The Dynamics of Elastic Structures

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PREFACE

This collection is called “Dynamics of elastic structures” and covers a wide range of theoretical and applied issues in regions: hydroacoustics, the dynamic theory of elasticity, theory of diffraction, radiation and sound propagation in liquid, gaseous and solid (isotropic and anisotropic) media, speaker patterns, acoustic monitoring.

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CHAPTER ONE

WAVE PROCESSES IN ANISOTROPIC ELASTIC STRUCTURES. SOUND SCATTERING BY RANDOMLY ORIENTED SPHEROIDAL BODIES

A. KLESHCHEV

1.1. The Dynamic Theory of the Elasticity for the Transversely Isotropic Medium

It is well known [1] that a pulsed sound signal, like a bunch of energy, propagates at a group velocity. This circumstance forces us to use the method of imaginary sources when studying the temporal characteristics of pulsed signals scattered by various bodies placed in a plane waveguide [2 – 7]. Wherein the spectral characteristics of the pulses dealing with continuous harmonic signals can also studied using the normal wave method [8].

When studying waves in anisotropic media, the initial equations are the dynamic equilibrium of continuous medium [9 – 13]:

$$\left.\begin{aligned}
&\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}+\frac{\partial\tau_{zx}}{\partial z}=\rho\frac{\partial^2 u}{\partial t^2};\\
&\frac{\partial\sigma_y}{\partial y}+\frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\tau_{zy}}{\partial z}=\rho\frac{\partial^2 \upsilon}{\partial t^2};\\
&\frac{\partial\sigma_z}{\partial z}+\frac{\partial\tau_{xz}}{\partial x}+\frac{\partial\tau_{yz}}{\partial y}=\rho\frac{\partial^2 w}{\partial t^2},
\end{aligned}\right\}\qquad(1.1)$$

We restrict ourselves to the consideration of plane monochromatic waves [13], the general expression of the displacement vector of such a wave can be written as:

$$\vec{u} = \vec{u}^0 \cdot e^{i(\vec{k}\,\vec{r} - \omega t)}, \tag{1.2}$$

where: $\vec{u}^0$ - is the constant vector (independent of either coordinates or time), called the vector amplitude of the wave.

The displacement vector (1.2) only in that case will satisfy the equations of motion (1.1) if its real and imaginary parts individually satisfy the same equation. If the vector amplitude $\vec{u}^\circ$ is real, then:

$$\vec{u} = \vec{u}^\circ\,(cos\,\varphi + i\,sin\,\varphi) = \vec{u}' + i\vec{u}'', \tag{1.3}$$

moreover $\vec{u}' = \vec{u}^\circ \cdot cos(\vec{k}\,\vec{r} - \omega t)$ and $\vec{u}'' = \vec{u}^\circ \cdot sin(\vec{k}\,\vec{r} - \omega t)$ are real solutions of the basic equations (1.1) in the form of plane monochromatic waves. Therefore, we can always choose any of them, for example, $\vec{u}'$, as a real solution. A plane monochromatic wave (1.2) will satisfy the equations of motion (1.1) not for any parameter values $\vec{u}^\circ, \vec{k}, \omega$. We rewrite the equations of motion (1.1) in another form, using the notation of the elastic modules as components of the 4th rank tensor [5]:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = c_{ijlm} \frac{\partial^2 u_m}{\partial x_j \partial x_l} \tag{1.4}$$

$(i, j, l, m = 1, 2, 3)$

Substituting (1.2) in (1.4) and considering that $\dfrac{\partial}{\partial x_j} e^{i\vec{k}\,\vec{r}} = \dfrac{\partial}{\partial x_j} e^{ik_l x_l} = i\,k_j e^{i\vec{k}\,\vec{r}}$, we obtain:

$$\rho\,\omega^2 u_i = c_{ijlm} k_j k_l u_m \tag{1.5}$$

We introduce instead c_{ijlm} of the tensor

$$\lambda_{ijlm} = \frac{1}{\rho} c_{ijlm}\,, \tag{1.6}$$

which we will call the reduced tensor of elastic moduli.

Considering that $\vec{k} = k\,\vec{n}$ ($\vec{n}$ is the unit vector); $\vec{n}^2 = 1$; $k = |\vec{k}|$; $k_j = k\,n_j$; $c = \omega/k$, we rewrite (1.6) in the form:

$$\lambda_{ijlm} n_j n_l u_m - c^2 u_i = 0 \tag{1.7}$$

If we introduce a tensor of the second rank:

$$\Lambda = \Lambda^{\vec{n}} = \left(\Lambda_{im}\right) = \left(\lambda_{ijlm} n_j n_l\right), \tag{1.8}$$

then equation (1.7) can be written in direct form:

$$\left(\Lambda - \lambda\right)\vec{u} = 0, \tag{1.9}$$

From (1.9) it follows that the displacement vector of plane wave $\vec{u}$ is an eigenvector, and the square of the phase velocity of the wave c^2 is an eigenvalues of the tensor Λ.

Vector equation (1.9) is the main one for the theory of elastic waves in anisotropic media and is called the Christoffel equation. Solving this equation reduce to finding the eigenvectors and eigenvalues of the tensor Λ.

A real symmetric positive definite (for any directions of the wave normal) tensor Λ in the general case has three different eigenvalues $\lambda_1 = c_1^2$; $\lambda_2 = c_2^2$; $\lambda_3 = c_3^2$, each of which has its own vector that determines the direction of displacement in the wave. Therefore, in anisotropic media in the general case, for any given direction of the wave normal, three waves with different phase velocities can propagate. We will call such three waves, having a common wave normal, isonor-mal.

Transversely isotropic elastic medium is characterized by five elastic modules: $A_{11}, A_{12}, A_{13}, A_{33}, A_{44}$, and the generalized Hooke's law for such a medium is written in the form [9 - 13]:

$$\left.\begin{aligned}
\sigma_y &= A_{11}\,\varepsilon_y + A_{12}\,\varepsilon_z + A_{13}\,\varepsilon_x;\\
\sigma_z &= A_{12}\,\varepsilon_y + A_{11}\,\varepsilon_z + A_{13}\,\varepsilon_x;\\
\sigma_x &= A_{13}\left(\varepsilon_y + \varepsilon_z\right) + A_{33}\,\varepsilon_x;\\
\tau_{yx} &= A_{44}\,\gamma_{yx};\\
\tau_{zx} &= A_{44}\,\gamma_{zx};\\
\tau_{yz} &= \frac{1}{2}\left(A_{11} - A_{12}\right)\gamma_{yz},
\end{aligned}\right\} \tag{1.10}$$

where: $\varepsilon_y, \varepsilon_z, \varepsilon_x, \gamma_{yx}, \gamma_{yz}, \gamma_{zx}$ - deformation components, $\sigma_y, \sigma_z, \sigma_x, \tau_{yx}, \tau_{yz}, \tau_{zx}$ - strain tensor compo-nents.

The problem will be solved in a flat setting, i. e. the displacement vector $\vec{U}$ has only two components other than zero *U* (on the *X* axis) and *W* (on the *Z* axis) and there is no dependence on the coordinate *y*. Taking this into account, the deformation components will be equal:

$$\left.\begin{aligned} &\varepsilon_x = \frac{\partial U}{\partial x}; \quad \varepsilon_y \equiv 0; \quad \varepsilon_z = \frac{\partial W}{\partial z}; \\ &\gamma_{yx} \equiv 0; \quad \gamma_{yz} \equiv 0; \quad \gamma_{zx} = \frac{\partial U}{\partial z} + \frac{\partial W}{\partial x}, \end{aligned}\right\} \tag{1.11}$$

And Hooke's law will be simplified:

$$\left.\begin{aligned} &\sigma_y = A_{12}\,\varepsilon_z + A_{13}\,\varepsilon_x = A_{12}\frac{\partial W}{\partial z} + A_{13}\frac{\partial U}{\partial x}; \\ &\sigma_z = A_{11}\,\varepsilon_z + A_{13}\,\varepsilon_x = A_{11}\frac{\partial W}{\partial z} + A_{13}\frac{\partial U}{\partial x}; \\ &\sigma_x = A_{13}\,\varepsilon_z + A_{33}\,\varepsilon_x = A_{13}\frac{\partial W}{\partial z} + A_{33}\frac{\partial U}{\partial x}; \\ &\tau_{zx} = A_{44}\,\gamma_{zx} = A_{44}\left(\frac{\partial U}{\partial z} + \frac{\partial W}{\partial x}\right) \end{aligned}\right\} \tag{1.12}$$

The equations of dynamic equilibrium for a flat formulation takes the form:

$$\left.\begin{aligned} &\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} + \rho_1\,\omega^2 U = 0; \\ &\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \sigma_z}{\partial z} + \rho_1\,\omega^2 W = 0, \end{aligned}\right\} \tag{1.13}$$

where: ρ_1 – transverse isotropic half – space density

1.2. Diffraction Pulse Sound Signal on the Soft Prolate Spheroid Located in Plane Waveguide with the Anisotropic Elastic Bottom

We turn to a familiar problem of the diffraction of pulses on spheroidal bodies in the plane waveguide [10 – 12], retaining the upper boundary condition Dirichlet, waveguide dimensions and scatterer with respect to boundaries, replacing only ideal hard boundary on the elastic isotropic bottom. Physical parameters of the lower medium will correspond to the isotropic elastic bottom, but in their values, they will be very close to parameters of transversely-isotropic rock – a large gray siltstone [9]. The longitudinal wave velocity in this material is 4750 *m/s*, the transverse wave velocity – 2811*m/s*. When used in this case the method of imaginary sources need to enter the reflection coefficient V for the each source [14], when displaying sources relative to the upper border sources, as before [7 – 10, 12, 14, 15], will change the sign on the opposite, which corresponds to a change of phase by π .

It is known to [14], that the imaginary sources method boundary conditions are not fulfilled strictly on any of borders of the waveguide even in the case of ideal boundary conditions of Dirichlet and Neumann. For the better fulfillment of these conditions in diffraction problems [7 – 10, 12, 14, 15] were introduced imaginary scatterers by mirroring their relative waveguide boundaries. Likewise, introduce imaginary scatterers and in our problem and compare the sequence of reflected pulses [7, 8, 12] in the case of ideal borders and in presence of a hard elastic bottom in the waveguide. [14] shows that the method of imaginary sources applicable in the case where the reflection coefficient V will be a function of the angle of the incidence of the wave from a source relative to the normal to the boundary. In our case, this angle will be determined by the mutual position of the source (real or imaginary) and the scatterers (real or imaginary), where the wave falls from the source. Since the receiver is combined with a real source Q , the sequence of reflected pulses will be determined by the quantity and amplitudes of reflected signals (from different scatterers) having the same propagation time from sources to scatterers and from scatterers to the point Q. Parameters of the waveguide, the position of the real source Q (combined receiver) and the real scatterer remained unchanged compared [7, 8, 12]: $L = 1000$ *m., H* = 400 *m*., the real source Q and real scatterer are located at the depth of 200 m., the scatterer is the ideal soft prolate spheroid with the semi-axes ratio $a/b = 10$ (a = 0,279 *m.)* and its axis of a rotation is perpendicular to the plane of the figure (see Fig. 1-1). The formula for the reflection coefficient V_{0N} , where N – the number of a source, is given in

[14]. For the calculation of first four of reflected pulses, the following reflection coefficients are V_{03} in the direction of the first (real) scatterer 01, V_{05} in the direction of the second (imaginary) scatterer 02. As a result of simple calculations with the help of [10] obtain: V_{03} = 0,8423 + i 0,5390; V_{05} = 0,8423 + i 0,5390.

Coefficients have turned complex, which means the total internal reflection at the boundary liquid – hard elastic bottom, therefore all three coefficients are equal 1,0 parts of the first two coefficients are close to +1,0, which is typical for the boundary liquid – absolutely hard bottom. The resulting sequence of calculations of first four reflected pulses is shown in Fig. 1-2. We compare them to the sequence in Fig. 1-3 for ideal boundaries [7, 8, 12]: 1st and

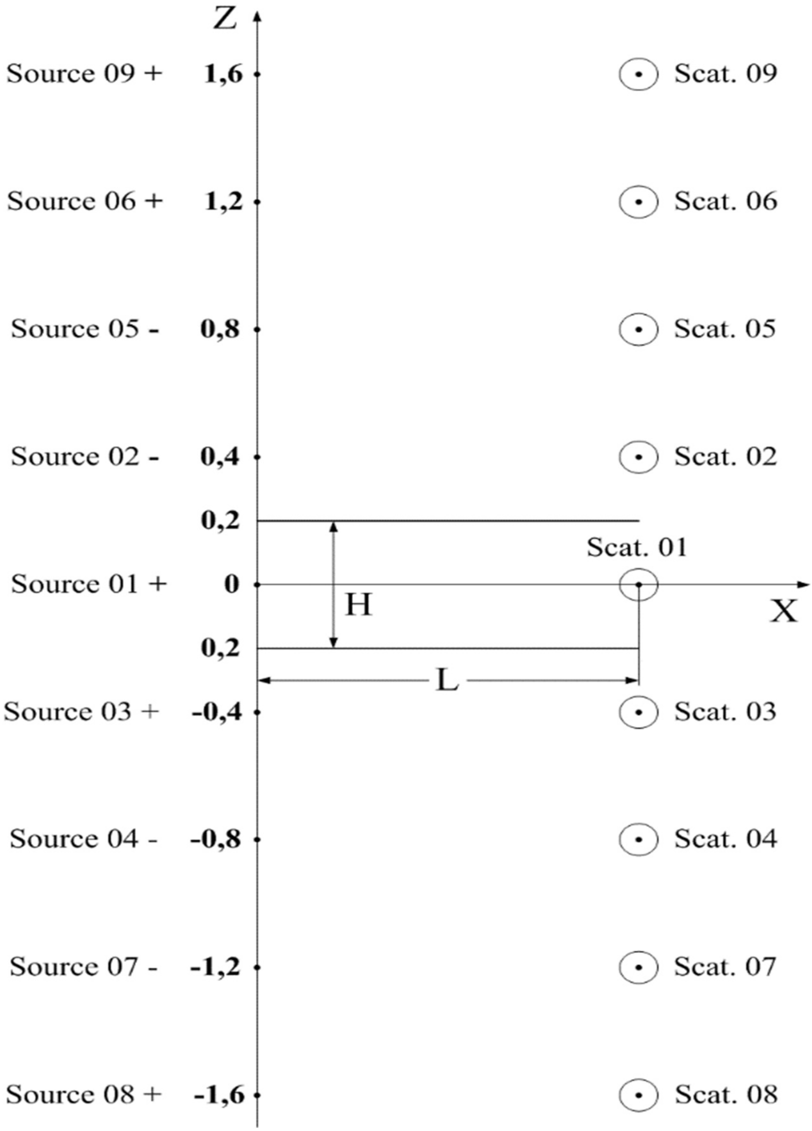

Figure 1-1: The mutual disposition of the pulse point-sources and scatterers in the plane waveguide

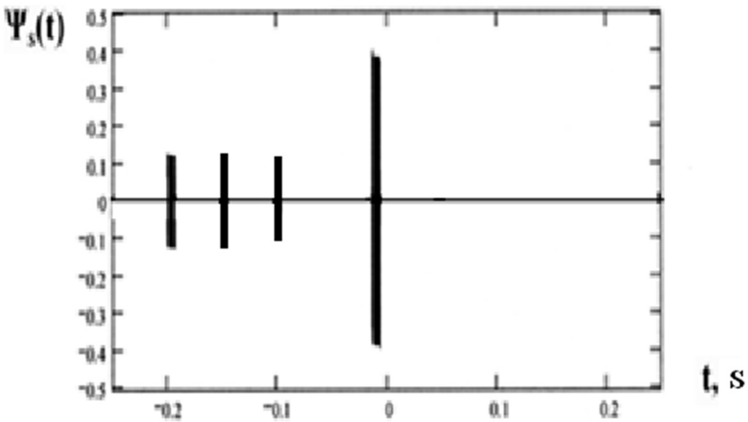

Figure 1-2: The normalized series of first four reflected impulses in the waveguide with the anisotropic elastic bottom

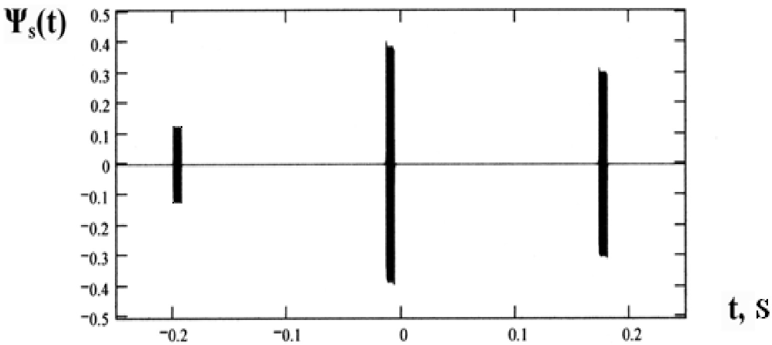

Figure 1-3: The normalized series of first three reflected impulses with the harmonic filling in the point Q

4th pulses 1-2 are identical with first and second pulses of Fig. 1-3, as for for 2nd and 3rd pulses in Fig. 1-2 in the case of ideal boundaries and symmetrical location of real a source and a scatterer relatively of boundaries of the waveguide, they are compensated each reflected pulses, i. e. 2nd and 3rd pulses (see Fig. 1-2) show the difference in sequences of reflected pulses when replacing an absolutely hard bottom on an elastic hard bottom.

A similar pattern is observed for anisotropic bottom, such as silicon, in wich quasi-longitudinal wave velocity of about 8300 *m/s* and quasi-transverse wave velocity of about 5700 *m/s*, with the second quasi-transverse wave do not occur because of the problem statement [12]. Because of the high velocities of quasi – longitudinal and quasi – transverse waves total internal reflection effect at the anisotropic bottom manifest itself even more strongly than the isotropic bottom.

In the first part of the review we investigate an interaction of a scatterer and an interface between media, it is shown that a main role in this is played by interference effects. The second part of the review is devoted to a study of a spectrum of a scattered field of an ideal prolate spheroid placed in an underwater sound channel with non – reflecting boundaries. In the third part of the review is determined the effect of a bottom structure on a series of pulses, reflected from a spheroidal body located in a plane waveguide.

As a result of the research we can draw three conclusions:

1) in studying propagation and diffraction of pulse signals in a plane waveguide need to use the method of imaginary sources as pulses like bundles of energy spread to any directions (including and along the axis of the waveguide) with the group velocity does not exceed the sound velocity, namely the group velocity based the method of imaginary sources;
2) replacing the hard elastic bottom on the absolutely hard bottom is acceptable to those sources (real and imaginary) from which waves in the fall to the hard elastic bottom try total internal reflection;
3) we have adopted the model of image sources and image scatterers is quite acceptable (due to internal reflection), at least, for first four calculated reflected pulses in a plane waveguide with hard elastic bottom.

1.3. The Dynamic Theory of the Elasticity for the Orthotropic Medium

Based at the use of the theory of the elasticity for the anisotropic medium and with the help of the hypothesis of the thin shells is determined the characteristic equation for wave numbers of elastic waves in the thin orthotropic cylindrical shell. Let's consider the infinite thin orthotropic cylind-rical shell. The harmonic elastic wave is spread along axis *Z*, that is the axis of the symmetry of the second order. The orthotropic elastic medium is characterized by the nine elastic modules [9, 11, 16, 17]: A_{11}, A_{12}, A_{13}, A_{22}, A_{23}, A_{33}, A_{44}, A_{55}, A_{66}. Hooke's generalized law for an orthotropic body is written in the form [9]:

$$\left.\begin{aligned}
\sigma_r &= A_{11}\varepsilon_r + A_{12}\varepsilon_\varphi + A_{13}\varepsilon_z\,;\\
\sigma_\varphi &= A_{12}\varepsilon_r + A_{22}\varepsilon_\varphi + A_{23}\varepsilon_z\,;\\
\sigma_z &= A_{13}\varepsilon_r + A_{23}\varepsilon_\varphi + A_{33}\varepsilon_z\,;\\
\tau_{\varphi z} &= A_{44}\,\gamma_{\varphi z}\,; \quad \tau_{rz} = A_{55}\,\gamma_{rz}\,;\\
\tau_{r\varphi} &= A_{66}\,\gamma_{r\varphi}\,,
\end{aligned}\right\} \tag{1.14}$$

where: $\sigma_r, \sigma_\varphi, \sigma_z, \tau_{\varphi z}, \tau_{rz}, \tau_{r\varphi}$ - stress tensor components, $\varepsilon_r, \varepsilon_\varphi, \varepsilon_z, \gamma_{rz}, \gamma_{\varphi z}, \gamma_{r\varphi}$ - strain components, which in turn are equal [16, 17]:

$$\left.\begin{aligned}
&\varepsilon_r = \frac{\partial U_r}{\partial r};\; \varepsilon_\varphi = \frac{1}{r}\frac{\partial U_\varphi}{\partial \varphi} + \frac{U_r}{r};\; \varepsilon_z = \frac{\partial U_z}{\partial z};\\
&\gamma_{\varphi z} = \frac{1}{r}\frac{\partial U_z}{\partial \varphi} + \frac{\partial U_\varphi}{\partial z};\; \gamma_{rz} = \frac{\partial U_r}{\partial z} + \frac{\partial U_z}{\partial r};\\
&\gamma_{r\varphi} = \frac{\partial U_\varphi}{\partial r} - \frac{U_\varphi}{r} + \frac{1}{r}\frac{\partial U_r}{\partial \varphi}.
\end{aligned}\right\} \tag{1.15}$$

Dynamic equilibrium equations in a circular cylindrical coordinate system have the form [16, 17]:

$$\left.\begin{aligned}
&\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\varphi}}{\partial \varphi} + \frac{\partial \tau_{rz}}{\partial z} + \frac{1}{r}\left(\sigma_r - \sigma_\varphi\right) + \rho\,\omega^2 U_r = 0;\\
&\frac{\partial \tau_{r\varphi}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_\varphi}{\partial \varphi} + \frac{\partial \tau_{\varphi z}}{\partial z} + \frac{1}{r}2\,\tau_{r\varphi} + \rho\,\omega^2 U_\varphi = 0;\\
&\frac{\partial \tau_{rz}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\varphi z}}{\partial \varphi} + \frac{\partial \sigma_z}{\partial z} + \frac{1}{r}\tau_{rz} + \rho\,\omega^2 U_z = 0,
\end{aligned}\right\} \tag{1.16}$$

where:

$$\left.\begin{aligned}
&\sigma_r = A_{11}\frac{\partial U_r}{\partial r} + \frac{A_{12}}{r}\frac{\partial U_\varphi}{\partial \varphi} + \frac{A_{12}}{r}U_r + A_{13}\frac{\partial U_z}{\partial z};\\
&\sigma_\varphi = A_{12}\frac{\partial U_r}{\partial r} + \frac{A_{22}}{r}\frac{\partial U_\varphi}{\partial \varphi} + \frac{A_{22}}{r}U_r + A_{23}\frac{\partial U_z}{\partial z};\\
&\sigma_z = A_{13}\frac{\partial U_r}{\partial r} + \frac{A_{23}}{r}\frac{\partial U_\varphi}{\partial \varphi} + \frac{A_{23}}{r}U_r + A_{33}\frac{\partial U_z}{\partial z};\\
&\tau_{\varphi z} = \frac{A_{44}}{r}\frac{\partial U_z}{\partial \varphi} + A_{44}\frac{\partial U_\varphi}{\partial z};\ \tau_{rz} = A_{55}\frac{\partial U_r}{\partial z} + A_{55}\frac{\partial U_z}{\partial r};\\
&\tau_{r\varphi} = A_{66}\frac{\partial U_\varphi}{\partial r} - \frac{A_{66}}{r}U_\varphi + \frac{A_{66}}{r}\frac{\partial U_r}{\partial \varphi}.
\end{aligned}\right\} \tag{1.17}$$

1.4. Phase Velocities of Elastic Waves in a Thin Orthotropic Cylindrical Shell

The components of the displacement vector in the elastic wave $\vec{U}\left(U_r, U_\varphi, U_z\right)$ traveling along the axis Z, we clean in the form of the following decompositions [16, 17]:

$$\left.\begin{aligned}
&U_r = e^{ikz}\sum_{m=0}^{\infty} cos(m\varphi)U_m(r);\\
&U_\varphi = e^{ikz}\sum_{m=1}^{\infty} sin(m\varphi)V_m(r);\\
&U_z = e^{ikz}\sum_{m=0}^{\infty} cos(m\varphi)W_m(r);
\end{aligned}\right\} \tag{1.18}$$

where k – desired wave number of elastic wave.

Substituting (1.17) in (1.18) we obtain the equations of dynamic equilibrium in displace-ments [16, 17]:

$$\frac{\partial^2 U_r}{\partial r^2} + \frac{a_1}{r}\frac{\partial^2 U_\varphi}{\partial\varphi\,\partial r} + a_2\frac{\partial^2 U_z}{\partial z\,\partial r} + \frac{a_3}{r}\frac{\partial^2 U_\varphi}{\partial\varphi\,\partial r} - \frac{a_3}{r^2}\frac{\partial U_\varphi}{\partial\varphi} +$$
$$+\frac{a_3}{r^2}\frac{\partial^2 U_r}{\partial\varphi^2} + a_4\frac{\partial^2 U_r}{\partial z^2} + a_4\frac{\partial^2 U_z}{\partial r\,\partial z} + \frac{1}{r}\frac{\partial U_r}{\partial r} + \frac{a_2}{r}\frac{\partial U_z}{\partial z} - \quad (1.19)$$
$$-\frac{a_5}{r^2}\frac{\partial U_\varphi}{\partial\varphi} - \frac{a_5}{r^2}U_r - \frac{a_6}{r}\frac{\partial U_z}{\partial z} + a_7 U_r = 0,$$

where:

$$a_1 = \frac{A_{12}}{A_{11}};\ a_2 = \frac{A_{13}}{A_{11}};\ a_3 = \frac{A_{66}}{A_{11}};\ a_4 = \frac{A_{55}}{A_{11}};\ a_5 = \frac{A_{22}}{A_{11}};\ a_6 = \frac{A_{23}}{A_{11}};\ a_7 = \frac{\rho\omega^2}{A_{11}}.$$

$$\frac{\partial^2 U_\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial^2 U_r}{\partial\varphi\,\partial r} + \frac{a_8}{r}\frac{\partial^2 U_r}{\partial r\,\partial\varphi} + \frac{a_9}{r}\frac{\partial^2 U_\varphi}{\partial\varphi^2} +$$
$$+\frac{a_9}{r}\frac{\partial U_r}{\partial\varphi} + \frac{a_{10}}{r}\frac{\partial^2 U_z}{\partial z\,\partial\varphi} + \frac{a_{11}}{r}\frac{\partial^2 U_z}{\partial\varphi\,\partial z} + a_{11}\frac{\partial^2 U_\varphi}{\partial z^2} + \quad (1.20)$$
$$+\frac{1}{r}\frac{\partial U_\varphi}{\partial r} - \frac{1}{r^2}U_\varphi + \frac{1}{r^2}\frac{\partial U_r}{\partial\varphi} + a_{12}U_\varphi = 0,$$

where:

$$a_8 = \frac{A_{12}}{A_{66}};\ a_9 = \frac{A_{22}}{A_{66}};\ a_{10} = \frac{A_{23}}{A_{66}};\ a_{11} = \frac{A_{44}}{A_{66}};\ a_{12} = \frac{\rho\omega^2}{A_{66}}.$$

$$\frac{\partial^2 U_r}{\partial z\,\partial r} + \frac{\partial^2 U_z}{\partial r^2} + \frac{a_{13}}{r^2}\frac{\partial^2 U_z}{\partial\varphi^2} + \frac{a_{13}}{r}\frac{\partial^2 U_\varphi}{\partial z\,\partial\varphi} +$$
$$+a_{14}\frac{\partial^2 U_r}{\partial r\,\partial z} + a_{16}\frac{\partial^2 U_z}{\partial z^2} + \frac{a_{15}}{r}\frac{\partial^2 U_\varphi}{\partial\varphi\,\partial z} + \quad (1.21)$$
$$+\frac{a_{15}}{r}\frac{\partial U_r}{\partial z} + \frac{1}{r}\frac{\partial U_r}{\partial z} + \frac{1}{r}\frac{\partial U_z}{\partial r} + a_{17}U_z = 0,$$

where:

$$a_{13} = \frac{A_{44}}{A_{55}};\ a_{14} = \frac{A_{13}}{A_{55}};\ a_{15} = \frac{A_{23}}{A_{55}};\ a_{16} = \frac{A_{33}}{A_{55}};\ a_{17} = \frac{\rho\omega^2}{A_{55}}.$$

If now the components of the displacement vector $\vec{U}\left(U_r, U_\varphi, U_z\right)$ are replaced by their expansions (1.18) and substituted (1.18) in (1.19)÷(1.21), then for radial functions $U_m\left(r\right)$, $V_m\left(r\right)$ $W_m\left(r\right)$ we obtain the following equations [16, 17]:

$$\frac{\partial^2 U_m}{\partial r^2} + \frac{m(a_1 + a_3)}{r}\frac{\partial V_m}{\partial r} + (a_2 + a_4)ik\frac{\partial W_m}{\partial r} - \frac{m(a_3 + a_5)}{r^2}V_m -$$
$$-\frac{a_3}{r^2}m^2U_m - a_4k^2U_m + \frac{1}{r}\frac{\partial U_m}{\partial r} + \frac{(a_2 - a_6)}{r}ikW_m + \left(a_7 - \frac{a_5}{r^2}\right)U_m = 0;$$
(1.22)

$$\frac{\partial^2 V_m}{\partial r^2} + \frac{m(1 + a_8)}{r}\frac{\partial U_m}{\partial r} - \frac{m^2 a_9}{r}V_m - \frac{m a_9}{r}U_m + \frac{1}{r}\frac{\partial V_m}{\partial r} -$$
$$-\frac{m(a_{10} + a_{11})}{r}ikW_m - a_{11}k^2V_m + \left(a_{12} - \frac{1}{r^2}\right)V_m - \frac{m}{r^2}U_m = 0;$$
(1.23)

$$(1 + a_{14})ik\frac{\partial U_m}{\partial r} + \frac{\partial^2 W_m}{\partial r^2} - \frac{m^2 a_{13}}{r^2}W_m + \frac{mik}{r}(a_{13} + a_{15})V_m -$$
$$-k^2 a_{16}W_m - ik\frac{(1 + a_{15})}{r}U_m + \frac{1}{r}\frac{\partial W_m}{\partial r} + a_{17}W_m = 0;$$
(1.24)

Boundary conditions on the absence of stresses $\sigma_r, \tau_{r\varphi}, \tau_{rz}$ on the external ($r = a$) and internal ($r = b$) surfaces of the shell are added to the equations (1.22)÷(1.24) [16, 17]:

$$\frac{\partial U_m}{\partial r} + \frac{a_1}{r}mV_m + \frac{a_1}{r}U_m + a_2 ikW\Big|_{\substack{r=a\\ r=b}} = 0; \quad (1.25)$$

$$\frac{\partial V_m}{\partial r} - \frac{1}{r} V_m - \frac{m}{r} U_m \Big|_{\substack{r=a \\ r=b}} = 0; \tag{1.26}$$

$$i k U_m + \frac{\partial W_m}{\partial r} \Bigg|_{\substack{r=a \\ r=b}} = 0; \tag{1.27}$$

For thin shells it is advisable to use expansion in degrees $\xi = z/R_0$, where $R_0 = \dfrac{a+b}{2}$ - is the average radius, and $z = r - R_0$ - is the coordinate measured from the middle surface [16 – 19]:

$$\left.\begin{aligned} U_m(r) &= \sum_{n=0}^{N_1} x_n \xi^n ; \\ V_m(r) &= \sum_{n=0}^{N_1} y_n \xi^n ; \\ W_m(r) &= \sum_{n=0}^{N_1} z_n \xi^n . \end{aligned}\right\} \tag{1.28}$$

We substitute decompositions (1.28) in the boundary conditions (1.25)÷(1.27), as a result we get 6 equations for $3(N_1 + 1)$ unknown coefficients x_n, y_n, z_n [16, 17]:

$$\begin{aligned} & R_0^{-1} \sum_{n=0}^{N_1} x_n n(\xi_1)^{n-1} + a_1 m \left(R_0 + \frac{h}{2} \right)^{-1} \sum_{n=0}^{N_1} y_n n(\xi_1)^n + \\ & + a_1 \left(R_0 + \frac{h}{2} \right)^{-1} \sum_{n=0}^{N_1} x_n n(\xi_1)^n + a_2 i k \sum_{n=0}^{N_1} z_n n(\xi_1)^n = 0, \end{aligned} \tag{1.29}$$

where:

$$\xi_1 = \frac{a - R_0}{R_0}; \; h = a - b;$$

$$R_0^{-1}\sum_{n=0}^{N_1} x_n n(-\xi_1)^{n-1} + a_1 m\left(R_0 - \frac{h}{2}\right)^{-1}\sum_{n=0}^{N_1} y_n n(-\xi_1)^n +$$
$$+ a_1\left(R_0 - \frac{h}{2}\right)^{-1}\sum_{n=0}^{N_1} x_n n(-\xi_1)^n + a_2 i k\sum_{n=0}^{N_1} z_n n(-\xi_1)^n = 0, \quad (1.30)$$

$$R_0^{-1}\sum_{n=0}^{N_1} y_n n(\xi_1)^{n-1} - \left(R_0 + \frac{h}{2}\right)^{-1}\sum_{n=0}^{N_1} y_n (\xi_1)^n -$$
$$- m\left(R_0 + \frac{h}{2}\right)^{-1}\sum_{n=0}^{N_1} x_n (\xi_1)^n = 0, \quad (1.31)$$

$$R_0^{-1}\sum_{n=0}^{N_1} y_n n(-\xi_1)^{n-1} - \left(R_0 - \frac{h}{2}\right)^{-1}\sum_{n=0}^{N_1} y_n (-\xi_1)^n -$$
$$- m\left(R_0 - \frac{h}{2}\right)^{-1}\sum_{n=0}^{N_1} x_n (-\xi_1)^n = 0, \quad (1.32)$$

$$i k\sum_{n=0}^{N_1} x_n n(\xi_1)^n + R_0^{-1}\sum_{n=0}^{N_1} z_n n(\xi_1)^{n-1} = 0, \quad (1.33)$$

$$i k\sum_{n=0}^{N_1} x_n n(-\xi_1)^n + R_0^{-1}\sum_{n=0}^{N_1} z_n n(-\xi_1)^{n-1} = 0. \quad (1.34)$$

The rest equations can be substitution of decompositions (1.28) in equations (1.22)÷(1.24) and by equated of coefficients at the fellow parameter ξ [16, 17]:

$$\begin{aligned}&x_{n+2}(n+2)(n+1)+x_{n+1}(n+1)(2n+1)+\\&+x_n\left[n^2-m^2a_3-a_5+R_0^2\left(a_7-k^2a_4\right)\right]+\\&+x_{n-1}2R_0^2\left(a_7-k^2a_4\right)+x_{n-2}R_0^2\left(a_7-k^2a_4\right)+\\&+y_{n+1}(n+1)m(a_1+a_3)+y_nm\left[n(a_1+a_3)-a_3-a_5\right]+\\&+z_{n+1}ik(a_2+a_4)R_0(n+1)+z_nR_0ik\left[2n(a_2+a_4)+a_2-a_6\right]+\\&+z_{n-1}R_0ik\left[(n-1)(a_2+a_4)+a_2-a_6\right]=0;\end{aligned} \tag{1.35}$$

$$\begin{aligned}&-x_{n+1}m(1+a_8)(n+1)-x_nm\left[n(1+a_8)+R_0a_9+1\right]-x_{n-1}R_0ma_9+\\&+y_{n+2}(n+2)(n+1)+y_{n+1}(2n+1)(n+1)+y_n\left[n^2-R_0m^2a_9-1+\right.\\&\left.+R_0^2\left(a_{12}-k^2a_{11}\right)\right]+y_{n-1}R_0\left(2R_0a_{12}-m^2a_9-2R_0a_{11}k^2\right)+y_{n-2}\times\\&\times R_0^2\left(a_{12}-a_{11}k^2\right)-z_nR_0ikm(a_{10}+a_{11})-z_{n-1}R_0ikm(a_{10}+a_{11})=0;\end{aligned} \tag{1.36}$$

$$\begin{aligned}&x_{n+1}ik(1+a_{14})R_0(n+1)+x_nikR_0\left[2n(1+a_{14})+1+a_{15}\right]+\\&+x_{n-1}ikR_0\left[(n-1)(1+a_{14})+1+a_{15}\right]+y_nikmR_0(a_{15}+a_{13})+\\&+y_{n-1}ikmR_0(a_{15}+a_{13})+z_{n+2}(n+1)(n+2)+z_{n+1}(n+1)(2n+1)+\\&+z_n\left[n^2-m^2a_{13}+R_0^2\left(a_{17}-k^2a_{16}\right)\right]+z_{n-1}2R_0^2\left(a_{17}-k^2a_{16}\right)+\\&+z_{n-2}R_0^2\left(a_{17}-k^2a_{16}\right)=0,\end{aligned} \tag{1.37}$$

It is necessary to use $3(N_1+1)-6$ of equations (1.35)÷(1.37), but for $n=0$ and $n=1$ coefficients with negative indexes are equal to zero. Then in common with equations (1.29)÷(1.34) the homogeneous system of $3(N_1+1)$ equations relative to coefficients x_n, y_n, z_n is formed. The we expand the determinant of this system and let this determinant is equal zero we receive the characteristic equation for wave numbers *k* of elastic waves with the mode *m* in the orthotropic cylindrical shell.

1.5. Sound Scattering by Randomly Oriented Spheroidal Bodies

Let the spheroidal scatterer is found in the centre of the sector group of the hydrophones and he is orientated accident, only hydrophone is appeared by the sound radiator (Fig. 1-4), he radiates the harmonic sound signal of the frequency ω .

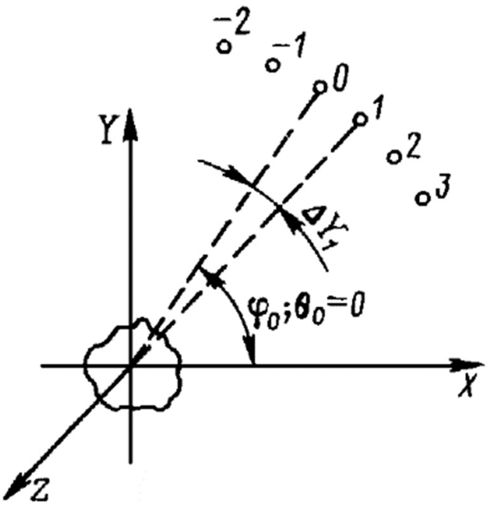

Figure 1-4: The orientation of the scatterer relatively of the source and the hydrophones

The reflected of the body the signal is the multiplicative signal, consisting of the accidental and determinate parts. The determinate component can separate from the accidental component. We will suppose the accidental component by fixed and homogeneous [20, 21]. The angular correlation function of the fixed and homogeneous accidental field has the following appearance [22, 23]:

$$R_{p_1,p_2}(\theta,\tau) = \overline{p^2(t)}\,\overline{R_{1,2}(\theta,\tau)} = R_{1,2}(\theta,\tau), \tag{1.38}$$

where: $\overline{p^2(t)}-$ the middle square of the sound pressure; $\tau-$ the temporal interval between the signals (by us $\tau = 0$); the line over the function $\overline{R_{1,2}(\theta,\tau)}$ means the averaging.

The angular space correlation function $R_{1,2}(\Delta\theta_p;0^0)$, setting the communication between hydrophones 0 and P, has appearance:

$$R_{1,2}(\Delta\theta_p;0^0) = \int_{\sigma_1}^{\sigma_2} D(\eta_0;\eta_0)D^*[\eta_0;\cos(\theta_0 - \Delta\theta_p)]d\eta_0, \quad (1.39)$$

where: $\Delta\theta_p = p\Delta\theta; \theta_0 = \arccos\eta_0 -$ the angle of the illumination; σ_1 and $\sigma_2 -$ the cosines of the critical angles of the illumination; $D(\eta_0;\eta_0)-$ the angular characteristic of the sound reflection in the direction at the source; * - means the complex conjugate quantity.

At the base (1.39) were calculated the angular correlation functions of the soft and hard, prolate and oblate spheroids. For the soft prolate spheroid the angular characteristic of the scattering will have appearance [15]:

$$D(\eta_0;\eta_0) = -\frac{2}{ik}\sum_{m=0}^{\infty}\sum_{n\geq m}^{\infty}(-1)^n \varepsilon_m \overline{S}_{m,n}(C,\eta_0)\overline{S}_{m,n}(C,\eta_0)\cos m\varphi \frac{R_{m,n}^{(1)}(C,\xi_0)}{R_{m,n}^{(3)}(C,\xi_0)};$$

(1.40)

$$D[\eta_0;\cos(\theta_0-\Delta\theta_p)] = -\frac{2}{ik}\sum_{m=0}^{\infty}\sum_{n\geq m}^{\infty}(-1)^n \varepsilon_m \overline{S}_{m,n}(C,\eta_0)\overline{S}_{m,n}[C,\cos(\theta_0-\Delta\theta_p)]\frac{R_{m,n}^{(1)}(C,\xi_0)}{R_{m,n}^{(3)}(C,\xi_0)},$$

(1.41)

where: $\varepsilon_m = 1$ by $m = 0$; $\varepsilon_m = 2$ by $m \neq 0$; $\overline{S}_{m,n}(C,\eta_0)-$ the angular spheroidal function; $R_{m,n}^{(1)}(C,\xi_0)$ and $R_{m,n}^{(3)}(C,\xi_0)-$ the radial spheroidal functions first and third genders; $C = kh_0 -$ the wave size, $h_0 -$ semi-focal distance, $k-$ the wave number in the liquid; $\xi_0 -$ the radial spheroidal coordinate of the scatterer.

The correlation functions were calculated in the limits from 0^0 to 30^0 with the step $h_{\Delta\theta} = 2,5^0$ for the wave sizes $C = 10,0$ and $C = 3,1$. The results of the calculations of the moduluses of the angular correlation functions of the sound scattering by the hard $[\left|R_{1,2}(\Delta\theta_p;0^0)\right|]$ (the curve 1) and soft $[\left|R_{1,2}(\Delta\theta_p;0^0)\right|]$ (the curve 2) oblate spheroids for $C = 10,0$ are introduced at Fig. 1-5 (the angles of the intensive scattering).

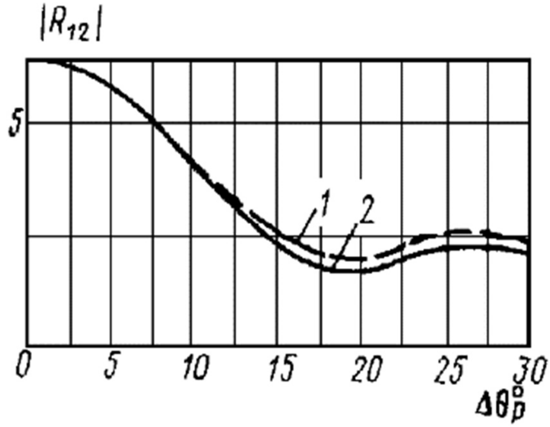

Figure 1-5: The modules of the angular correlation functions of the oblate spheroids

For the examination of the hypothesis against homogeneous of the accident scattered field (in limits of the sector $0^0 - 30^0$) for the prolate spheroid were calculated the correlation functions $R'_{1,2}(\Delta\theta_p;\Delta\theta_2;0^0)$ and $R_{1,2}(\Delta\theta_p;\Delta\theta_2;0^0)$ relatively the zero hydrophone $(\Delta\theta_2 = 0^0)$ and hydro-phones with the angular position $\Delta\theta_2 = 10^0, 20^0, 30^0.$ At Fig. 1-6 are shown the modules of the angular correlation functions of the prolate spheroid (soft and hard) by $\Delta\theta_2 = 0^0[\left|R'_{1,2}(\Delta\theta_p;0^0)\right|]$ (the curve 1) and $[\left|R_{1,2}(\Delta\theta_p;0^0)\right|$ (the curve 2) and by $\Delta\theta_2 = 30^0[\left|R'_{1,2}(\Delta\theta_p;\Delta\theta_2;0^0)\right|]$

(the curve 3) and $[\left|R_{1,2}(\Delta\theta_p;\Delta\theta_2;0^0)\right|]$ (the curve 4) for the angles of the intensive scattering by $C = 10,0$.

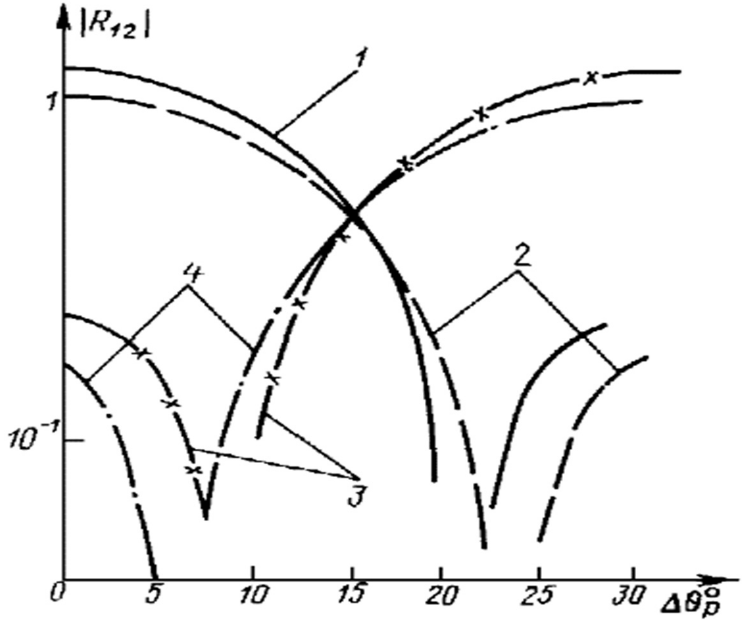

Figure 1-6: The modules of the angular correlation functions of the prolate spheroids (by the angles of the intensive scattering)

How may see from the plots, the modules of the angular correlation functions equally depend on the angle $\Delta\theta_p$, only the maximums of this functions displace at the angle 30^0, what confirms of the hypothesis against of the homogeneous of the accident scattered field in this case. For the angles of the weak scattering the hypothesis against of the homogeneous of the accident scattered field don't appear just already by $\Delta\theta_2 = 10^0$. From the curves of the Fig. 1-7, relating to this condition, we see, what by the transition from $\Delta\theta_2 = 0^0$ by $\Delta\theta_2 = 10^0$ is reversed the form of the angular correlation functions.