Fundamentals of the Theory of Planning the Search for Space Objects

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^{By} Stanislav S. Veniaminov

Cambridge Scholars Publishing



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This book first published 2020

Cambridge Scholars Publishing

Lady Stephenson Library, Newcastle upon Tyne, NE6 2PA, UK

British Library Cataloguing in Publication Data A catalogue record for this book is available from the British Library

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ISBN (10): 1-5275-4601-2 ISBN (13): 978-1-5275-4601-1

TABLE OF CONTENTS

Abstract	vii
Reviewer's Foreword	viii
Author's Preface	. xi
Abbreviations	xiv
Chapter One	1
Introduction	
Chapter Two The essence of the problem	8
 2.1 On available a-priori orbital information on the sought-for space object 2.2 Basic definitions	8 . 17 . 18 . 26
Chapter Three Cycling objects and the search for them	. 30
3.1 Initial concepts and notation	. 31
3.2 The equivalence curves	. 32
3.3 The case of search for an object by one censor within a cycle	. 37
3.4 On an important specific form of the motion law	. 41
3.5 Optimum planning of search for an object with the help	
of a censor group	. 48
3.6 Optimum planning of search for a group of objects with the help)
of a censor group	. 33
3.7 Optimum planning of search by the criterion of detection time	. 60
5.6 The case of unknown probability distribution of object's position	11 71
3.9 The influence of an error in the object motion period upon	. / 1
nlanning its search	74
pramming no search	/ -

Chapter Four
Detection of high orbital space objects
4.1 Specifics of the problem
4.2 The search by argument of latitude
4.3 Treatment of the basic concepts of the equivalence curves
principle and its practical use
4.4 Transition to discrete search plans
4.5 Presentation of real observation conditions in the <i>tu</i> -plane
4.6 Assumption of presence of errors in all orbit parameters
4.7 The search by the criterion of detection time
4.8 The search for an object (a spacecraft) after its maneuver 108
4.9 Accounting for the error in orbital period
4.10 The search for a GEO satellite after its orbit correction 117
4.11 The search for a group of space objects with the help of a
group of sensors
4.12 On the efficiency of planning procedure
Chapter Five 140
The most general statement and solution of the search problem
5.1 The space object current position uncertainty domain and its
temporal transformation
5.2 Basic notions, concepts, definitions, and notations
5.3 Temporal transformation of a point in the picture plane
5.4 Generalization of the equivalence principle of the search plan
elements
5.5 The search plan degradation during its realization
5.6 Available ways for mitigation of the search plan degradation
phenomenon negative consequences
5.7 Planning the search for a space object with partial compensation
of the search plan degradation negative consequences
5.8 Generalization of the ephemeris notion for acquisition of a weak
intelligence signal
5.9 Challenging aspects of the developed theory impact
on the modernization of space surveillance facilities
and reorganization of their management 171
-
Chapter Six 177
Conclusion
References

ABSTRACT

The basic tenets of a new theory of planning the search for space objects, using imprecise a-priori information on their orbit parameters, are developed. This is a second edition, corrected, remade, and enlarged. The main notion and the stem of the theory is the principle of equivalence of the search plan elements for different times. The basis of the theory includes the set-theoretical presentation of the space object current position uncertainty domain and its dynamics, the formulation in these terms of the equivalence principle of the search plan elements for different times. All the main search situations taking place in the space surveillance practice were considered and investigated on the base of this theory. The most search efficiency can be achieved in terms of this theory, first of all, for narrowangle and narrow-beam sensors and for a weak signal from the sought-for space object. The phenomenon of the search plan degradation in the process of its realization is revealed and mathematically rigorously substantiated and described. This gave the theory a certain finality and completeness. This phenomenon entails the appearance of errors of the 1st and 2nd kind (the formation of gaps between the elements of the plan which implies the probability of the loss of the sought-for space object and the excessive reviewing of the already viewed areas of the space object current position uncertainty domain). A number of constructive ways to mitigate and compensate for the negative consequences of this phenomenon in the construction of search plans are suggested. As a result, the developed theory made it possible to construct mathematically well-founded search plans for space objects (including optimal ones) practically for any character of errors of the initial data on the state vector of the sought-for space object. Some qualitative and quantitative estimates of the proposed search methodology efficiency (obtained both theoretically and by practical realization) are given.

REVIEWER'S FOREWORD

The topic of the monograph by the well-known in Russia and abroad expert in this sphere – the search for an SO by rough a priori data on its orbit parameters – is very urgent and timely both at present and in prospect of space research. Namely the imperfectness of usually applied search methods in optical range largely accounts for the limited content of the high orbit SO dynamic catalogs. The latter are referred to as concerning the main products of Space Surveillance Systems (SSS) functioning in the USA, Russia and, in prospect, of the European SSS which is under way. The most important aspect of the problem, in our opinion, is detecting and cataloging SOs of small sizes (microsatellites (cubesats) and elements of space debris) and those having faint brightness.

The problem of searching for SOs with faint brightness is versatile. In this monograph, one of the key moments of the problem is considered – the foundations of the theory of planning the search for an SO with the help of narrow-angle sensors, using imprecise a priori information, which helps not only just reduce the search region but also optimize the process of sounding and sensing this region.

The appearance of this book is very timely, more so because the results presented in it are a pioneering contribution to the field of space research concerned. The significance of them was highly appreciated by the American Association for the Advancement of Science with the international award for 2005 (for the first time for Russians).

Hitherto, such a theory with so high degree of mathematical substantiation did not exist.

A decisive author's choice in his approach to construction of the theory was his suggestion of the set-theoretical representation of the SO current position uncertainty domain and temporal structural transformation of the latter. Namely thanks to such an approach, the success was achieved in suppression of the 1st and 2nd kinds of the search planning errors.

For constructing the theory and practical methods, the principle of equivalence of the search plan elements for different times was introduced in the monograph which has a fundamental importance for solving the problem at hand.

As a very important practical application of the theory, in the monograph there was developed a particular (though having important and highly effective applied significance) conception of planning the search for an SO for the case of primary growth of positional errors down track.

Of special importance in the author's investigations are his discovery and strict mathematical formulation of the phenomenon of the search plan degradation throughout its realization. It is essential that this phenomenon is not only theoretically investigated in detail but also there were worded and formalized some mitigating and compensatory devices and constructive recommendations convenient for use by astronomers-observers in their practice.

From the outset of observations of SOs, a problem of detecting smallsized SOs by imprecise ephemerides arose. At present, this problem became yet more important on account of the necessity of monitoring the near-Earth space with respect to elements of space debris and maneuvering SOs for which propagation of their expected positions is connected with substantial uncertainty. The theory and related practical methods of planning the search for SOs suggested in the monograph raise hope that the progress will be gained in this field because they show efficiency in detection of faint signals.

The idea of accumulating the intelligence signal energy in one point of the sensor's receiver is well known and has been used for a long time in the practice of astronomic observations. However, application of this device was successful only when the state vector was provided with a very high accuracy or through the laborious sorting out of lots and lots of possible state vectors. In this monograph, as an important author's achievement, there developed possible ways for providing the appropriate compensation of the relative motion of the intelligence signal and the receiver for the case of imprecise data on the state vector as well.

So, the work considered in this reference is not only just a monograph, including a compilation of well-known approaches and methods. It contains a lot of new scientific results oriented to enhancing the efficiency of space surveillance and clearing the way for new capabilities in the field of observational astronomy (first of all, we mean the phenomenon of degradation of the search plan throughout its realization).

The author's long-term experience in space surveillance and his participation in the creation of the Russian Space Surveillance System stipulated the wide and profound scope of the problem investigated, proximity of the results to serving the needs of concrete and, chiefly, urgent tasks of space surveillance practice. In fact, all the important search situations present in the space surveillance practice are considered.

Many scientific results obtained in the monograph (in particular, the search methods in an algorithmic form) were implemented in real acting

systems as regular dedicated programs. The positive experiences of their operation were repeatedly presented and discussed at several international workshops and conferences (the US/Russian space surveillance workshop, the European Conference on Space Debris and others) and were published in their Proceedings. The principal theses and theoretical propositions of the monograph have been discussed and approved at two scientific seminars in the Institute of Astronomy of Russian Academy of Sciences.

Compared with the author's earlier publication on this topic, the modern edition is considerably expanded, the theory looks more complete, the presentation of the material has become more perfect, didactic, and available for mastering. A number of inaccuracies and misprints have been corrected, and some suggestions of readers have been taken into account. In fact, this is a new independent work.

One could mention some demerits of the monograph, but they do not touch the scientific content and scientific substantiation of the theories presented in it.

We believe advisable to publish the monograph under consideration as a scientific edition in the series of mathematics, astronomy, space surveillance, observations of artificial Earth satellites.

> Dr. Sci. in math. and phys. Lydia V. Rykhlova Dr. Sci. in math. and phys. Alexander V. Bagrov

AUTHOR'S PREFACE

In recent years, in connection with the worldwide trend towards miniaturization of spacecraft, the continuing contamination of near-Earth space (NES), and the growth of the small-sized fraction of space debris, the problem of search for space objects (especially small and weaklycontrasting) has become much more acute and even more urgent.

At the forefront of space researchers experiencing the pressure of this problem are astronomers-observers. At the same time, all specialists involved in space activities, developers, operators, and owners of space assets are interested in its effective solution.

This theory and the monograph describing it were conceived as a result of the observers' numerous unsuccessful attempts to find (using traditional methods) a number of lost objects, characterized by rough available data about their orbits.

This monograph appeared to be a finale of the author's researches in the field of space surveillance and inventory of space objects performed during about the last 40 years. The main results were obtained in the process of the author's participation in creation and perfection of the Russian space surveillance system. Some of them were developed in the frame of the US/Russian Space Surveillance Workshops which exist since 1994. And these workshops as well as participation in the sessions of Inter-Agency Space Debris Coordination Committee (IADC) gave the author an impetus for intensive work on this topic.

The main stimulus for emergence of this monograph was absence of a whole common mathematical theory for planning the search for space objects, using only incomplete and imprecise a-priori information on their state vectors. Such a theory could essentially enhance the efficiency of search and detection of space objects which must make themselves felt in the practice of space surveillance both in Russia and other space-faring nations. For a long time, numerous results in the field of space surveillance, namely, in solving the search problems produced the eclectic sum total of search methods – often empirical ones and not connected theoretically with each other.

During development of such a theory, a certain success was attained in constructing a common methodology of planning the search for a space object by rough a-priori orbital information. This result was obtained owing to use in the planning process of the set-theoretical approach to presentation, analysis, and due regard of the temporal structure transformation of the sought-for space object current position uncertainty domain. At the same time, no less important and fruitful was the formulation of the principle of equivalence of search plan elements for different times, then its generalization, development of the equivalence curves apparatus, and its introduction into the practice of planning the search.

All this together with the developed theory allow constructing search plans (including optimum) on the strict mathematical basis practically for any character of the errors distribution in a-priori data on the sought-for space object's orbit parameters. The more so, in this monograph, a more general theory was developed that allows planning the search for an abstract object (not only the space one) moving with a given law, some simple restrictions and assumptions being kept. At the same time, permitting some natural and practically justified assumptions on the character of the errors distribution (for example, taking into account prevailing growth of errors only along the track and neglecting the others) leads to essential simplification of the planning procedure and, as a result, to simple distinct visual constructive layouts for getting optimum and suboptimum search plans. This makes it very convenient for astronomers-observers to organize the planning process for the search for space objects. Obvious refinement and natural character of these schemes witness the adequacy and correctness of the developed theory. Practical search methods constructed on the base of this theory were implemented at the real operating electro-optical sensors. They have been successfully running for several years and exposed their high performance. The results of their testing were presented at the international conferences and workshops [1, 2].

Practical background for the theory and methods and the examples of the search plans construction and their application were considered in this monograph predominantly in terms of optical and electro-optical sensors. Despite this, there are no limitations for applying the theory and methods to radars, lasers, and other sensors.

A distinctive feature of the suggested methodology of planning the search for a space object, using imprecise a priori information, is that its optimization scheme appears reiterative, multiform, and rather versatile. These properties allow obtaining optimum solutions not only by different criteria, but also in different forms and in several stages. For example, after having constructed the optimum search plan at a given optimization level and in a certain formulation of the problem or on a given conditions, one can proceed with the optimization process in an alternative form, on alternative conditions, or at the next optimization level, that is, by principle *"optimum optimorum"*. And all this is on the set-theoretical base within the frame of the united mathematical apparatus of the equivalence curves.

In addition to the fact that the theory and methods proposed here make it possible to significantly improve the efficiency of the search for space objects in terms of saving the search resource of sensors (both the sensors itself and the employment of the operating personnel), detection of small and weakly-contrasting space objects becomes available. So, the search algorithms turn out to be less critical to the relative velocity of the soughtfor objects.

The author expresses his gratitude to Dr. Sci. Prof. O. Yu. Aksenov, Dr. Sci. R. R. Nazirov, Dr. Sci. L. V. Rykhlova, Dr. Sci. A. V. Bagrov, the author's colleagues and other specialists who took part in the discussion of the material of the monograph, made a number of useful comments and suggestions which were gratefully taken into account by the author in finalizing the text of the monograph.

ABBREVIATIONS

ACR – additional cutting rule

BB - "branch-and-bound" (approach, scheme, method (of))

CCD – charge coupled device

CPUD - current position uncertainty domain

DOP - discrete optimization problem

DSO – deep-space object

EC – equivalence curve

ECA – equivalence curves apparatus

ECA – effective checked area

ECS – effective checked surface

ECT – equivalence curves tool

ERS - effective reflecting surface

ESA – European Space Agency

FoV - field of view

GE – generalized ephemeris

GEO - geostationary orbit

GEOSO – geostationary SO (an SO in a GEO)

HEO – highly eccentric orbit

HEOSO – an SO in a HEO

HO – high orbit

HOSO – a high orbital SO, a deep space object

HOSC – a high orbital constellation of SOs

IADC - Inter-Agency Space Debris Coordination Committee

ISS – International Space Station

LEO – low Earth orbit

LEOSO - a low orbital SO

LoT – loss of track

MEO – medium Earth orbit

MEOSO – a medium orbital SO

NASA - National Aeronautics and Space Administration

NES – near Earth space

OD - orbital debris (US) = SD (Eur.)

OSC - operating spacecraft

OSP - optimum search plan, optimum search planning

OSR – optimum search region

PE – principle of equivalence

PP – picture plane

Q.E.D. – quod erat demonstrandum (*lat.*)

RCS – radar cross section

RF – radio frequency

r.m.s. – root-mean-square (error, criterion)

RSSS – Russian SSS

SC - constellation of SOs, space system

SD - space debris

SMA – short measuring arc

SMAF - small measuring arc factor

SO - space object

SOCPUD – space object current position uncertainty domain

SP - search plan, search planning

SS – space surveillance

SSh-spaceship, spacecraft

SSN – space surveillance network

SSS – space surveillance system

ST - search theory

TOPS – theory of optimum planning the search

TST of SOCPUD - temporal structural transformation of SOCPUD

TV-television

CHAPTER ONE

INTRODUCTION

Although the volume of space surrounding the Earth has never been empty and permeated with natural objects, in the 60's of the 20th century with the launch of the first Earth satellite, near-Earth space (NES) began to be filled with space objects (SO) of artificial origin.

Domestication of space not only brings progress to humanity. It also has great negative consequences littering NES with space debris (SD) which not only poses a threat to operating spacecraft (OSC), but also violates the ecological balance around the Earth that was set over millions (and possibly billions) of years [3, 66].

The technogenic contamination of NES which has piggy-backed at the good intentions of man is dangerously progressing. And we do not have effective means not only to stop this dramatic process, but even to significantly slow it down. And this is in spite of the fact that a million army of scientists came out to fight against it and the states allocate huge sums for this.

So that, humanity has a new important care (trouble) and even the duty - to keep the process of technogenic "settlement" of NES under close attention and control, that is, to continuously conduct tireless and careful space monitoring [3]. Actually, in technical terms, this means the need to have constantly updated kinetic parameters (positional coordinates and corresponding velocity components) for as many SOs as possible.

Without this, it is impossible to maintain stable communication with existing active satellites and spaceships, to perform operational control from the Earth in real time, to monitor the position and movement of space debris in NES in order to prevent the collision of its elements with operating spacecraft, and predict the possible reentry and fall of large debris to Earth, not to mention a full-fledged comprehensive scientific study of the process of technogenic pollution of NES.

So, to ensure space activities, including its security, close monitoring of the near-Earth environment is essential for which costly professional (dedicated) means of space surveillance are being created. Two space surveillance systems (SSS) have been built and are in operation for many years – in Russia (RSSS) and in the US (US SSN). The creation of a European SSS under the auspices of the European Space Agency (ESA) is under way [4]. A significant contribution to the implementation of the outer space monitoring function is made by independent (not dedicated) observation facilities (radar, optical, optoelectronic, passive RF) – assets of various countries. The main product of this monitoring is the dynamic catalog of space objects which is maintained and updated due to measurements obtained from the means of observation. The most complete and accurate SOs catalogs containing all principal characteristics of relatively large SOs that are updated in real time are those of space surveillance systems.

A lot of measurements from all the censors serve as a source not only for updating and refining the kinetic parameters of SOs that were previously cataloged, but also for detecting new SOs. The latter function (especially for high-orbit SOs) should be served by special search modes and programs of the surveillance censors (in contrast to the routine modes and programs for observation of specific SOs with the help of precise target designations).

As for the general history of the appearance of the search problem for space objects, it originated mainly with the appearance of high-orbit SOs (HOSOs). To detect low-orbit SOs (LEOSOs) it was irrelevant, because the existing radar network continuously monitors the vast low area of outer space. And this radar network is constantly growing all over the world and their capacity and capabilities are increasing.

The large number of LEOSOs passes through the operation ranges of the radars due to the significant number of the latter, their high productivity, and the short period of an LEOSO circulation. This circumstance easily solves the problem of obtaining a sufficiently dense flow of measurements for all LEOSOs in order to realize the principle of their passive detection (without search).

In accordance with these conditions, the orbit parameters of new SOs are obtained as "production wastes" of the refined parameters of cataloged SOs. And so, special search modes of the dedicated sensors are not required in this case (except the case of very small and faintly reflecting SOs).

However, it should be borne in mind that what has been said applies only to relatively large objects that are available for detection by the network of specialized facilities with a wide operation range. At the same time, in NES, there is a huge population of small and weakly-contrasting SOs (including LEOSOs) inaccessible for detection by radars operating in

Introduction

normal, routine modes. And this population of small-sized objects is constantly growing [3, 65, 66]. In view of the enormous velocities of the motion of LEOSOs (by the way, the relative velocity of head-on collisions can be twice that of each SO), a big danger is the collision not only with large SOs controlled by the radar networks but with small ones as well [3, 63, 64, 67, 71].

Hence, monitoring of the fine fraction of SD is also urgently needed. The existing network of radars cannot provide this working in normal, routine modes. A more or less satisfactory solution to this last problem requires narrow-beam radars (not to mention the use of newest optical telescopes capable of detecting LEOSOs) and special search modes of their operation. In this monograph such search modes are being developed.

In addition to this, it should be borne in mind that the operational slant range of radar is rather limited. Therefore, the situation with the solution of the HOSO detection problem is quite different from that of LEOSO detection. And this is another side of our common large problem. The practical impossibility of creating a sufficiently powerful solid and continuous in time electromagnetic field in deep space and overlapping all possible HOSOs' trajectories (that is, impossibility to provide a complete radar coverage of deep space) leaves the only admissible way for dealing with the HOSO detection problem – namely, the principle of active search for them with the help of optical, electro-optical sensors, and specialized narrow-beam radars.

However, it is not easy to realize this. An effect of some specific high orbits' factors complicates the managing of HOSOs' surveillance. It leads to the appearance of search situations not only at the stage of initial detection of new SOs, but also at the next stages of their motion. Then it stipulates some difficulties in obtaining measurements, and their deficiencies cause excessive energy-consuming and labor-consuming character of deep space surveillance and its high instability.

We list here the main factors:

- all operating spacecraft are forced to maneuver both to maintain the correspondence of their orbits to specified flight programs and in order to avoid dangerous collisions with other objects;
- technological orbital corrections for geostationary, geosynchronous, and some other HO satellites occur regularly and with a relatively high frequency;

- too long periods of circulation of HOSOs cause the factor of a short measuring arc which is expressed in the really unattainable necessity of using very long (in time and space) measuring intervals for attaining acceptable accuracy of the initial orbit determination and renovation of its parameters;
- it is extremely important for optical sensors that there are satisfactory meteorological and astroclimatic conditions, and the phase of illumination of the object being observed;
- there are a substantial remoteness and a great volume of the space region of high orbits (hundreds and thousands of times larger than those of low orbits);
- the level of the intelligence signal is usually very low which may become still fainter because of unfavorable observation conditions (due to the influence of the weather, the phase of illumination, remoteness, and so on);

the output capacity of present sensors is rather limited.

Hence, it is clear that the methodology for detecting and sustaining mass traffic control of HOSOs must be significantly different from that of LEOSOs. Until recently, it was not systematically developed. The search methods used were quite an eclectic collection. The universal mathematical apparatus and the theory of optimum planning the search for HOSOs which should be the basis of this methodology were also absent.

The beginning of the space age which put forward the process of filling the near space with the rapidly growing SO population and, as an inevitable consequence of it, the need for careful surveillance and control of this population posed a whole series of serious theoretical problems. Some of them were quickly and successfully resolved. Others were hard pressed to get their solution. At the same time, in the process of further development and domestication of space, the essences of many problems changed and were refined (sometimes radically). Periodically, there arose also new problems. While one of the basic problems of monitoring SOs' motion – creation of the mathematical model of motion (the propagator) taking into account all substantial perturbations and the determination of their orbits using a set of measurements – has been quite satisfactorily solved in works [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 56, 60], the problem of optimum planning the search for SOs with due regard to imprecise a priori information on their orbits has remained unsolved hitherto [16].

Despite the enormous covering and operational power of the SSSs and the presence of many other facilities for monitoring the SOs in such a rapid space exploration and the catastrophic progress of contamination of

Introduction

NES given the performance and operational capacity of existing surveillance censors being used and prospective ones, using an out-of-date ideology of searching for and discovering new and lost SOs, will not last long enough. So, the most urgent problem of modernizing this ideology is on the agenda.

All the methods of search for SOs and search situations arising in the space surveillance routine and in astronomer-observers' practice can be conditionally broken up into two large categories:

- the survey in consecutive order in a given search region (that is, scanning the part of sky allotted to the sensor) and detecting "whatever we come across";
- 2) the search for a particular SO, using some a priori orbital information (as a rule significantly imprecise), with the help of special individual search programs developed especially for that a priori information.

These categories cover almost all search situations.

In the first case, the sensor carries out scanning a large celestial sphere region which is fixed and motionless in every search cycle. It uses the constant strategy chosen beforehand and detects every SO in the sensor's field of view (FoV) the signal of which has the energy enough for its acquisition by the sensor's receiver (which is determined by the technical performance of the latter). The methods of regularly sequentially scanning the sky region and setting optical and radar fences developed and published in works [17, 18, 19, 20] appeared rather limited for space surveillance practice.

The methods of the first category are useful at the initial stage of cataloging SOs when a priori information on most of SOs' orbits is practically absent and then in a mode of periodical surveys. The application of these methods is limited by

their high energy intensity,

low economy,

- long duration and connectedness of the whole scanning cycle and hence low operating rate,
- inevitability of obtaining and identification of many unnecessary measurements,

low effectiveness of acquisition of faint intelligence signals,

just unfitness for some important search situations,

Chapter One

significant inefficiency in case of using narrow-angle and narrow-beam facilities.

These methods cannot provide high enough operating rate of detecting a particular SO, high accuracy of initial orbit determination when acquiring the first signal in the end of a cycle of survey, detection of SOs with small effective reflecting surfaces or their radar cross sections (RCS), in cases when the SO is in a poor phase of illumination, and others. They lay high claim to the output capacity of the sensors and altogether leave untouched significant reserves of enhancing efficiency of exploitation of sensors and therefore cannot be considered universal and, moreover, promising.

In contrast to the first case, in the second one, the sensor is checking (by a special search plan) a strictly limited and continuously drifting space domain – the sought-for SO current position uncertainty domain (CPUD) [25, 61, 62]. The size, position, aspect angle, and motion parameters of the SOCPUD in space are determined by given a priori information on the SO orbit and the quality of the latter.

What has been said and a number of additional considerations entail that the necessity of development of optimum search methods on the basis of effective use of imprecise a priori orbital information is stipulated by

- the claims of sufficient completeness, high efficiency, and stability to monitoring SOs' motion;
- toughening of requirements to operating rate and reliability of detecting new, lost, and maneuvered SOs;
- economy reasons and demands for more effective use of sensors;
- need of planning provision of observation for deep space objects with the help of narrow-angle and narrow-beam sensors as well as targeting the special lighting-up (backside illumination) facilities;
- difficulties of the search, detection, and observation problems when dealing with SOs having faint brightness and rapidly moving across the sensor's FoV;
- presence of the problem of guaranteed control of very important SOs' motion during their launch, after the maneuver, during worsening of the observation conditions, and in emergency.

The development of a methodology for optimum and suboptimum search planning and its theoretical validation becomes especially urgent when solving the problem of searching for and detecting small and

Introduction

weakly-contrasting SOs and using narrow-angle optical and narrow-beam radar sensors for these purposes.

The concept of "imprecise orbital information" used above is rather conditional. The same information can be treated as accurate enough for one sensor and imprecise for another. And one need to decide in each case. It depends particularly on the size of FoV or width of the beam. For definiteness, let's settle to call some a priori orbital information on the sought-for SO precise if FoV of the sensor used for the search completely covers the SOCPUD and imprecise in the opposite case. (The term "completely" here should not be comprehended absolutely. And about this, please, see later.) It is clear that if we have the precise information on the SO's orbit, its search degenerates into its particular case – observation of the SO by the accurate targeting data. Namely in this sense, one may not make difference of principle between these two modes (see it later in detail).

This monograph is devoted to the investigation of the second category of search methods for SOs – the search, using rough a priori information about their orbits – and the development of the corresponding theory foundations for search planning. It is namely this category that is most relevant and interesting from the mathematical point of view as well.

CHAPTER TWO

THE ESSENCE OF THE PROBLEM

2.1. On available a-priori orbital information on the sought-for space objects

In case of total absence of a priori information about the orbit of the sought-for SO, the observer will prefer the first category of search situations and he has nothing to do but to turn to the survey methods. The theory developed here presupposes the indispensable presence of some, albeit very crude, initial metric information about the sought-for SO. This information which will later be used to construct the search plans may have different forms and origins, for example:

information on the time and place of launch; designed or expected nominal values of the SO orbit parameters; statistic data on SOs' orbits of different classes; estimates of the orbit parameters by imprecise measurements, by few ones, or by measurements at the short measuring arc;

available overaged information on the orbit;

information on the orbit adjustment or on the SO's maneuver;

information on the orbital structure of the SOs' constellation (SC).

The orbit parameters are getting overaged in time due to the evolution of the state vector errors. And if by some reason the influx of measurements for a given SO has been stopped, then its orbital information (both metric and non-metric) gradually loses the accuracy in due course. This is one of the most natural ways of obtaining imprecise orbital information on an SO.

Further, let us take notice that the SO catalog maintained in the SSS serves as another natural source of imprecise orbital information on SOs even if the orbital parameters considered as very accurate from many points of view appears imprecise for narrow-angle and narrow-beam sensors or for carrying out some special operations with the proper SOs, perhaps, using some special auxiliary instruments.

In absence of publications of the orbit parameters values, if there is a need of detection of a HOSO for instance in highly eccentric orbit (HEO), it would be advisable to use the following a priori information.

- 1. The position of the orbit plane (longitude of ascending node Ω) when the launch site is known can be calculated, using the time and date of launch and regarding the optimum energetic strategy of putting the SO into its orbit.
- 2. In terms of stableness of the HEO apogee position, its inclination *i* should be as close as possible to its critical value [13].
- 3. Based on the structure of the SC, the features of the functional purpose, and the flight program of its spacecraft, the inclinations of their initial orbits have their own specifics for different SCs (see Fig. 2-1-1).
- 4. In terms of providing the best conditions of radio visibility and the survey of the North hemisphere, the value of argument of perigee ω should be comprised within the interval 270 290° [21, 22].
- 5. Practically all the tasks performed by functional HEOSOs demand periodical repetition of the SO trace on the Earth surface which can be achieved only with multiplicity of the SO nodal period T_{Ω} to the sidereal day (24 hours) with no regard for the orbit parameters evolution ($T_{sid}/T_{\Omega} = 1, 2, ...$). For reasons of providing more size of the optical and radio visibility zone, long enough duration of covering the particular territory, and better conditions of radio communication, the preferable orbits are those with nodal period $T_{\Omega} = 11^{h}57^{m}45^{s}$ which provides the repetition of the same Earth trace.
- 6. To ensure that the perigee of the orbit with such a period of circulation is outside the dense layers of the atmosphere, the eccentricity should not exceed the value

$$e_{cr} = 1 - \frac{R_{E} + h_{a}}{a} \approx 0.74$$

where e_{cr} is the critical value of eccentricity, R_E – the Earth radius, h_a – the height of atmosphere (relative).

For the sake of enlarging the operational part of orbit and reducing energy expenditure for putting the SO into the basic circular orbit, the eccentricity should be as great as possible. That is why for the majority of HEOs with a nearly 12-hour nodal period the designed eccentricity is within the interval 0.7 ... 0.74.

7. The value of argument of latitude can also be approximately estimated by a priori information on the program of placing the HEOSO into the operational orbit. It is possible to approximately determine the time t_a of an SO coming to the apogee proceeding from the typical scheme of putting into orbit (with minimum time of phasing the SO in the transfer orbit), the time and site of launch being known:

$$t_a = t_{st} + \Delta t_{act} + \Delta t_{in} + \Delta t_{trans} + T_{in}/2$$

where t_{st} – time of start,

 Δt_{act} – duration of active motion ($\approx 8 \dots 10 \text{ min}$),

 Δt_{in} – duration of motion in the initial (basic) circular orbit,

 Δt_{trans} – duration of motion in the transfer orbit,

 T_{in} – the initial period of the SO revolution.

Then, based on the available experience, we can suppose that the sum of three items $\Delta t_{act} + \Delta t_{in} + \Delta t_{trans}$ approximately equals 27 min (for example, for HEOSOs started from the Vandenberg base, that is SOs of series SDS, program 711, and so on).

One should mean also that during the launch of HEOSOs, the boosters usually move in the same HEOs. When an SO is put into the geostationary orbit (GEO), its booster remains in the transfer HEO having its inclination equal to the latitude of the launch site and the argument of perigee equal to 0° or 180° . Sometimes, the booster is to be put into GEO. A priori information on the time and site of launch for search of an SO in a circular 12-hour orbit is, as a rule, more definite than that for a HEOSO.

Taking into account the peculiarities of evolution of the state vector errors, detection of HEOSOs and 12-hour circular SOs comes to the search by argument of latitude u. The strict statement of this problem and optimum methods for its solution are given in Chapter 4. If there are appreciable uncertainties in other orbit elements, the problem also comes to the search by argument of latitude u in several passes (section 4.6) or to the search task in a more common statement (Chapter 5).

The search problem with a priori information of a different kind, for example, the out of date state vector, also comes to the search by argument of latitude. Such a search situation very often arises in the space surveillance practice when for some reason the needed measurement information is absent for a long time. In such a case, the SO position error grows most rapidly along the track (that is, in argument of latitude) [11]. One can see this from Table 2-1-1 in which the state vector error evolution



Fig. 2-1-1. Histograms of the initial orbit inclination distributions for some HOSCs

is presented in the radius-vector (r), along the track (l), and in the binormal (b) when propagating the motion of a geosynchronous satellite with a given initial state vector determined by 4 highly precise 3-dimensional measurements (a slant range and two angles) at the measuring arc 360°. The appearance of a problem of search by argument of latitude u, the orbit plane and its position in space being given, in this case is evident.

There is one more source of imprecise a priori information for addressing the search. Typical for space surveillance practice is scheduling the observations of a HOSO by the imprecise ephemeris calculated from the initial state vector determined by measurements obtained on a small measuring arc. Such a situation often appears, for example, because of dependence of successful operation of optical systems upon the time of day and the atmosphere transparency, in case of short-term stay of the SO in the radar zone (for instance, at the edge of the zone), on account of dependence of the passive RF sensor efficiency upon the duration of the irradiating spacecraft operation, and so on. Here the condition for application of the search methods by argument of latitude is still more favorable. This fact is corroborated by the data of Table 2-1-2 where the geosynchronous SO position propagation error evolution is given, the initial state vector being determined by 4 two-dimensional measurements with the root-mean-square (r.m.s.) error 2" located at the measuring arc 45°.

The essence of the problem

Table 2-1-1

State vector error evolution in radius-vector (r), along the track (l), and in binormal (b) when propagating the motion of a geosynchronous SO with a given precise state vector (the measuring arc is 360°)

180	0.207	87.3	1.69						0.006	13.5	0.12	
60	0.153	29.1	1.51						0.005	4.5	0.11	
30	0.138	14.6	1.47						0.005	2.29	0.11	
20	0.134	9.7	1.46						0.005	1.53	0.11	
10	0.129	4.8	1.44						0.005	0.77	0.11	
3	0.125	1.45	1.43						0.005	0.25	0.11	
7	0.124	1.0	1.43						0.004	0.17	0.11	
	0.124	0.6	1.43						0.004	0.09	0.11	
0	0.123	0.5	1.43	3.44	6.0	10		0.16	0.004	0.017	0.11	0.025
n interval,	in r, km	in <i>l</i> , km	in b , km	in \dot{r} , cm/s	in \dot{l} , cm/s	in \dot{b} ,	cm/s	in T_{Ω} , c	$M\Delta r$, km	$M\Delta l, \mathrm{km}$	$M\Delta b,$ km	$M\Delta T_{\Omega}, s$
Prediction days	r.m.s. error						estimate bias					

Chapter Two

 Table 2.1.2

 Geosynchronous SO position propagation error evolution (in r, l, b) given an imprecise initial state vector (the
 corresponding measuring arc is 45 %

10	9.7	1552	0.39			
S	6.3	755	0.38			
ŝ	4.36	465	0.38			
6	3.36	310	0.38			
-	3.64	155	0.38			
0	3.67	0.08	0.38	6.8	29.5	2.9
ys.				s,	s	/s
erval, da	in r, km	in <i>l</i> , km	in b , km	in ŕ, cm	in <i>l</i> , cm/	in \dot{b} , cm
on inte		1	1	1	I	1
Predicti				ror	a .s.u	1.1

14