Ω at the beginning of the above interval first decreases and then increases in magnitude. In the latter case, the angle φ_Ω attains its minimum value at a time of one thousand years before 30.12.1949. Unlike other parameters, the perihelion angle φ_p varies non-uniformly.

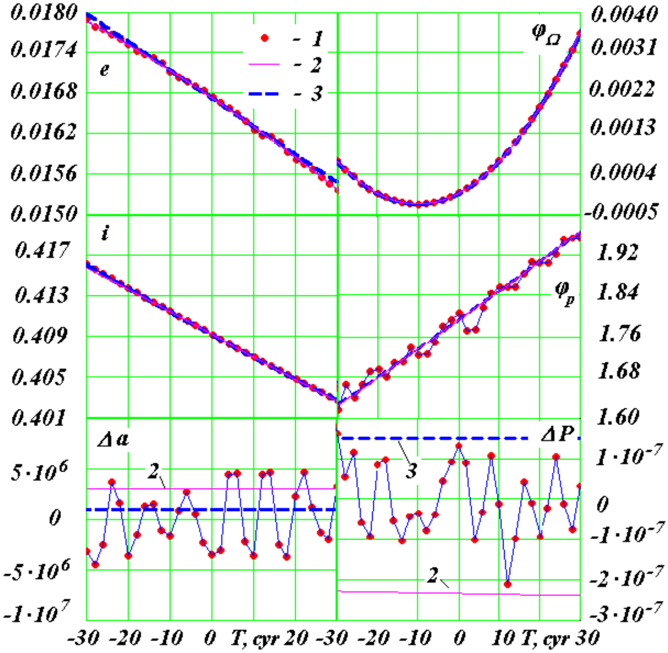

Figure 3. Secular variations of the Earth's orbit (points *1*) and the comparison with approximations of the observational data of (Newcomb, 1895) and (Simon *et al.*, 1994) (lines *2* and *3*, respectively): e is the eccentricity; i is the inclination of the orbital plane of the Earth to its equatorial plane by 2000.0; φ_Ω is the angular position of the ascending node of the orbit with respect to the x axis by the epoch of 2000; φ_p is the angular position of perihelion in the orbital plane reckoned from the ascending node γ_0; Δa and ΔP are the deviations of the semi-major axis and the orbital period, respectively, from their mean values over six thousand years; the angles are given in radians, Δa in meters, ΔP in centuries, and the time T is counted from 30.12.1949 in centuries; the interval between the points is 200 years; and 1 *cyr* is a century.

The deviations Δa and ΔP show that the semi-major axis of the Earth's orbit a and the period P of the Earth's revolution around the Sun both oscillate within some narrow intervals around the mean values of those quantities. Lines *2* and *3* in Figure 3 show the average changes of those parameters according to the observations by S. Newcomb (Newcomb, 1895) and J. L. Simon *et al.* (Simon *et al.*, 1994). From the graphs of e, i,

φ_Ω, and φ_p, it is seen that the calculated data are well supported by observations, with the ranges of observed oscillations, Δa and ΔP, being within the typical observational error (Newcomb, 1895; Simon *et al.*, 1994).

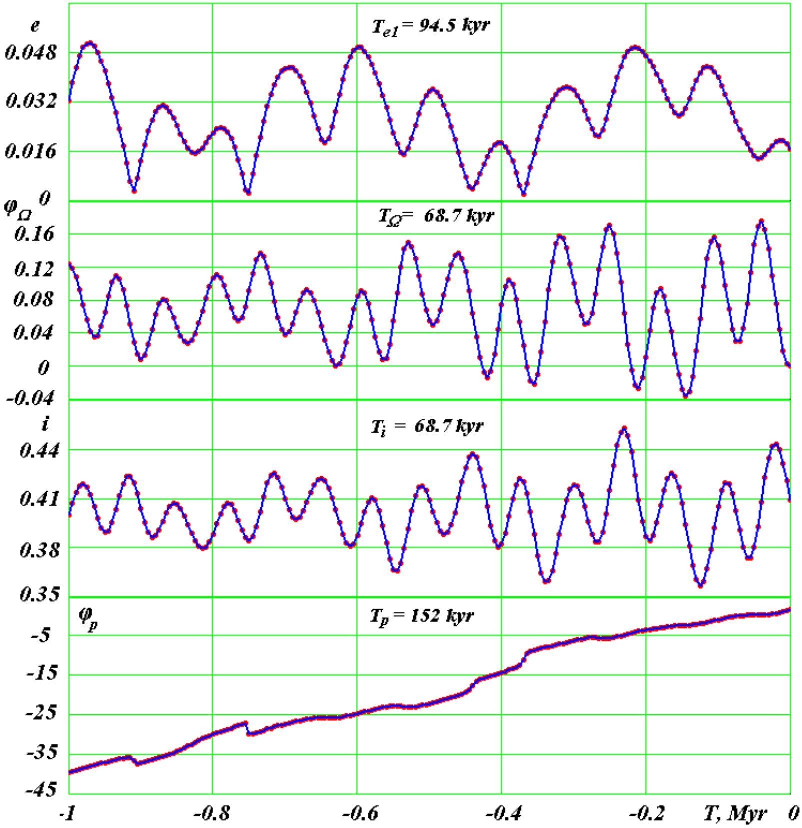

Figure 4. The evolution of the Earth's orbital elements for the last one million years: e is the eccentricity; φ_Ω is the angle of the ascending node of orbit; i is the orbit's inclination; φ_p is the perihelion angle; T is the time in million years counted from 30.12.1949; T_{e1}, T_Ω, and T_i are the least periods (in thousand years) of the oscillations of eccentricity, ascending node, and orbital inclination, respectively; and T_p is the average (over one million years) period of perihelion rotation.

Figure 4 shows the evolution of the Earth's orbital elements, e, φ_Ω, i, and φ_p, over the past one million years. The shortest period of eccentricity oscillations is T_{e1} = 94.5 thousand years, and the longer periods are T_{e2} = 413 thousand years and T_{e3} = 2.31 million years. The angle of the ascending node of the orbit φ_Ω and the inclination angle of the orbit i both oscillate with a period of $T_\Omega = T_i$ = 68.7 thousand years. The behavior of the change in angle φ_p, increasing towards the future (see Figure 3), reflects the non-uniform rotation of perihelion in a counterclockwise direction with an average (over one million years) period T_p = 152 thousand years. As seen from the graph, periods of a reverse exist, i.e., in a clockwise direction—movement of perihelion.

The oscillations of angles φ_Ω and i reflect the rotation of the orbital axis $\vec{S}$ (Figure 2) around a fixed vector $\vec{M}$ with a period of 68.7 thousand years (Smulsky, 2003). This vector is the sum of the angular momenta of all the bodies in the Solar system. The plane perpendicular to the momentum vector $\vec{M}$ is called the Laplace plane. The angle of its inclination and the angle of the ascending node with respect to the fixed equatorial plane xOy (Figure 2) are i_M = 0.401834 radian and φ_M = 0.0680946 radian, respectively. The rotation, or, in other words, the precession of the axis $\vec{S}$ around the vector $\vec{M}$, proceeds in a clockwise direction. In addition to the precession, the orbital axis $\vec{S}$ executes oscillations with respect to the vector $\vec{M}$ with periods of 97.35 thousand years, 1.164 million years, and 2.32 million years. Here, the angle between the vectors $\vec{S}$ and $\vec{M}$ never exceeds 2.94°. All these movements are reflected in the behavior of the angles φ_Ω and i in Figure 4.

The evolution of the orbit over a span of 100 million years is discussed in Appendix B. According to the presented results, the evolution of the Earth's orbit proceeds as a result of four movements (Figure 2): 1) precession of the orbital axis $\vec{S}$; 2) oscillations of the orbital axis $\vec{S}$; 3) oscillations of eccentricity e; and 4) rotation of the orbit in its plane (rotation of perihelion).

Evolution of the Earth's rotational motion

The evolution of the Earth's rotational motion is considered in a fixed coordinate system $x_e y_e z_e$ attached to the stationary plane of the Earth's orbit *3* (Figure 2). The inclination angle θ and the precession angle $\psi = \gamma_0\gamma_1$ define the position of the moving equator *4* with respect to the fixed plane of the Earth's orbit *3*. The differential equations of Earth's rotational motion (8)–(10) are given in Appendix C. As a result of the solution of those equations (Smulsky, 2011; Smulsky, 2014; Smulsky, 2016b; Smulsky, 2020), the laws of the change of the inclination angle $\theta(T)$ (Figure 2) and the angle $\psi(T)$ of precession of the equatorial plane *4* in time were obtained.

Figure 5 shows the change over a span of one million years in the difference $\Delta\psi = \psi - \psi_a$, where ψ is the precession angle, and $\psi_a = \psi_0 + \dot{\psi}_m \cdot T$ is the change of this angle at mean velocity $\dot{\psi}_m$. In the indicated time interval, the difference $\Delta\psi$ oscillates between -0.184 and 0.233 radian, with the full oscillation range being 0.417 radian.

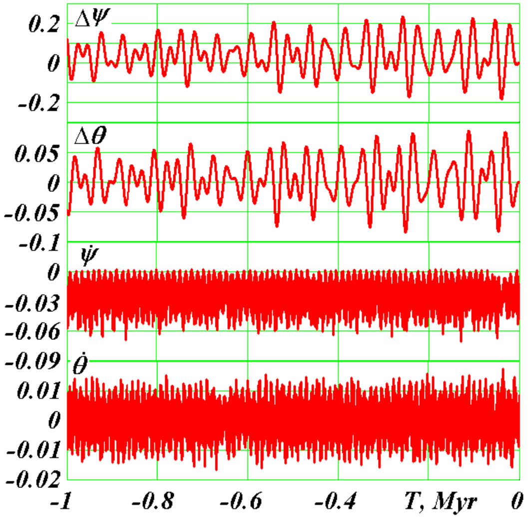

Figure 5. The evolution of the Earth's rotational motion over the last one million years. The differences for the precession angle, $\Delta\psi$, and for the inclination angle, $\Delta\theta = \theta - \theta_0$, are given in radians; the derivatives $\dot{\psi}$ and $\dot{\theta}$, in radians per one hundred years; and the time *T*, in million years.

The precession angle ψ decreases in an oscillatory manner into the future according to a linear law with an average velocity $\dot{\psi}_m = -2\cdot\pi/P_{pr}$. Here, P_{pr} = 25.738 thousand years is the average period of the Earth axis' precession for one million years (Smulsky, 2014; Smulsky, 2016b; Smulsky, 2020). The negative sign of $\dot{\psi}_m$ implies that the unit vector $\vec{N}$ of the Earth's rotation axis precesses in clockwise direction (Figure 2).

The precession of the Earth’s rotation axis $\vec{N}$ proceeds around the second stationary vector $\vec{M}_2$. The angles of the plane perpendicular to vector $\vec{M}_2$ relative to the xOy plane are as follows: the inclination angle is i_{M2} = 0.417728, and the angle of the ascending node, φ_{M2} = -0.0664662. The angle between the fixed vectors $\vec{M}_1$ and $\vec{M}_2$ is 3.201402°.

The above-mentioned difference for the inclination angle, $\Delta\theta$, is given in Figure 5 with respect to the initial value θ_0 = 0.40904645 for epoch T = 0. The value of $\Delta\theta$ oscillates similarly to $\Delta\psi$, but in a narrower range, from -0.0845 to 0.0855, so that the full oscillation range of $\Delta\theta$ is 0.17 rad. Thus, the oscillation amplitude for angle θ is 2.45 smaller than that for angle ψ. Besides, the oscillations of $\Delta\theta$ do not coincide in phase with the oscillations of $\Delta\psi$, being instead shifted along the time axis by a value of -7.5 thousand years.

As evident from Figure 5, the derivatives $\dot{\psi}$ and $\dot{\theta}$ steadily oscillate over the indicated time interval in the ranges from -0.0704 to 0.0027 and from -0.0166 to 0.0175, respectively, the oscillation amplitude of $\dot{\theta}$ being 3.8 times smaller than that of $\dot{\psi}$. The main oscillation period of both $\dot{\psi}$ and $\dot{\theta}$ is half a month, or 13.659 days. These oscillations are modulated by oscillations with larger periods: a half-year period and a period of 18.6 years. With the increasing duration of a period, the oscillation amplitude increases.

The steady oscillations of $\dot{\psi}$ and $\dot{\theta}$, and with no other trend in the variation of those derivatives, indicate a good precision of the method that was used for integrating the differential equations of rotational motion (equations [8]–[10] in Appendix C). With lesser accuracy in the integration procedure, an increase in the amplitudes emerges, and some trend may be manifested in the variation of $\dot{\psi}$ and $\dot{\theta}$.

Evolution of the orbital motion of the Earth relative to its rotational motion

The amount of heat coming to the Earth from the Sun depends on the parameters of the Earth’s orbital motion relative to the moving equatorial plane *4* (Figure 2). From the parameters of the orbital motion, i, φ_Ω, and φ_p, and from the parameters of the rotational motion, ψ and θ, one can determine the obliquity ε and the perihelion angle $\varphi_{p\gamma}$ of the moving orbital plane *5* with respect to the moving equator *4*. The spectrum of oscillations of the angle $\varphi_{p\gamma}$ is rather wide, with the oscillation of $\varphi_{p\gamma}$ contributing to the variation of angles φ_p, φ_Ω, i, ψ, and θ. The mean variation of the angle $\varphi_{p\gamma}$ follows a linear law, $\varphi_{p\gamma m} = \varphi_p - (2\pi \cdot T/P_{pr})$.

Figure 6 shows the variation of angle ε over five different time intervals In. Over the short time intervals, the variations of θ and ε are almost identical. Shown in the graphs are the main periods T_{ni} and the amplitudes of inclination-angle oscillations, θ_{ai} and ε_{a4}, proceeding with half-month period T_{n2}, half-year period T_{n3}, and a period T_{n4} = 18.6 years.

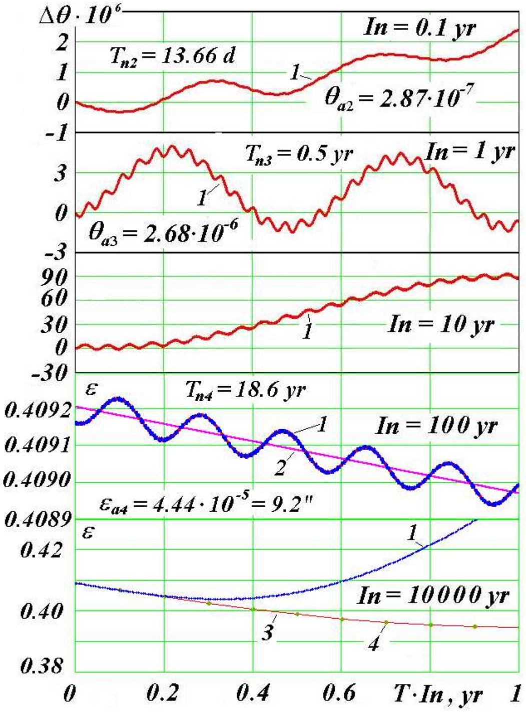

Figure 6. Dynamics of obliquity ε (in radians) during five time intervals In: yr – year; $\Delta\theta \approx \varepsilon - \varepsilon_0$; ε_0 is the inclination angle at the initial epoch of 12.30.1949. T_{n2}, T_{n3}, T_{n4} and θ_{a2}, θ_{a3}, ε_{a4} are the oscillation periods and the amplitudes of obliquity. *1*, numerical solution of (Smulsky, 2014; Smulsky, 2016a); *2*, approximation of the observational data of (Newcomb, 1895; Simon *et al.*, (1994); *3*, numerical solution of (Laskar *et al.*, 2004a); and *4*, solution by Sh.G. Sharaf and N.A. Budnikova (1969).

These oscillations are called nutational. The precession angle ψ exhibits similar oscillation periods, with the oscillation amplitudes being 2–3 times greater.

Over the interval *In* = 0.1 year, half-month oscillations are seen, and daily oscillations can be traced: over the interval of *In* = 1 year, half-year oscillations appear; over the interval *In* = 10 year, an oscillation trend with a 18.6-year period emerges; and over the interval *In* = 100 year, oscillations with the latter period prevail.

Over the interval *In* = 100 year, the calculated angle ε (line *1*) oscillates around its mean value (line *2*) according to the observational data by S. Newcomb (Newcomb, 1895; Duboshin, 1976) and by Simon *et al*. (1994). The amplitude of the oscillations with the period T_{n4} = 18.6 year and $\varepsilon_{a4} = 9.2''$ is also consistent with the reported observations. In astronomy, this amplitude is called the nutation constant.

Figure 6 shows the variation of angle ε during the time interval *In* = 10 kyr. Over larger time intervals, the evolution of angle ε will be shown in subsequent graphs. The evolution of perihelion angle $\varphi_{p\gamma}$ and the periods of revolution of the perihelion relative to the moving equatorial plane *4* or relative to the Earth’s rotation axis $\vec{N}$ (see Figure 2) will be considered below over an interval of -5 Myr.

The obtained parameters ε and $\varphi_{p\gamma}$ of the orbital motion of the Earth relative to its rotational motion, and the eccentricity *e* of the Earth’s orbit, allow one to calculate the amount of solar heat arriving at the Earth at different latitudes (Smulsky & Krotov, 2013; Smulsky & Krotov, 2014); the latter quantity is also called the Earth’s insolation *Q*. The Earth’s insolation *Q* is the amount of heat that comes from the Sun to a 1-m^2 area of the Earth’s surface during a certain period—a year or a half-year. Part of this heat is reflected from the Earth’s surface, and part is re-emitted by the Earth. Those losses of the solar heat falling onto the Earth’s surface are not taken into account in our analysis. Milutin Milankovitch (1939) proposed using the insolation for summer half-year at latitude 65° in Northern Hemisphere, Q_s^{65N}, as a reference characteristic of the Earth’s climate. As a result of our research (Smulsky, 2015), this characteristic has indeed proven itself to be a good indicator. That is why in subsequent graphs we will consider the evolution of angle ε and insolation Q_s^{65N} at the latitude 65°N.

The amount of heat Q is proportional to the solar constant J_0. Here, the value J_0 = 2 cal/(cm^2·min), which was adopted by M. Milankovitch (1936), is used. In other units, this quantity is J_0 = 83.736 kJ/(m^2·min) = 1395.6 W/m^2. As the space absolute radiometric reference (SARR), a solar-constant value J_0 = 1366.22 W/m^2 (Crommelynck *et al*., 1995) can be adopted. The insolation values Q given here can be recalculated in proportion to the values of J_0.

CHAPTER TWO

EARTH'S INSOLATION, ITS STRUCTURE, AND EVOLUTION FOR 200 KA

Geometrical characteristics of the Earth's insolation

Figure 7 shows the sky as viewed from the middle-latitude zone in Northern Hemisphere. At the center of the celestial sphere *1*, an observer M is located. The observer's horizon intersects the celestial sphere along the circle HH'. The perpendicular to the plane of the horizon intersects the celestial sphere at the point of zenith Z. The Earth's rotation axis, denoted with the Earth's angular velocity vector $\vec{\omega}_E$, intersects the celestial sphere at the point of the North Pole N. The angle φ of the arc between the vector $\vec{\omega}_E$ and the plane of the horizon is the observer's latitude. Recall that the angle of the arc of a sphere's great circle is equal to the central angle between the radii of the arc's ends so that, for instance, the arc φ is equal to $\angle HMN$.

The Sun S, as it executes its annual motion in a counterclockwise direction, draws on the celestial sphere *1* an ecliptic circle EE'. The latter circle intersects the equator circle AA' at points γ and γ'. The Sun's longitude λ is reckoned from point γ, the vernal equinox point, with the Sun receding at this point in spring. The distance between the Sun and the equator AA' is defined by declination δ.

The Earth rotates around the axis MN in a counterclockwise direction. Together with the Sun, the celestial sphere performs its daily rotation around this axis relative to the observer in a clockwise direction. Therefore, the daily movement of the Sun proceeds along the circle $S_rM_dS_s$, which is parallel to the equatorial circle AA'. At point S_r, the Sun rises over the horizon, at point M_d it arrives at noon, and at point S_s it falls over the horizon. The Sun, invisible to the observer, arrives at point M_n at midnight. The hour angle of the Sun, ω, is reckoned from the meridian that passes through the noon M_d.

It should be noted here that, in the caption to Figure 7, both the equatorial plane AA' and the ecliptic plane EE' are called moving planes because, with the passage of time, those planes change their position in space.

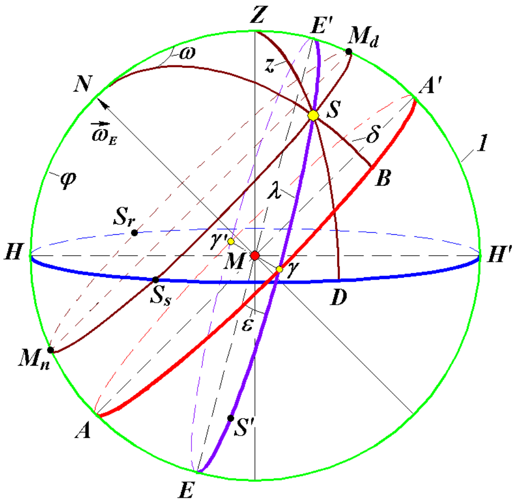

Figure 7. The main geometric characteristics of the Sun S upon irradiating a point M on the Earth’s surface: *1* is the celestial sphere; HH' is the horizon plane; N is the North Pole; AA' is the plane of moving equator; EE' is the plane of moving ecliptic, with ε being the angle between the AA' and EE' planes; Z is the zenith of the point M, with $z = \angle ZMS$ being the zenithal angle of the Sun; the arc $HN = \varphi$ is the geographical latitude of point M; $\omega = \angle MdNS$ is the Sun’s hour angle reckoned from noon M_d; and $\delta = SB$ and $\lambda = \gamma S$ are the Sun’s declination and longitude.

The length of the day is proportional to the length of arc $S_rM_dS_s$, whereas the length of the night is defined by the length of arc $S_sM_nS_r$. In the shown yearly position of Sun S, the day is longer than the night. If the Sun S is at the equator, at point γ or γ', then during the day it will move along the equatorial circle AA'. In the latter case, the day is as long as the night. If the Sun S' is in the southern part of celestial sphere, then its path under the horizon HH' is longer than the path over the horizon, and the night will be longer than the day.