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***Everyone can do whatever he would desire in his life.***
***But he does desire only what he can do.***

# PREFACE

At the present time, we are all witnessing the sharp-plotted scenarios played in politics, business, and industry in connection with the problems of extraction and redistribution of the oil and gas resources. Additional freshness is contributed into this atmosphere by “shale revolution” broken out owing to development of new technologies of super-deep and long distance curvilinear drilling. Now, instances of vertical well boring to depths exceeding 10 km are not rare, and the record distance from the drilling rig of the horizontal well has reached more than 13.5 km. Such wells are drilled at the limit of current industrial capabilities at maximum velocities, hydrostatic pressures, and temperatures values as well as strength and wear parameters of drill string materials under the significant impact of violent vibration effects and entire system instability. These processes are often accompanied by emergency and failure situations, including:

- Bifurcation buckling of vertical strings in the type of compressed-stretched, twisted, rotating tubular rods with internal fluid flows.
- String resonance (bending) vibrations.
- Self-excitation of torsional auto vibrations with slip-stick motions caused by non-linear frictional forces between the bit and the rock being destroyed.
- Whirling vibrations of the bit self-excited as a result of frictional and kinematic (nonholonomic) rolling of the bit over the surface of the hole bottom.
- Deadlock states in wells with geometric imperfections caused by increasing contact friction forces between the string and the borehole wall.
- Bifurcational buckling of the strings in curvilinear boreholes accompanied by a deterioration in the conductivity of the actuating cutting torque and axial force to the bit, increased wear of the string pipe, an increase in system power consumption, an increase in the intensities of stress fields and strains in the strings, and an increase in the probability of their destruction.

These emergency situations are mainly caused by three factors. First, it is the long string length. In terms of geometric similarity, it is similar to human hair. Therefore, a phenomenon occurring at one end of the string can influence, have little or no effect on the phenomena occurring at the other end. In mathematics, the equations describing such phenomena are called singularly perturbed. They are characterised by poorly converging solutions, the forms of which have singularities in the shape of edge effects, internal harmonic wavelets or may contain irregularities.

The second factor is related to the special character of frictional effects that appear in extended curvilinear boreholes. The fact is that the frictional force in them depends on the pressing force of contacting bodies. However, in curved sections, the force of pressing the string against the borehole wall is determined by the axial tension of the string. In this case, it is said in mechanics that frictional forces have a multiplicative (i.e., are multiplied) and not additive (i.e., not added) nature, as happens when the body slides over a rough flat surface.

The third factor is that the issues of mathematical modelling of mechanical phenomena accompanying the drilling processes are multi-parametric since they depend on a large number of geometric and mechanical quantities and can hardly be solved in a general formulation. Therefore, it is very important to consider the implementation of these mechanical phenomena for specific values of the determining values and establish the general regularities of their behaviour. It should be noted that due to the great complexity of the tasks and the uncertainty of the initial data, the proposed drill string dynamic models reflect only the qualitative aspects of the phenomena considered, which are also of no small importance. Here, it is appropriate to recall Hemming who noted that in mechanics, when modelling, it is not the number but the understanding that is important. At the same time, it can be asserted that mathematical models of frictional phenomena and the processes of drill string buckling in curvilinear boreholes largely reflect also their quantitative aspects. In this regard, they can be directly used both in the design of wells and when drilling them. Ultimately, the practical application of methods for the computer simulation of abnormal situations arising in deep-well drilling can contribute to their prevention.

The research given herein was discussed with specialists of the National Transport University and Smart Energy LLC (Kiev). The authors express their gratitude to all of them.

### Selected publications of the authors

1. Gulyayev V., Glazunov S. and Vashchilina O. Frequency analysis of drill bit whirlings on uneven bottoms of deep bore-holes. 2017. Journal of Mathematics and System Science, 7 , 14 – 24 (**USA**)
2. Gulyayev V.I., Gaydaychuk V.V., Andrusenko E.N., Shlyun N.V. Modelling the energy-saving regimes of curvilinear bore-hole drivage. 2015. Journal of Offshore Mechanics and Arctic Engineering. V.137, №1, 011402−1−011402−8. (**USA**)
3. Gulyayev V.I., Gaydaychuk V.V., Andrusenko E.N., Shlyun N.V. Critical buckling of drill strings in curvilinear channels of directed bore-holes. 2015. Journal of Petroleum Science and Engineering, 129, 168−177. (**USA**)

4. Gulyayev V.I., Andrusenko E.N. Theoretical simulation of geometrical imperfections influence on drilling operations at drivage of curvilinear bore-holes. 2013. Journal of Petroleum Science and Engineering.V.112. 170 – 177. (**USA**)
5. Gulyayev V.I., Gaydaychuk V.V., Solovjov I.L., Gorbunovich I.V. The buckling of elongated rotating drill strings. 2009. Journal of Petroleum Science and Engineering.V.67, P.140 – 148. (**USA**)
6. Gulyayev V.I., Borshch O.I. Free vibrations of drill strings in hyper deep vertical bore-wells. 2011. Journal of Petroleum Science and Engineering. V. 78, P. 759 – 764. (**USA**).
7. Gulyayev V.I., Hudoly S.N., Glovach L.V. The computer simulation of drill string dragging in inclined bore-holes with geometrical imperfections. 2011. International Journal of Solids and Structures. V.48, P.110–118. (**USA**)
8. Gulyayev V.I., Glushakova O.V. and Glazunov S.N. Stationary and non-stationary self-induced vibrations in wave guiding systems. 2014. Journal of Mechanics Engineering and Automation. V. 4(3).P. 213-224. (**USA**)
9. Musa N., Gulyayev V., Shlun N., Aldabas H. Critical buckling of drill strings in cylindrical cavities of inclined bore-holes. 2016. Journal of Mechanics Engineering and Automation. V. 6. P. 25 – 38. (**USA**)
10. Gulyayev V.I., Shlun N.V. Influence of friction on buckling of a drill string in the circular channel of a bore hole. 2016. Petroleum Science. V. 13. P. 698 – 711.(**China**)
11. Gulyayev, V.I., Tolbatov E.Yu. Forced and self-excited vibrations of pipes containing mobile boiling fluid clots. Journal of Sound and Vibration 2002. – 257(3), 425–437. (**Great Britain**)
12. Gulyayev, V.I., Tolbatov E.Yu. Dynamics of spiral tubes containing internal moving masses of boiling liquid. Journal of Sound and Vibration 2004. – 274(2), P. 233–248. (**Great Britain**)
13. Gulyayev V.I., Hudoliy S.N., Glushakova O.V. Simulation of torsion relaxation auto-oscillations of drill string bit with viscous and Coulombic friction moment models. 2010. Journal of Multi-body Dynamics. V. 225. P. 139 – 152. (**Great Britain**)
14. Gulyayev V.I., Shevchuk L.V. Nonholonomic dynamics of drill string bit whirling in a deep bore-hole. 2013. Journal of Multi-body Dynamics. V. 227(3). P. 234 – 244. (**Great Britain**)
15. Gulyayev V.I., Shlyun N.V. Global analysis of drill string buckling in the channel of a curvilinear bore-hole. 2017. Journal of Natural Gas Science and Engineering. V.40. P.168-178. (**Netherlands**)

16. Gulyayev V., Andrusenko E., Glazunov S. Computer simulation of dragging with rotation of a drill string in 3D inclined tortuous bore-hole. 2019. SN Applied Sciences. V. 126(1). (**Switzerland**)
17. V.I. Gulyayev, O.V. Glushakova, S.N. Khudoliy. Innovational attractors in wave models of torsional vibrations of deep drilling strings. 2010. Proceedings of the Russian Academy of Sciences. – Mechanics of Rigid Body No. 2. 134-147. (**RF**)
18. Gulyayev V.I., Andrusenko E.N., Shlyun N.V. Theoretical modelling of post - buckling contact interaction of a drill string with inclined bore-hole surface. 2014. Structural Engineering and Mechanics, V. 49(4), 427-448. (**South Korea**)
19. Gulyayev V.I., Khudoliy S.N., Andrusenko E.N. Sensitivity of resistance forces to localised geometrical imperfections in movement of drill strings in inclined bore-holes. 2011. Interaction and Multiscale Mechanics. V.4(1), P.1–16. (**Taiwan**)
20. Gulyayev V.I., Glushakova O.V. Large-scale and small-scale self-excited torsional vibrations of homogeneous and sectional drill strings. 2011. Interaction and Multiscale Mechanics. V. 4(4), P. 291 – 311. (**Taiwan**)
21. Musa N.W., Gulyayev V.I., Shevchuk L.V., Hasan A. Whirl interaction of a drill bit with the bore-hole bottom. Modern Mechanical Engineering.-2015.-V.5.-P.41-61. (**Hong Kong**)

# CHAPTER 1. PROBLEMS OF THE THEORETICAL MODELLING OF EMERGENCY SITUATIONS IN DEEP-WELL DRILLING

## *1.1. Technical aspects of deep-drilling problems*

Energy problems, which are becoming ever more acute in the 21st century, are caused by the approaching exhaustion of oil and gas resources and the fact that their production has become more complex. As a result of prolonged and inefficient extraction and consumption using low-cost technologies, the time of light oil and gas ended in the 20th century [5]. Therefore, deposits found in shale rocks and at depths of up to 10,000 m are now very promising. For example, in the United States, the possibility of extracting fuel from a depth of 30,000 feet (9,150 m) is being studied, and goals are being set for developing inclined and horizontal offshore wells with a distance of up to 15 km from the drilling platform [4].

Taking into account the increase in the depth and range of drilling, the cost of these wells already exceeds $50 million [3], and every third well is an emergency one; but reliable methods for the theoretical modelling of their functioning have not yet been developed, the conclusion can be drawn on the importance of theoretical forecasting of critical states of drill strings (DS) and the price of the forecast error.

The main global energy consumers are [2, 6] industry, transport, agriculture, the residential sector, and commercial and public services. The most energy-intensive is transport, followed by industry and the residential sector.

The energy type to be used by these consumers is selected based on six main factors: 1) fuel and energy feedstock calorific value; 2) ease of production; 3) ease of transportation; 4) ease of energy production and use; 5) associated hazard to health and life of employees; 6) presence of waste and potential environmental contamination. Given these factors, we can conclude that oil and gas will remain the most attractive energy sources in the coming decades.

An important circumstance contributing to the complication of the situation in the oil and gas industry is that under normal conditions usually only 40% of hydrocarbon fuels filling the cracks and pores of underground reservoirs can be extracted using conventional production techniques. One of the ways to increase the volume of fuels extracted from underground reservoirs is associated with the drilling of curvilinear boreholes penetrating the oil-bearing and gas-bearing beds along their laminated structure and, therefore, covering large areas of fuel intake [7]. Since, when using this technique, the total number of wells drilled is reduced, and the flow rate of curvilinear boreholes turns out to be significantly higher than that of vertical wells, in the near future drilling directed holes will become the main technique in much of the world. The development of curvilinear drilling techniques is also facilitated by the need to extract hydrocarbons from shale formations.

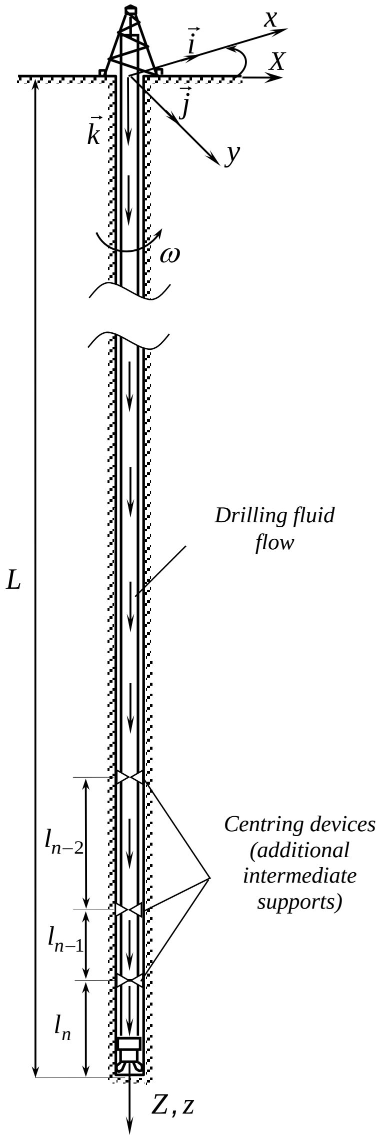



*Fig. 1-1 Geometrical layout of a drill string in a deep well*

In accordance with economic requirements, the geological conditions of the deposit and the process capabilities of oil and gas companies, vertical, directional, horizontal, and multilateral oil and gas wells of different depths are now being drilled. However, the practical introduction of drilling techniques for deep wells of different spatial orientation is associated with the need for the theoretical modelling of mechanical phenomena occurring in the drilling equipment structures to prevent critical modes of operation. In addition, one of the most important aspects of this area is the theoretical modelling of quasi-static and dynamic behaviour of deep-drilling strings. However, this task is significantly complicated by the fact that under conditions of geometric similarity the drill string is similar to human hair, and at the same time it is subject to intensive loads and a large number of different factors complicating the methods of theoretical modelling.

The most popular oil and gas well-drilling technique is the rotary method when the rock is cut using a bit attached to the bottom end of the drill string suspended in the borehole at the top end. In this case, the bit rotates because of the entire drill string rotation caused by the effect of actuating torque on its top end (Fig. 1-1).

A drill string is assembled from pipes 12–15 m long using threaded connections. Drilling efficiency and quality are determined mainly by its regime and the bottom hole assembly

(BHA) structure with reamers, centralisers, and stabilisers. The generation of high axial loads ensuring stabilisation and controllability of the wellbore trajectory is associated with the use of multi-bearing BHA, where the number of centralising components does not usually exceed five.

The drill string bottom structure also includes weighting agents, calibrators, and other elements.

To remove rock particles chopped as a result of being cut with a bit from the borehole, drilling fluid is supplied inside the string by a special pumping system that—rising in the outer space between the string and the borehole wall—entrains and carries away these particles. The drilling fluid also plays other important functional roles. It is known that rocks in the interior of the earth experience significant three-dimensional stresses caused by a comprehensive compression of upstream solids by gravity forces. Due to the continuity of the rock, these stresses counterbalance each other, much as hydrostatic forces in the liquid. However, rock discontinuity during well drilling leads to a redistribution of these stresses in the vicinity of the well, rock imbalance, and destruction. Should the well be filled with liquid with a specific gravity equal to that of the rock, its hydrostatic pressure on the walls will balance the imbalance of forces in the rock, and it will remain stable (using the language of drillers). It is also important to maintain a balance between the average density of the liquid and rock (especially for deep wells). In addition, the drilling liquid plays an important role in the formation of frictional interaction between the string and the borehole wall. Finally, the drilling liquid serves to cool the drill tools.

## *1.2. Abnormal phenomena accompanying deep-drilling processes*

When suspended, the drill string is exposed to distributed gravity forces. They generate a tensile axial force therein, which reaches a maximum at the suspension point and decreases to zero at its bottom end. When operating, the string is resting against the bottom of the well, it is exposed to the vertical reaction compression force; therefore, the entire string is in a stretched-compressed state of stress.

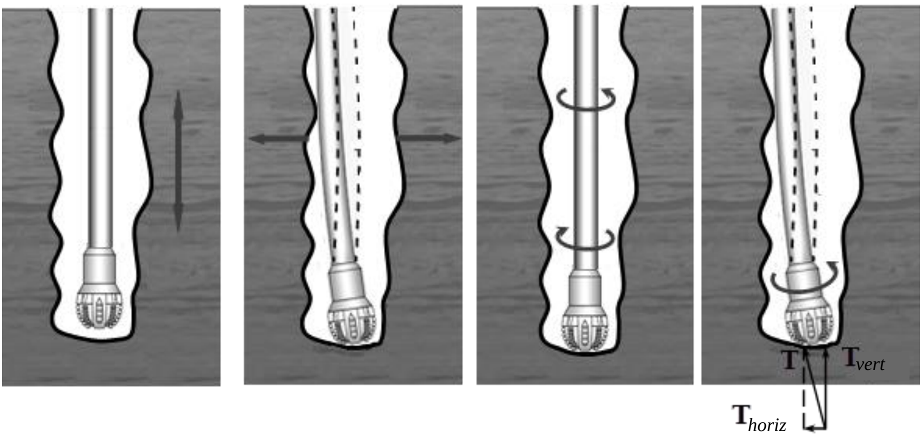
To impart a rotary motion to the string, torque is applied to its top end. For shallow wells, it can be assumed that in the stationary state this moment is equal to the cutting moment applied to the bit, and then the torque in the string itself remains constant along its length. However, if the string is deep, then to calculate the actuating torque at the suspension point, the distributed moments of frictional forces between the walls of the borehole well and the string shall be added to the cutting torque. Then, the internal torque in the string becomes variable. And it turns out to be significantly variable at longitudinal and twisting vibrations of the string, when the bit approaches and terminates to contact with the rock at the borehole bottom, and the cutting process becomes intermittent.

The angular velocity value has a significant influence on the cutting moment (rotation moment) value. As the speed change causes non-linear alteration of the torque, self-excitation of torsional vibrations leading to emergency situations is possible in the drill string.

An important factor affecting the quasi-static and dynamic behaviour of the drill string is its rotation. For strings with geometric imperfections and with mass imbalances, this leads to the appearance of centrifugal forces of inertia that significantly affect the stability of the drill string's straight shape. For bending vibration excitation, the rotation is the generating source of the gyroscopic (Coriolis) forces of inertia. These forces connect different types of movements (rotational and linear) and lead to the disturbance of the equiphase condition of vibrations.

Very complex effects in drill strings are generated by external and internal drilling fluid flows. Their motion is first associated with additional friction forces that affect the drill string dynamics. Second, for DS bending vibrations, internal flows (similar to rotational motion) also generate centrifugal and gyroscopic forces of inertia destabilising the straight shape of the string and changing its own bending vibration spectrum. The features of these forces have been studied in detail in the pipelines theory. Their manifestation in the DS has not been sufficiently investigated.

It should be noted that, in essence, all the above forces and effects can occur simultaneously with different combinations of their intensities and lead—depending on the DS length—to various unacceptable modes.




*(a) (b) (c) (d)*

*Fig. 1-2 Vibration modes of the drill string structure bottom part: a = bit axial beating; b = transverse string beating; c = torsional self-excited vibrations of the bit and the string; d = string bottom whirling motion*

Therefore, when extracting fuels from great depths, the increased drilling efficiency of vertical wells using a rotary system is associated with the issue of identifying critical drill string operation modes and developing measures aimed at reducing their negative impact on the drilling process. Such phenomena negatively affecting the drilling process include:

- helical buckling failure of the straight shape of the DS in its bottom part in the form of an extra-long compressed-stretched, twisted rotating rod,
- excitation of DS longitudinal vibrations when exposed to various process disturbances (Fig. 1-2, a),
- excitation of DS resonant bending vibrations due to geometric imperfections and imbalance of the whole system and its individual parts (Fig. 1-2, b),
- parametric build-up of DS slip-stick vibrations caused by non-linear frictional forces between the cutting tool (bit) and the rock being processed (Fig. 1-2, c),
- self-excitation of the bit whirling motion associated with its rolling around the system axis under the conditions of frictional or nonholonomic interaction of the bit with the well bottom surface (Fig. 1-2, d),
- sidewall sticking (loss of mobility) in long curvilinear boreholes with geometrical imperfections (dead lock states) as a result of a sharp increase in the forces of contact and frictional interaction,
- bifurcation buckling of the drill string in the curvilinear borehole channel with non-predictable zones of buckling localisation.

These phenomena can lead to emergencies accompanied by breakage of the DS pipe, sticking of the cutter tool in the rock cutting zone and mashing of the DS sections into the rock, unscrewing of the DS pipes, vertical deviation of the borehole axis and its unplanned distortion, as well as the loss of stability of the borehole walls and their destruction.

The drilling process parameters when critical states occur can be determined using mathematical model methods. However, attempts to conduct practical mathematical experiments on the prediction of the DS critical states are associated with significant computational challenges. Above all, this is because of the features of the relations between the DS geometric parameters.

So, for example, as the diameter of a long string is equal to a $10^{-5}$ part of its length, theoretically it appears to have negligible bending and torsional stiffness. Therefore, mechanical models of strings or absolutely flexible cables are often used for their theoretical investigation. At the same time, to correctly describe the edge and local effects of the DS bending deformation, they shall be calculated based on the beam theory; therefore, using this theory for lengths of several kilometres leads to the appearance of a so-called ‘computational rigidity accompanied by a significant deterioration in the convergence of computational algorithms. In mathematics, the equations modelling these effects are called singularly perturbed [1].

The second complication of the task of the DS quasi-static and dynamic behaviour modelling is associated with a complex combination of forces and influences that affect their quasi-statics and dynamics. Therefore, the problems under consideration are significantly multiparametric. In a general formulation, such problems are unsolvable. So, one of the most rational approaches to their solution is to separate the bending, longitudinal, and torsional movements of the string, consider them separately, and establish the most common patterns of the processes with the determination of their critical states.

### *1.3. Mathematical aspects of deep-drilling string mechanic issues*

The issues of mathematical modelling of static and dynamic mechanical phenomena and critical states arising in deep-drilling strings are associated with considerable theoretical difficulties. Above all, these difficulties are due to the complex nature of the static and dynamic impacts on the drill string and the complexity of the mechanical processes generated by them. Second, the factor of the large drill string length, which leads to a virtual loss of its bending stiffness, has a significant (and determining) effect on the specifics of the processes, the formulation of tasks, and the methods for their solution. Therefore, resolving DS bending equation by integration methods on large integration segments turns out to be difficult to implement. Most noticeably, these difficulties occur while attempting to solve tasks concerning DS bifurcational buckling and free vibrations. So, the problems regarding DS bending stability are given in two formulations.

The first statement is based on the Sturm-Liouville problem formulation on large length $L$ of the string, where the so-called ‘computational rigidity’ phenomenon is very noticeable. This is caused by the fact that the resolving functions of transverse displacements $u(x)$, $v(x)$ of non-trivial solutions vary rapidly with large derivatives on a small segment adjacent to the DS bottom end and have small values with small derivatives on the remaining part. The extraordinary complexity in the study of the combined equations, determining functions $u(x)$, $v(x)$, is due to the fact that the points of the onset of rapid changes in the resolving functions are not known beforehand. In this case, the effects of ‘computational rigidity’ are caused by the high degree of the differential equations and small factors implicitly presenting before the high order derivatives. The smallness of these coefficients appears when scaling integration length $L$ of the equilibrium and vibration equations to a unit segment. Then, the terms with the fourth derivatives shall be divided by $L^4$ and the role thereof in the overall balance of internal forces and moments is significantly reduced. As a result, the solution acquires areas of fast (such as boundary layer) and slow (regular) changes. Should this solution be combined in the form of a superposition of particular solutions that increase and decrease exponentially, then on large integration intervals the first group solution values tend to infinity, the second group solutions go to zero, and the problem of constructing the required solutions of the initial equations becomes impractical even for the two-point boundary value problems. In mathematics, such systems are called singularly perturbed [1].

Due to the above difficulties, the issues of long DS bending stability and natural vibration study have been virtually unexplored. This book offers a technique for their solution based on the application of the initial parameters method in conjunction with the Godunov orthogonalisation procedure (for vertical wells) and the finite-difference procedure with very small step (for curvilinear boreholes).

The formulation of the Sturm-Liouville problem used in the investigation of DS stability makes it possible to determine the beginning of the bifurcation bending process initiation. When it is implemented, the string protrudes and comes into contact with the borehole wall. At this stage, the second DS stability loss step is implemented, at which its supercritical state is examined, and the DS element equilibrium beyond the range of stability is studied for a given (usually, a regular spiral) deformation geometry. Problem formulations based on this approach are widely used in the world literature. They are based on the application of the flexible curved rods theory and are usually related to a number of simplifying assumptions on the nature of the supercritical DS behaviour, which substantially reduce the value of the results obtained on its basis.

This paper considers the initial stage of DS bifurcation buckling (first formulation), for the analysis of which the corresponding Sturm-Liouville problems are formulated along the entire length of the DS in a vertical well. Particular attention is paid to the DS stability calculation, based on singularly perturbed equations using the so-called integrated design schemes, which leads to multipoint boundary-value problems for continuous strings (Chapter 2). It is shown that the loss of stability of strings usually occurs in the shape of spiral wavelets.

Even greater computational difficulties are associated with the problems of DS free vibrations in vertical wells. In Chapter 3, dispersion analysis methods are used to establish that - in contrast to the vibrations of ordinary rods of infinite length that allow for solutions in the mode of standing plane waves - the vibrations of infinite rotating twisted tubes with fluid flows can only occur in the modes of progressive spiral waves (waves with circular or cylindrical polarisation). In addition, each progressive wave length corresponds to four phase-velocity values, different for the left and right spirals and depending on the wave direction. This circumstance indicates that free bending vibrations of a long (albeit finite) DS can only be implemented in spatial modes.

Due to the large length of the DSs, their torsion self-excited vibrations can also assume a special shape (Fig. 1-2, c). They are caused by a significantly non-linear dependence (with extremum points) of the bit rock cutting moment on its angular velocity and are generated through the cycle birth bifurcation. In mathematics, Henri Poincare was among the first to pay attention to the bifurcation nature of self-excited vibrations. A. Andronov considered the likelihood of these effects appearance in mechanical oscillatory systems with nonlinear friction. Later, Hopf provided a mathematical justification for this theory. Therefore, the effects of the build-up of vibrations in non-linear frictional systems were named the Poincare-Andronov-Hopf bifurcations.

Applied mathematics and physics distinguish two types of self-excited vibrations - that is, Thomson and relaxation. Thomson self-excited vibrations proceed in modes close to harmonic ones; relaxation self-excited vibrations are described by periodic or quasi-periodic broken-line functions with almost discontinuous derivatives (velocities). As our analysis shows, torsional self-excited vibrations of drill strings can be classified as relaxational ones. In this regard, their modelling involves much greater computational difficulties. Chapter 4 offers three models of torsional self-excited vibrations in drill strings: wave, vibration model with distributed parameters, and vibration model with a single degree of freedom. The calculations showed that, although the wave model involves quantum nature solutions, in the integral sense the solutions virtually coincided in all cases. However, in the case of self-excited vibrations in extended inclined wells with significant friction forces, the model with distributed parameters turns out to be more accurate.

Apparently, a special type of mechanical vibration called ‘whirling’ is only observed in drilling practice. When they build-up, the bit deviates from its balanced state and starts rolling—generally with slip and torsion—around the system axial line (Fig. 1-2, d). In these cases, the constraints imposed on the bit movement may be nonholonomic. As a result, the bit torsion can be stable and unstable, occurring in forward and backward rotation directions, or the bit centre can move along the most intricate trajectories. Nonholonomic mechanics and geometry methods should be applied to describe these types of vibrations (Chapter 5).

Drilling curvilinear super long boreholes is associated with heavy technical and theoretical difficulties. When drilling, the string can be in rather severe conditions caused by contact and frictional forces. During drilling or tripping operations (for example, to change the bit), these forces reach very high values, especially in places of geometric irregularities of the well centreline in the shapes of dog legs and harmonic or spiral wavelets (Fig. 1-3). They are often the main cause of drilling technique disturbance and lead to the DS sticking. To model these effects, the theory of curvilinear flexible rods, differential geometry, and computational mathematic methods (Chapter 6) shall be applied. Of particular interest is the matter of interfacing two well sections with different curvatures. This book shows that the use of the minimal curvature method, based on the soft string drag & torque model, for this operation is irrational, but it is necessary to match the well sections by introducing small sections in the shape of the Cornu spiral (clothoid) or a cubic parabola. To model this effect, it is advisable to use the stiff string drag & torque model developed by the authors.

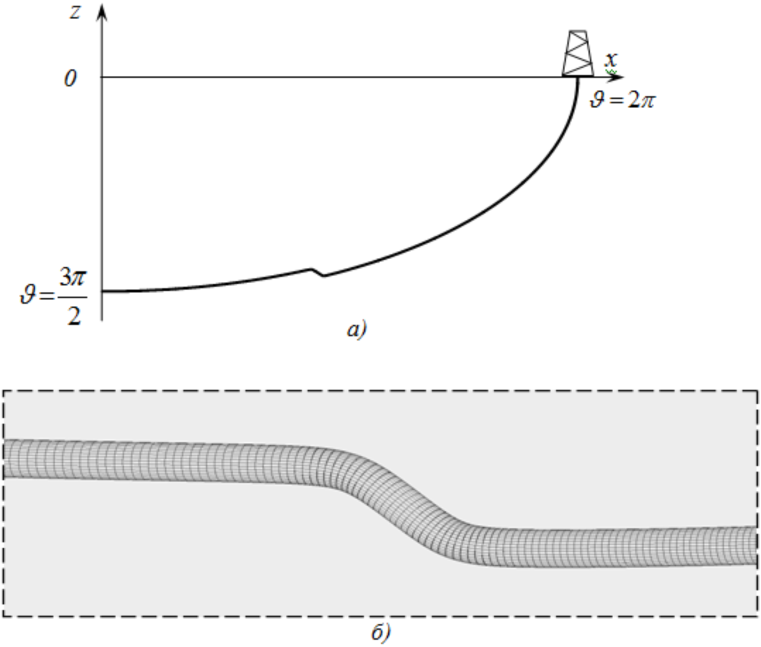



*Fig. 1-3 Axial dog leg path (a), dog leg shape (b)*

Finally, in terms of mechanics and mathematics, the matter of the DS Eulerian (bifurcational) buckling in a curvilinear borehole channel is of great interest. Above all, it must be considered on the basis of the Sturm-Liouville problem formulation for a curved elastic rod. In this case, it is broken down into two separate problems. First, using the stiff string drag & torque model, axial force and torque functions shall be built, and then, using them as coefficients, eigen value equations for the string along the entire length shall be formulated. As the problem formulated is also singularly perturbed, the buckling modes implemented based on the solution have the shape of localised harmonic wavelets, whose locations are not known in advance. In this regard, to solve the problem, an integrated (global) approach should be applied, and localised bulges should be searched for along the entire DS length. In addition, as the bulged string movements are limited by the borehole channel walls, the formulation of the problem requires consideration of non-linear constraints.

We eliminate the constraint equations using differential geometry methods, the channel surface theory and using a special mobile trihedron. The solution of this problem makes it possible to find the critical loads for the string in a curvilinear borehole channel and indicate the location of its buckling zone (Fig. 1-4).

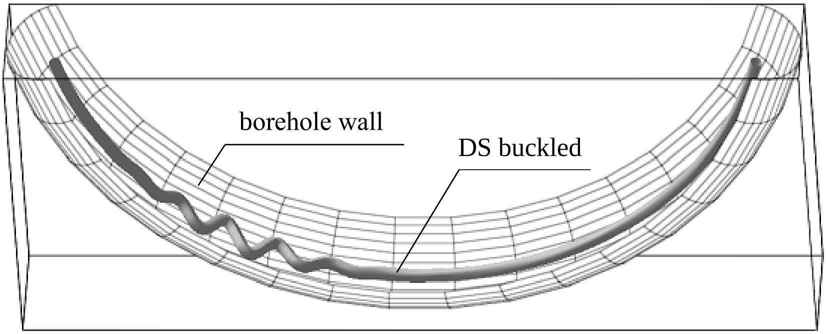



*Fig. 1-4 Diagram of drill string local buckling a curvilinear borehole channel*

Mathematical features of the formulated problems set forth in this subsection make them both rather time-consuming and appealing for a scientist. Some of these problems are solved in the papers of authors (see References [1–20] of Preface).

*References to Chapter 1*

1. Chang K., Howes F. Nonlinear Singular Perturbation Phenomena: Theory and Applications M., Mir. – 1988. – 247 p.
2. Editorial. Avoiding an oil crunch // Science. – 1999. – V.286, № 5437. – P. 47.
3. Iyoho A.W., Meize R.A., Millheim K.K., Crumrine M.J. Lessons from integrated analysis of GOM drilling performance // SPE Drilling & Completion. – March 2005. – P. 6–16.
4. Jonggeun Choe, Jerome J.Schubert, Hans C. Juvkam-Wold. Well-control analyses on extended-reach and multilateral trajectories // SPE Drilling & Completion. – June 2005. – P. 101–108.
5. Kerr R.A. Bumpy road ahead for world’s oil. // Science, 18 Nov.2005, Vol. 310.– P. 1106–1108.

6. Leonardo Maugeri. Oil: never cry wolf- why the petroleum age is far from over// Science. – May 2004. – V.304.– P. 1114–1115.
7. Myslyuk M.A., Rybchich I.I., Yaremiychuk R.S. Well Drilling. Vol. 3, Vertical and directed drilling – Kyiv: “Interpress LTD”. – 2004. – 294 p. (in Ukrainian).

# CHAPTER 2. STABILITY OF THE COMPRESSED-STRETCHED, TWISTED, ROTATING STRINGS WITH INTERNAL FLUIDS IN VERTICAL WELLS

## *2.1 Basic relations of quasi-static equilibrium and vibrations of rotating, twisted drill strings with internal fluid flows*

The bifurcation buckling of a rotating DS is described by the equations of its neutral equilibrium in a perturbed state, which are composed accounting for the presence of internal longitudinal tensile and compression forces, torques, and forces of inertia due to rotation and motion of the internal fluid flow.

Let us formulate a problem of the dynamic equilibrium of a DS in operating conditions, considering the effect of its quasi-static buckling as a special case of its motion. Let the drill string rotates at angular velocity $\omega$. To formulate the equations of its motion, let us introduce inertial coordinate system $OXYZ$ with the origin at the point of suspension and coordinate system $Oxyz$ with unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ associated with the string and rotating together with it (Fig. 1-1).

In the initial undeformed state, axes $OZ$ and $Oz$ coincide with the longitudinal axis of the string. We will investigate the stability of the straight shape of the string in rotating coordinate system $Oxyz$. Assume that the elastic displacements of its elements along axes $Ox$ and $Oy$ are equal to $u$ and $v$, respectively, displacements along $Oz$ axis will be ignored.

Consider that the drill string (Fig. 1-1) is an elastic tubular rod loaded by longitudinal force $T$ and torque $M_z$, which rotates at constant angular velocity $\boldsymbol{\omega}$ about its longitudinal axis. In the pipe channel, fluid with density $\rho_f$ flows at velocity $V$. We will investigate the rod vibration in rotating coordinate system $Oxyz$ with axis $Oz$ directed along the longitudinal axis of the undeformed rod.

To derive the dynamics equations, separate a pipe element $dz$ long and consider the equilibrium of internal moments relative $Oy$, $Ox$ axes system. For the considered combination of forces, these moments include [10, 11, 14, 17] the increments of elastic moments $dM_y$, $dM_x$; moments $-Q_x dz$, $-Q_y dz$ of shear elastic forces $Q_x$, $Q_y$ with arms $dz$; moments $-Tdu$, $-Tdv$ of internal axial force of prestress $T$ generated by increments $du$, $dv$ in interval $dz$ of transverse displacements $u$, $v$ along axes $Ox$, $Oy$; bending moments $-M_z d(dv/dz)$, $M_z d(du/dz)$ caused by the alteration of torque $M_z$ orientation due to increments $d(dv/dz)$, $d(du/dz)$ of angles of rotation $dv/dz$, $du/dz$ in interval $dz$.

Let us sum up these moments

$$
\begin{aligned}
&dM_y - Q_x dz - Tdu - M_z d\left(\frac{dv}{dz}\right) = 0,\\
&dM_x - Q_y dz - Tdv + M_z d\left(\frac{du}{dz}\right) = 0.
\end{aligned}
\tag{2.1}
$$

The bending moments of elastic forces, being a part of this system, are calculated based on beam theory formulas

$$
M_y = -EI\frac{d^2u}{dz^2}, \qquad M_x = -EI\frac{d^2v}{dz^2}. \tag{2.2}
$$

The equilibrium of forces applied to the element in the direction of axes $Ox$, $Oy$ is described using equations

$$
dQ_x + q_x dz = 0, \qquad dQ_y + q_y dz = 0, \tag{2.3}
$$

where $q_x$, $q_y$ are the internal distributed forces directed along the corresponding axes.

Let us change system (2.2), (2.3) to the following form:

$$
\begin{aligned}
&Q_x = \frac{dM_y}{dz} - T\frac{du}{dz} - M_z\frac{d^2v}{dz^2}, \qquad Q_y = \frac{dM_x}{dz} - T\frac{dv}{dz} + M_z\frac{d^2u}{dz^2},\\
&\frac{dQ_x}{dz} + q_x = 0, \qquad \frac{dQ_y}{dz} + q_y = 0.
\end{aligned}
\tag{2.4}
$$

With equations (2.2), system (2.4) may be simplified to the system of two bending equilibrium equations of a rod prestressed by longitudinal force $T$ and torque $M_z$,

$$
\begin{aligned}
&EI\frac{\partial^4 u}{\partial z^4} - \frac{\partial}{\partial z}\left(T\frac{\partial u}{\partial z}\right) - \frac{\partial^2}{\partial z^2}\left(M_z\frac{\partial v}{\partial z}\right) = q_x,\\
&EI\frac{\partial^4 v}{\partial z^4} - \frac{\partial}{\partial z}\left(T\frac{\partial v}{\partial z}\right) + \frac{\partial^2}{\partial z^2}\left(M_z\frac{\partial u}{\partial z}\right) = q_y.
\end{aligned}
\tag{2.5}
$$

The right-hand sides of these equalities contain distributed forces $q_x$, $q_y$. Taking into account that the DS is not exposed to active forces, transverse load $\boldsymbol{q}$ based on d'Alembert's principle shall be taken to be equal to the forces of inertia caused by rod motion $\boldsymbol{q}_r$ and liquid flow $\boldsymbol{q}_f$, that is

$$
\boldsymbol{q} = \boldsymbol{q}_r + \boldsymbol{q}_f .
$$

For the rod element, distributed force of inertia $\mathbf{q}_\mathrm{r}$ shall be calculated as follows:

$$\boldsymbol{q}_r = -\rho_r F \boldsymbol{a}_r,$$

where $\rho_r$ is the linear density of the rod; $F$ is its cross section area; $\boldsymbol{a}_r$ is the absolute acceleration of the element.

In rotating coordinate system $Oxyz$, absolute acceleration $\boldsymbol{a}_r$ shall be calculated from the Coriolis formula [8, 21]

$$\boldsymbol{a}_r = \boldsymbol{a}_r^e + \boldsymbol{a}_r^r + \boldsymbol{a}_r^c,$$

where $\boldsymbol{a}_r^e$, $\boldsymbol{a}_r^r$, $\boldsymbol{a}_r^c$ are the bulk, relative, and Coriolis acceleration vectors, respectively.

Bulk acceleration vector $\boldsymbol{a}_r^e$ shall be calculated from the formula

$$\boldsymbol{a}_r^e = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}), \tag{2.6}$$

where $\boldsymbol{r} = u\boldsymbol{i} + v\boldsymbol{j} + z\boldsymbol{k}$ is the radius-vector of a beam element in coordinate system $Oxyz$.

Having performed the corresponding vector operations, we obtain

$$a_{r,x}^e = -\omega^2 u, \qquad a_{r,y}^e = -\omega^2 v, \qquad a_{r,z}^e = 0. \tag{2.7}$$

The components of the relative acceleration vector in the directions of the coordinate system $Oxyz$ axes are determined by equalities

$$a_{r,x}^r = \frac{d^2u}{dt^2}, \qquad a_{r,y}^r = \frac{d^2v}{dt^2}, \qquad a_{r,z}^r = 0. \tag{2.8}$$

Coriolis acceleration $\boldsymbol{a}^c$ vector of the rod element shall be calculated from the formula

$$\boldsymbol{a}_r{}^c = 2\boldsymbol{\omega} \times \boldsymbol{V}_r{}^r, \tag{2.9}$$

where $\boldsymbol{V}_r{}^r$ is the relative velocity vector of the element with components

$$V_x^r = \frac{du}{dt}, \qquad V_y^r = \frac{dv}{dt}, \qquad V_z^r = 0. \tag{2.10}$$

Accounting for equalities (2.9) and (2.10), we have

$$a_{r,x}^c = -2\omega \frac{dv}{dt}, \qquad a_{r,y}^c = 2\omega \frac{du}{dt}, \qquad a_{r,z}^c = 0. \tag{2.11}$$

By adding the obtained component values of accelerations (2.7), (2.8), (2.11), we obtain the components of the rod element rotational motion inertial force vector

$$q^{\omega}_{r,x} = -\rho_r F(-\omega^2 u - 2\omega \frac{dv}{dt} + \frac{d^2u}{dt^2}),$$
$$q^{\omega}_{r,y} = -\rho_r F(-\omega^2 v + 2\omega \frac{du}{dt} + \frac{d^2v}{dt^2}). \qquad (2.12)$$

The distributed force of inertia acting on the moving fluid element shall be calculated as follows:

$$\boldsymbol{q}_f = -\rho_f F_f \boldsymbol{a}_f , \qquad (2.13)$$

where $\rho_f$ is the linear density of the fluid; $F_f$ is the cross section area of the pipe channel; $\boldsymbol{a}_f$ is the absolute acceleration of the fluid element. It consists of rotary acceleration together with the rod and acceleration due to self-motion in the pipe channel.

The first component is calculated according to the pattern of formulas (2.6)–(2.12). When calculating the second component, let us take into account that the element occupies a new position on the beam at each instant; therefore, its velocity, for example, along $Ox$ axis is determined not only by the velocity of the rod point, where the element is located, but also by the fact that the fluid moves to an adjacent point on the rod with a different $z$ coordinate. Then, we can write

$$\frac{dx}{dt} = \frac{\partial x}{\partial t} + \frac{\partial x}{\partial z} \cdot \frac{\partial z}{\partial t} = \dot{x} + x' V_f . \qquad (2.14)$$

Here, $V_f$ is the fluid velocity along axis $Oz$. The dot is used to indicate the differentiation with respect to $t$; the prime mark, with respect to $z$.

The presentation of velocity $dx/dt$ as (2.14) has an analog in field theory, where operator $d/dt$ is called the substantial time derivative; $\partial/\partial t$, the local derivative operator, and the expression $(\partial x/\partial z) \cdot (\partial z/\partial t)$ is referred to the convective component of the field value alteration.

By differentiating again both parts of expression (2.14) with respect to time, we can find the transverse component of the fluid element absolute acceleration

$$\frac{d^2x}{dt^2} = \frac{\partial^2 x}{\partial t^2} + 2\frac{\partial^2 x}{\partial z \partial t} V_f + \frac{\partial x}{\partial z} \cdot \frac{\partial V_f}{\partial t} + \frac{\partial^2 x}{\partial z^2} V_f^2 + \frac{\partial x}{\partial z}\frac{\partial V_f}{\partial t} =$$
$$= \ddot{x} + 2\dot{x}' V_f + V_f x' \frac{dV_f}{dz} + x'' V_f^2 + x' \dot{V}_f \qquad (2.15)$$

This formula can be compared with the Coriolis theorem formula for the absolute acceleration of a material point, where $\ddot{x}$ constitutes the relative acceleration; $2\dot{x}'V_f$, the Coriolis acceleration; $x''V_f^2$, the centrifugal acceleration; $\dot{x}'$, the angular rotation velocity of the rod element.

Using our notations $u$, $v$ for displacements along axes $Ox$, $Oy$ and assuming that the fluid flows inside the tubular rod at constant speed $V_f$, we obtain expressions for the accelerations due to its motion in the oscillating (not rotating) tube

$$\begin{aligned} \frac{d^2u_f}{dt^2} &= \frac{\partial^2 u_r}{\partial t^2} + 2V_f \frac{\partial^2 u_r}{\partial z \partial t} + V_f^2 \frac{\partial^2 u_r}{\partial z^2}, \\ \frac{d^2v_f}{dt^2} &= \frac{\partial^2 v_r}{\partial t^2} + 2V_f \frac{\partial^2 v_r}{\partial z \partial t} + V_f^2 \frac{\partial^2 v_r}{\partial z^2}. \end{aligned} \tag{2.16}$$

Here, $u_f$, $v_f$ are the transverse displacements of the fluid element; $u_r$, $v_r$ are the transverse displacements of the rod element.

As the fluid also participates in the rotational motion along with the tube, it is also exposed to distributed forces of form (2.12); the complete components of the distributed forces of inertia applied thereto are determined by the following relations:

$$\begin{aligned} q_{f,x} &= -\rho_f F_f \left[ \left( -\omega^2 u - 2\omega \frac{\partial v}{\partial t} + \frac{\partial^2 u}{\partial t^2} \right) + \left( 2V_f \frac{\partial^2 u}{\partial z \partial t} + V_f^2 \frac{\partial^2 u}{\partial z^2} \right) \right], \\ q_{f,y} &= -\rho_f F_f \left[ \left( -\omega^2 v + 2\omega \frac{\partial u}{\partial t} + \frac{\partial^2 v}{\partial t^2} \right) + \left( 2V_f \frac{\partial^2 v}{\partial z \partial t} + V_f^2 \frac{\partial^2 v}{\partial z^2} \right) \right]. \end{aligned} \tag{2.17}$$

Substituting the right-hand sides of these equalities into equation (2.5), we obtain the vibration equations of a rotating tubular rod prestressed by force $T$, torque $M_z$ and containing fluid flows

$$EI \frac{\partial^4 u}{\partial z^4} - \frac{\partial}{\partial z}\left( T \frac{\partial u}{\partial z} \right) - \frac{\partial^2}{\partial z^2}\left( M_z \frac{\partial v}{\partial z} \right) - \left( \rho_r F_r + \rho_f F_f \right)\omega^2 u -$$
$$- 2\left( \rho_r F_r + \rho_f F_f \right)\omega \frac{\partial v}{\partial t} + V^2 \rho_f F_f \frac{\partial^2 u}{\partial z^2} + 2V\rho_f F_f \frac{\partial^2 u}{\partial z \partial t} + \left( \rho_r F_r + \rho_f F_f \right)\frac{\partial^2 u}{\partial t^2} = 0,$$

$$EI \frac{\partial^4 v}{\partial z^4} - \frac{\partial}{\partial z}\left( T \frac{\partial v}{\partial z} \right) + \frac{\partial^2}{\partial z^2}\left( M_z \frac{\partial u}{\partial z} \right) - \left( \rho_r F_r + \rho_f F_f \right)\omega^2 v +$$
$$+ 2\left( \rho_r F_r + \rho_f F_f \right)\omega \frac{\partial u}{\partial t} + V^2 \rho_f F_f \frac{\partial^2 v}{\partial z^2} + 2V\rho_f F_f \frac{\partial^2 v}{\partial z \partial t} + \left( \rho_r F_r + \rho_f F_f \right)\frac{\partial^2 v}{\partial t^2} = 0. \tag{2.18}$$