Quantifying the Sustainability of Public Debt

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Time-Varying Evidence from a Developing Country

By

Cansın Kemal Can
and Necmiddin Bağdadioğlu

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To Our Mothers

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[1] This book is the published version of the author's Hacettepe University doctoral thesis.

INTRODUCTION

The developing countries like Turkey mostly have long histories of economic instability induced mostly by indebtedness and malfunctioning fiscal policies. In particular, frequent unfavourable movements in the public debt dynamics are among the most profound hindrances to economic development in those countries. Thus, to offer fiscal policy recommendations for those countries, proper scrutiny and investigation of the disruptions in the fiscal policies and underlying grounds for the public debt fluctuations are crucial for reducing the excessive upward movements in the public debt so as to establish fiscal sustainability. Therefore, it is the very purpose of this book to implement a technical appraisal of this sort to contribute to the literature by gauging the degree of Turkish public debt sustainability in a historical perspective to comprehend the past trajectory of the indebtedness and offer caveats to avoid future challenges.

Is there a practical way of quantifying the sustainability of public debt? Is the current public debt sustainable in Turkey? Has the public debt always been sustainable in Turkey during the near economic history, or are there episodes of fiscal sustainability? Based on the findings related to the past and current status of public indebtedness, are there any alarming issues that raise concerns about the future trajectory of public debt in Turkey? These are some of the questions which were attempted to answer in this book. Nevertheless, the techniques used for the analysis can be adapted to other developing country data as well.

In order to find answers to these questions in various levels of stringencies, a special form of the fiscal reaction function was estimated in different settings in this book, including the time-varying setup. Succinctly speaking, this function tests the existence of sufficient fiscal reaction in the form of primary balance generation to establish and preserve the sustainability of public debt, which constitutes a practical way of concretizing and quantifying the rather ambiguous concept of public debt sustainability.

One of the interesting findings of the time-varying analysis is that sustainability has been achieved through a fiscal transformation, but it did not exist in each sub-periods of the near economic history in Turkey. Also,

another noticeable point to make is the heightened concerns about the near future based on the worrisome pattern of parameters. The time-varying estimation indicates a downward movement in the fiscal reaction parameter, potentially jeopardising public debt sustainability in the following years.

In view of these arguments, this book's primary motivation is to enrich the academic literature on public debt sustainability in developing countries by introducing a time-varying fiscal reaction function using Turkish data.

The book is developed under three chapters;

The first chapter discusses the theoretical issues related to public debt sustainability. It sheds light on the relevant terminology and formal derivations of the model's underlying equations. This chapter also goes into the consequences of a failure in reacting to rising public debt appropriately. The chapter also covers the extent to which the government's fiscal reactions can be performed and how the timing of responses can be determined, and why they are essential. In short, the first chapter can be thought of as an overture to the fiscal reaction function.

The second chapter is devoted to the review of the related literature. The theoretical and empirical contributions to the subject are evaluated in this chapter to comprehend the current status of the literature on which the analyses of the book are grounded to make the academic contributions more distinguishable. This chapter also discusses the alternative models which could have been used instead of the fiscal reaction function, examines their pros and cons and evaluates how the fiscal reaction function outperforms them in concretizing the analysis of public debt sustainability. This comparison provides the rationale for preferring the fiscal reaction function over the other models as a public debt sustainability analysis tool.

The third chapter deals with the empirics of the subject. Along with the design and implementation of the empirical analysis, this chapter also introduces the data and its distinctive properties and a brief outline of Turkey's economic history in the last fifty years to facilitate the interpretation of the empirical findings. The chapter also briefly introduces the econometric techniques used for estimation and ends with a discussion of the empirical results.

Lastly, the final part provides an overall conclusion. It gives some cautions about the potential risks associated with Turkey's future public debt sustainability based on the empirical findings derived in the third chapter.

CHAPTER ONE

THEORETICAL ISSUES REGARDING PUBLIC DEBT SUSTAINABILITY

This chapter aims to briefly consider the key concepts and definitions pertinent to public debt sustainability along with the discussion of some other relevant topics. Some derivations of debt dynamics that guide the book's methodological part are also included within this chapter. In addition to that, the chapter also discusses the costs and dangers associated with high and unsustainable debt.

For this purpose, it is worthwhile to start with the definition of debt and continue with outlining the meaning of public debt sustainability from academic as well as pragmatic perspectives. The key concepts to be discussed include solvency, debt overhang, doom loop, adjustment fatigue, fiscal space, snowball effect, Ponzi scheme, transversality condition, intertemporal budget constraint etc.

Besides, this chapter also provides an introduction to the fiscal reaction function. The underlying algebra of the indicated function is analysed in this section which facilitates the comprehension of the results in the empirical section. Moreover, this chapter clarifies the distinction between solvency and sustainability. These two terms are frequently used interchangeably in the literature; however, solvency is only a prerequisite for sustainability, and sustainability is a far broader concept.

Finally, the chapter covers the shortcomings and limitations of the debt sustainability framework.

1.1 An Appraisal of Debt Sustainability Concept and its Importance

This section purposes to transform the rather vague concept of debt sustainability into a more concrete and clear-cut notion. Due to the opaque character of the concept, the theoretical and operational definitions of public

debt sustainability are abundant in the literature. The inconsistency between the statistical and academic definitions of the concept and the existence of various alternative definitions make real-world sustainability assessments more arduous. Taking these challenges into account, the purpose of this section is to outline the concept of debt sustainability and some other relevant terminology in order to present the background required to comprehend the econometric estimation results in the third chapter.

1.1.1 Defining the Public Debt Sustainability

Throughout economic history, borrowing has been an inevitable source of financing by enabling the countries to finance public needs beyond their budgetary capacity since they have scarce financial resources compared to the size of the society's financial needs. Long-run infrastructure investments, for instance, are mostly financed through borrowing as they require a significant amount of funding. Also, borrowing allows the cost of borrowing to be borne by the next generations who will, to a large extent, reap the benefits of such long-term investments.

However, despite its beneficial aspects, borrowing can also have impairing effects on the economy when used abusively by the fiscal authorities. The deteriorating effects of public debt get even harsher if the government fails to implement proper financial management strategies to prevent it from reaching unrepayable levels compared to governments' capacity to pay. Thus, it is quite an essential task for governments to monitor their existing borrowing and design their future financing needs wisely and thoroughly. The normative judgements about the debt profile will be misleading in the absence of descriptive monitoring of debt realizations and associated fiscal policy reactions. Yet, misleading arguments related to debt management might beget devastating outcomes in the future as they will lead to wrong policy choices. For this reason, a proper and firm grasp of the term needs to be acquired so as to implement debt sustainability appraisal appropriately.

Public debt can very briefly be described as an obligation of the government to make return payments to the debt holders. The government acquires the command over the financial source for a certain period until redemption. In exchange for this right, the government promises to make payments at a particular date in the future. These payments generally include an interest payment as well. The debt holder can resell the bonds in the secondary market or wait until the maturity date. The length of the maturity period determines the liquidity of the debt, and the shorter the maturity term, the more liquid the securities become. The array of debt portfolio is oftentimes

quite significant since the government borrows in various maturities. The government can shorten this period by repurchasing the stakes in the market or wait until the date of payoff (Buchanan & Wagner, 1967, pp. 3-4). Literally, debt is permanent and inevitable for every country. However, the real problem is not the existence of the debt but the perpetual need to roll over the debt accumulation. The difficulties regarding public debt management occur when the existing debt matures, forcing the government to find new financing sources. The exigent financing required for continuous social expenditures is a major source of the debt management problem, and if prolonged, it leads to debt sustainability concerns. Retiring or reducing the existing public debt is not a permanent solution to the debt management problem. The country will still have to acquire extra financial resources to refinance the remaining debt and make expenditures beyond the budgetary capacity.

Besides debt reduction, the government can also alter the maturities of the existing debt; however, even then, the debt-induced management problems might keep appearing in the economy. The government can indeed employ several techniques to reduce the detrimental effects of malfunctioning debt management, but the point here is that regardless of the existing amount and the maturity structure of outstanding debt, the government has to service the debt on a continuous basis. Failing to do so brings about debt sustainability problems.

Thus, rational debt management is a principal requirement for modern economies. However, outlining the causes and consequences of public debt's unsustainability and portraying the boundaries for the phenomenon is quite a challenge for theorists as it has a very multi-faceted nature (Wyplosz, 2011, p. 4). Moreover, one of the most displeasing and a rather intrinsic issue relevant to the public debt sustainability analysis is the lack of any cut and dried operational definition of debt sustainability in the literature (Chalk & Hemming, 2000, p. 3). This weakness of the concept hinders the provision of normative guidance as the scope for the judgement is considerably large (Debrun, 2015, p. 2).

In fact, the vagueness of the term debt sustainability originates from the lack of a proper definition of the term *sustainability*. Generally speaking, the concept of sustainability inherently refers to the processes which are maintained for long periods. The word "maintain" presumably reveals the intuition behind sustainability. Using this analogy, it is safe to conclude that public debt sustainability is, to a large extent, related to preserving (or maintaining) the value or the composition of debt throughout a large time

span (IMF, 2011, p. 6). According to Salsman (2017), the sustainability of the public debt is the government's capacity to borrow prudently and affordably to provide public goods and services without sacrificing its sovereignty or the rights, liberties, and prosperity of citizens.

In the literature, economists have numerous other attempts to define debt sustainability, but none of them has been universally accepted thus far. Part of the reason for the lack of a clear-cut description of public debt sustainability is that for most of the history of economic thought, the solvency rather than sustainability of the debt has been the popular term to discuss among economists. Nevertheless, even though these two terms are closely linked, solvency is not a prerequisite for sustainability. An insolvent policy might still give rise to sustainability, provided that the government is capable of altering policies accordingly. As a result, commitment and policy reversal capabilities are crucial for a solvent government to have a sustainable fiscal policy (Horne, 1991, p. 2).

Another reason why there is no clear-cut definition of public debt sustainability is the complexity within the scope of public debt profiles of modern economies. In modern economies, the magnitude of the debt is not as critical as the credibility of the country since the latter allows for a more comprehensive comparison among countries, unlike the former. Also, many economies in the world have a sizeable amount of contingent liabilities that are very difficult to measure and far above their outstanding assets. Contingent liabilities are postponed borrowing for the government, and similarly, public debt can be thought of as postponed taxation. The existence of such contingent liabilities makes debt management more difficult to control. Hence, the implicit leverages of most countries are far below their explicit leverages. As a result, effectively, the debt structure of a country appears to be far more complicated than what mainstream theories suggest.

In economic history, the theorists had primarily concentrated on the government's solvency rather than debt sustainability. The Keynesian paradigm, for instance, gave rise to radical departures from the classical view in public financial management as far as solvency was concerned. An important milestone in this respect was obviously the Great Depression in the 1930s, which led to the collapse of all the postulates of the classical view. After the recession, the income-generating properties of the debt were more prominently revealed by Keynesian economists.

The Great Depression transformed the economic view profoundly and, in a way, characterized the modern fiscal policy framework. In the aftermath of

this recession, Keynesians impinged on the Classical view from various aspects. For example, instead of perceiving the debt as a burden on the economy, the new idea was to see it as an overall asset of the entire nation. According to this new paradigm, during harsh recessions, the borrowing could reignite the economy if the borrowed funds are conveyed to the real economy through heightened public expenditures.

In addition, unlike Classics, Keynesians advocated deficits in the budget as it allowed the government to make extra spending, which could stimulate the economic prosperity in the country. Through an unbalanced budget, financial resources higher than the budgetary capacity could be diverted to the real economy by the government via their fiscal policy tools to stimulate the effective demand in the overall economy. In other words, insolvency (but not unsustainability) is in a way recommended by Keynesians, let alone refrained from (Buchanan J, 1999, pp. 94-97). Among these fiscal policy tools, debt instruments were particularly proposed to absorb excess funds during a boom and to pump liquidity into the economy during a recession (Salsman, 2017). Consequently, according to this view, the overall size of the debt is not essential for sustainability considerations. In fact, what matters is the capacity of the country to service the public debt without being highly indebted in the long run. In other words, this modern theory emphasizes the importance of the income-generating potential of the economy rather than the existing amount of public debt.

1.1.2 Why is Public Debt Sustainability Important?

One of the critical consequences of persistently high public debt is the potential vulnerability to sudden stops of financial flows as high debt leaves the country unguarded against financial risks and unfavourable economic events. Inflows of funds into the country can stop abruptly for various reasons, including shifts in global risk preferences or an adverse shock originating from international markets. Especially for developing countries that need excessive external financing, such an instant halt in the financial flow can have impairing effects as the economy effectively demands continuous international finance to roll over the existing debt. Most of the time, such stops materialize very swiftly, and the country is often caught off-guard. These sudden stops can be so severe that they might even lead to an outflow of existing capital by reducing the sovereign credit rating. The decline in this rating, in turn, might potentially have a permeating impact on the country as a whole, such as capital account restrictions, drastic cuts in public expenditures, currency crisis, banking crisis, recession and even a default (Eichengreen & Gupta, 2016, p. 3).

Along with the above-mentioned scenario, a sharp upswing in the risk premium of the economy brings about a substantial surge in the interest rate, which can crowd out the private investment. In addition to that, from a public finance perspective, when the debt level is already high and unsustainable, the government falls short of adequate fiscal space in the case of a downturn. As the debt level is currently high, the government loses the flexibility to increase the expenditures when it needs to implement fiscal policies for social purposes. The result of the above-mentioned scenario is usually a *"debt overhang"*. It is the situation where the expected tax burden arising from the existing level of debt is so high that the investors no longer have the willingness to perform new investments as they are concerned about potential default of the country (Sachs & Huzinga, 1987, p. 41). In this case, the consumers face a disincentive to increase their consumption for the same reason, and the result will be a drag on economic activities. In this case, the economic actors perceive the creditors as the sole benefiter of the upcoming stream of primary surpluses. Being the sole bearer of the costs of harsh policy adjustments, they have a high reluctance to make new investments, thereby boosting economic activity since the loss will be inevitable and the expected tax payments will be considerably increased.

Consequently, the decline in investments gives rise to lower economic growth and consequently to lower government revenues. The end result of this process is an insufficient amount of funds for the social and economic functions of the budget. Under these circumstances, the concerns about the default in the country rise with a higher risk of insolvency because the way the government will be able to finance itself becomes questionable. These circumstances create a vicious circle between low growth, low revenues, higher borrowing needs, higher risk of default, and lower investments (Krugman, 1989, p. 6).

As a result, economic activities plunge and fiscal balances deteriorate dramatically. In other words, the contagion of an economic downturn in specific sectors of the economy eventually leads to debt sustainability problems in the economy. In addition to this, the spillover effect echoes back to the private sector via channels of the higher risk premium and lower credit ratings of the sovereign. Thus, to avoid such an unpleasant sequence of economic events, monitoring the sustainability of the public debt thoroughly is crucial from the economic stability perspective.

1.2 A Prelude to Fiscal Reaction Function

Now that the importance of public debt sustainability and the associated potential problems have been clarified, the next step is to choose an appropriate tool to scrutinize the existence of fiscal sustainability. The fiscal reaction function comes into play at this point. Simply, this function gauges the strength of the reaction pursued by the fiscal authorities when the debt level moves away from its sustainable path. In other words, this function tests the strength of the primary surplus reciprocations to debt realizations.

For the purpose of comprehending the logic behind this function, the underlying mechanism of debt dynamics needs to be evaluated in detail. The objectives of exploring those mechanisms are twofold: The first objective is to reach the debt dynamics by which the debt evolves over time. The second objective is to formulate the formal solvency condition. To accomplish these objectives, another variable, namely the primary balance, is incorporated into the analysis so as to reveal the nexus between debt and deficit. This new term can be defined as the difference between non-interest revenues and non-interest expenditure. This indicator measures the influence of current fiscal policies on the indebtedness of the country. The interest reimbursements occur due to past debt and deficit realizations, and by excluding these payments from fiscal accounts, primary balance reveals the current fiscal policies in a more straightforward way (Archibald & Greenidge, 2006, p. 7).

1.2.1 How Does Debt Evolve Over Time? A Law of Movement for Public Debt and Conditions for Solvency

In this part, to accomplish the above-mentioned objective, the debt dynamics are covered in a simple yet insightful formal model for a stylized closed economy whose only type of debt is in local currency. Those dynamics constitute the background for the fiscal reaction function. Therefore, their formal derivations are important for public debt sustainability considerations.

The first component of the public debt dynamics is the intertemporal budget constraint. The central intuition behind the budget constraint is that the existing debt stock of the country mirrors its past deficits incurred. In fact, the existing stock of debt is the summation of former stock of debt, current deficit and other flows. Since there is an interest payment associated with any debt, the debt accumulation continues unless the deficit is financed with other methods. If the deficit and interest payments are serviced with new

borrowing, the country faces a vicious circle of debt and deficit (Domar, 1944, p. 799).

Formally, this vicious circle can be formulated through the notation listed below.

D_t	: Debt Stock	$D_t = D_{t-1} + \Delta D_t$
I_t	: Interest Expenditure	$I_t = i_t D_{t-1}$
R_t	: Government Revenues	
G_t	: Primary Spending	
PB_t	: Primary Balance	$PB_t = R_t - G_t$
i_t	: Nominal Interest Rate	
r_t	: Real Interest Rate (Fisher Equation)	$r_t = \left(\frac{1+i_t}{1+\pi_t}\right) - 1$
π_t	: Inflation	
$P_t Y_t$	: Nominal GDP	$P_t Y_t = (1+\pi_t)(1+g_t)P_{t-1}Y_{t-1}$
g_t	: Growth Rate	

Using the above notation, the vicious circle described by Domar (1944) can be formulated as follows:

$$G_t + iD_{t-1} - R_t + OT_t = (D_t - D_{t-1}) \qquad (1)$$

Equation (1) formally exhibits the public debt dynamics. From this equation, the nexus between budget constraint and the law of motion of debt dynamics can be extracted assuming that the debt is equal to the sum of the current debt accumulation or change in debt which is the right-hand side of the equation. Additionally, interest spending is the second component of the left-hand side of the equation, which is nothing but the interest rate times past debt. On the left, R_t represents the government revenues excluding interest earnings and G_t denotes government spending. The OT_t on the left-hand side refers to other flows of debt dynamics, including expenditures not included in G_t such as contingent liabilities and financing sources outside borrowing, including privatization and seigniorage. However, it is assumed that such flows are non-existent for the sake of simplicity for the time being.

Taking all these issues into account, Equation (1) may be rephrased as:

$$-PB_t + i_t D_{t-1} = (D_t - D_{t-1})$$

Solving for D_t;

$$D_t = (1+i_t)D_{t-1} - PB_t \qquad (2)$$

Equation (2) is a simple and powerful illustration of how debt relates to past debt and primary balance. This equation states that public debt is the summation of the past debt, the interest paid on initial debt and the primary balance. It is an expression revealing the change in debt over time. In other words, it is a law of motion for debt dynamics. This expression is essentially a reorganized budget constraint for one period. Thus, it enables to calculate the level of debt, provided that the values on the right-hand side of the equation are known. Moreover, this equation explains what happens to the debt over time. It can also be defined as a reorganized budget constraint. It relates the debt in the period $t-1$ to the debt in period t. Manipulating this expression with some forward substitution, the intertemporal budget constraint and the solvency condition can be derived.

Iterating for the first two periods:

$$D_1 = (1+i)D_0 - PB_1$$
$$D_2 = (1+i)D_1 - PB_2$$

Plugging the first row in the second one;

$$D_2 = (1+i)((1+i)D_0 - PB_1) - PB_2$$
or
$$D_2 = (1+i)^2 D_0 - (1+i)PB_1 - PB_2$$

Following this sequence and repeating the iteration for each subsequent year, the intertemporal constraint for the terminal year N can be found as;

$$D_N = (1+i)^N D_0 - \sum_{j=1}^{N} (1+i)^{(N-j)} PB_j \qquad (3)$$

This expression relates the debt level at time N to debt level at the beginning , D_0, and the primary balances between periods 1 and N. Dividing each side of the Equation (3) by $(1+i)^N$ and solving for D_0;

$$D_0 = \sum_{j=1}^{N} \left(\frac{1}{1+i}\right)^j PB_j + \left(\frac{1}{1+i}\right)^N D_N \qquad (4)$$

Equation (4) simply illustrates how the initial debt is related to the terminal debt and the discounted value of primary balances. Here, the discount factor for primary balances and debt involves the number of years until they occur and the corresponding interest rate. However, in order to reach a more intuitive expression for the analysis, a restriction on the terminal debt is needed. Otherwise, any level of primary balance will be consistent with the

above equation, and from a debt sustainability perspective, there would be no conclusion to draw.

The constraint to be imposed is the disallowance of the *Ponzi scheme,* which can basically be defined as financing the existing debt by issuing new debt in every round on a continuous basis. Obviously, such a scheme is not a solution to accumulated debt but only a temporary suspension, which is clearly not sustainable since it is literally impossible to find new investors every time new funds are needed. As the debt is effectively never paid back through primary balances, the new investors will be unwilling to finance the ever-increasing debt of the country with mounting risk of default. In the literature, this restriction is also called the *transversality condition*. This condition essentially implies the non-existence of a Ponzi scheme. By imposing the transversality condition, the Ponzi scheme is invalidated, forcing the government to deplete the existing debt in the terminal period by generating primary surpluses rather than relying solely on new investors. More formally, as time goes to infinity, the last term in Equation (4) needs to be equal to zero.

$$\lim_{N\to\infty}\left(\frac{1}{1+i}\right)^{N} D_N = 0 \qquad (5)$$

The restriction that Equation (5) imposes does not prevent the terminal debt from being positive and does not rule out increasing debt. Nevertheless, if the initial public debt is greater than zero, the economy has to run primary surpluses on a continuous basis to be solvent and should not rely on a Ponzi game for financing. In other words, the transversality condition prohibits the creation of excessive public debt without covering the initial debt and compounding the interest burden thereon (Burnside, 2005, p. 13). Thus, for the government to be solvent, the principal has to be serviced through the discounted primary surpluses occurring in the subsequent periods.

Hence, the solvency condition for the government reads:

$$D_0 = \sum_{j=1}^{\infty}\left(\frac{1}{1+i}\right)^{j} PB_j \qquad (6)$$

In the absence of the transversality condition on the intertemporal budget constraint, any array of primary balance would be consistent with the solvency condition. In other words, only after restricting the government by inhibiting the Ponzi game, an insightful solvency condition can be reached.

1.2.2 Key Conditions and Equations for Modelling Debt Sustainability: Augmenting the Formal Framework

The above-mentioned intertemporal solvency condition states that the sum of the initial debt and future stream of primary expenditures should amount to the present value of the future flow of revenues. According to this term, a government is solvent if it is able to repay its existing debt via future primary surpluses. However, this solvency condition depends on how future events will unfold. Countries with very high debt ratios might still be deemed sustainable provided that they rely solely on the governments' ability to generate a future stream of primary surpluses. The government might also opt to attract new investors whenever a new fund is needed for repayment (Ponzi Scheme). However, such a scheme gets riskier in every round and will eventually fail since it is impossible to bring new investors into the system forever.

Nevertheless, using these two terms, a simple but intuitive academic definition of public debt sustainability can be made. Formally, if the policymaker can satisfy the intertemporal solvency condition (i.e. No Ponzi Scheme), then we can infer that the public debt is sustainable from a narrow perspective. In this case, the expected future primary surpluses cover the existing accumulation of debt. It might be stated that this is a relatively soft requisite for sustainability since, according to this definition, governments with a high amount of deficit and debt burden may still be deemed sustainable. Also, intertemporal solvency largely depends on the uncertain future realizations of primary surpluses. It renders sustainability contingent on expectations about the potential course of upcoming events that might not occur in reality. In practice, the indicated higher primary balance might be achieved by a tax hike, a spending cut or by a combination of both. Another possibility is a money supply increase by the central bank to achieve an effective negative interest rate through higher inflation which is high enough to outweigh the nominal interest rate leading the actual value of the public debt to shrink (Jha, 2012, p. 21).

The proposition that the future stream of primary surpluses must match the current debt is also called a Ricardian Regime in the literature. In this case, future revenues are assumed to be equal to the existing public debt. However, in the non-Ricardian regimes, the government does not make a binding commitment to cover the current debt with the future flux of primary balances as a certain portion of the public debt will be covered by financial repression caused by money creation (Greiner & Fincke, 2015, pp. 2-6).

Nevertheless, such deliberate alterations in the debt level are not considered appropriate by many economists and the authorities. According to the IMF and the World Bank, for instance, *"Debt is sustainable if the country (or its government) does not, in the future, need to default or renegotiate or restructure its debt or make implausibly large policy adjustments"* (Hassine, 2015, p. 4). This approach states that the public debt is not sustainable if;

- A debt restructuring is required,
- The pace of accumulation of debt is swifter than the growth of the government's ability to repay (GDP)
- Some level of painful economic policy adjustments in the form of retrenchment will be needed in the future.

In modern economic understanding, however, what matters is not the level of debt but its position against the financial potential of the economy. Here, the capacity to repay is generally measured by some macroeconomic indicators such as GDP and this ratio is also called *"public leverage"*. In this sense, the debt scaled by the capacity to pay, i.e. *Debt/GDP ratio,* is a very useful and powerful tool used for public debt sustainability analysis. The path of debt/GDP ratio gives quite a hint about the sustainability of the public debt in the country. It is a very commonly used indicator of capacity to repay because, calculated by the value-added approach, it reflects the sum of all economic activities provided in the economy. Oftentimes, it proves to be a more useful measure for analysing the solvency position of the economy compared to the sole magnitude of the public debt. For this reason, despite its restricted capacity as an indicator, the debt/GDP ratio is prevalently used in the literature.

From this perspective, public debt is sustainable if the ratio of current debt level to capacity to pay is steady or declining and is not too high. If the ratio of existing debt to capacity to repay (GDP) is remaining high and/or increasing, then the debt is deemed unsustainable. Additionally, suppose the debt ratio is quite high initially. In that case, even a continuous decline in the ratio does not imply sustainability until the ratio reaches a certain level that is sufficiently low (Jha, 2012, p. 22). Thus, to add some more insight to the analysis, the debt/GDP ratio can be incorporated into the formal derivations to account for the above-mentioned economic dimensions brought by the level of public debt scaled by the capacity to pay.

The debt dynamics equation was,

$$D_t = (1 + i_t)D_{t-1} - PB_t \qquad (2)$$

Following the above-mentioned definition of the evolution of debt, this expression can be scaled by GDP using the notation P_tY_t.

$$\underbrace{\frac{D_t}{P_tY_t}}_{d_t} = \frac{(1+i_t)}{(1+\pi_t)(1+g_t)} \underbrace{\frac{D_{t-1}}{P_{t-1}Y_{t-1}}}_{d_{t-1}} - \underbrace{\frac{PB_t}{P_tY_t}}_{pb_t}$$

where $P_tY_t = (1+\pi_t)(1+g_t)P_{t-1}Y_{t-1}$

Incorporating the Fisher equation;

$$r_t = \left(\frac{1+i_t}{1+\pi_t}\right) - 1$$

the scaled debt dynamics equation above can be simplified as:

$$d_t = \frac{(1+r_t)}{(1+g_t)} d_{t-1} - pb_t \qquad (7)$$

The influential factors on debt dynamics can be analysed from different angles using this equation. A higher primary balance, for instance, lead to a lower d_t. A higher initial debt, on the other hand, might give rise to a higher d_t. On the contrary, a higher growth rate brings about a lower d_t as it improves the capacity to pay. Finally, a higher real interest rate results in higher d_t by increasing the interest expenditure payments.

According to Bohn (1998), this equation is quite significant for the stationarity of the debt series. It makes it harder for the formal tests to determine the (non)existence of a unit root. For instance, if r_t= 0.02 and $g_t = 0.04$, then, $\frac{(1+r_t)}{(1+g_t)}$ will be 0.98, which is very close to unity but still stationary. Hence, the debt dynamics equation involving r_t and g_t generates a challenge for the formal stationarity tests to produce precise results in distinguishing the sustainable debt from unsustainable ones (Burger et al., 2012, p.8).

Using an abbreviation for the term $\frac{(1+r_t)}{(1+g_t)}$ in Equation (7) and denoting the whole term as ϕ_t, the budget constraint or debt dynamics can be rewritten as follows:

$$d_t = \phi_t d_{t-1} - pb_t \qquad (8)$$

It simply denotes the current level debt/GDP as a function of its lagged values and the current primary balance scaled by the capacity to pay.

Equation (8) can also be illustrated in a phase diagram to clarify debt dynamics. On the phase diagrams below, the vertical axis depicts the debt in the current period, whereas the horizontal axis shows the debt level of the previous period. For simplicity, it is assumed that there is a linear relationship between d_t and d_{t-1} in that pb and ϕ are constant. The value of ϕ determines the explosiveness of the debt in the economy. If $\phi < 1$, as illustrated in Figure 1 below, the initial level of debt, d_0, converges to d^* where d_t and d_{t-1} are equal and remain at this equilibrium level thereafter. The d^* is the sustainable level under the primary balance level pb_t.

Nevertheless, in the explosive debt case below (Figure 2), the real interest rate is higher than the growth rate ($r > g$), and therefore for any positive level of $d_0 > d^*$ the debt/GDP ratio deviates from sustainable level unboundedly, and the speed of growth can be unexpectedly high.

Differencing Equation (8) reveals more insights about debt dynamics.

$d_t = \phi_t d_{t-1} - pb_t$ (8)

Subtracting the previous period debt d_{t-1} from both sides;

$$d_t - d_{t-1} = \left[\frac{1+r_t}{1+g_t} - 1\right] d_{t-1} - pb_t \quad \text{or,}$$
$$\Delta d_t = (\phi_t - 1) d_{t-1} - pb_t \qquad (9)$$

The momentum of debt and the governments' effort for stabilisation can be observed from Equation (9). The term in the brackets simply indicates the proportion of the debt accumulation arising due to interest reimbursements of the former period. The equation simply states that if the real growth rate is smaller than the real interest rate, then the debt is deemed explosive. In that case, the public debt automatically rises even in the absence of new borrowing. When this happens, high levels of primary surpluses (fiscal reactions) are needed for offsetting the explosiveness in debt dynamics. Hence, the magnitude of pb_t the ratio is a good indicator for detecting governments' ability to trim explosiveness in debt dynamics.