Planar Trusses

Planar Trusses:

Schemes and Formulas

By

Mikhail Kirsanov

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CONTENT

Introduction

Engineers have been consistently interested in new layouts for statically determinate trusses for a couple of reasons. Firstly, they desire optimal solutions for durability, rigidity and cost savings on materials and installation, and secondly, their interest in the architectural expressiveness of the structures of bridges, coatings, and buildings. Importantly, not all engineered bridge designs feature simple triangular, diagonal or lattice layouts. Here are some examples of well-known original bridges based on statically determinate trusses (Fig. 1 – 3).

Fig. 1. Bridge over the Firth of Forth, Scotland [1]

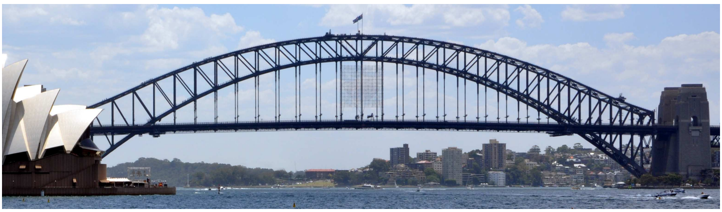

Fig. 2. Harbor Bridge, Sydney, Australia

Hutchinson R.G. and Fleck N.A. have engaged in a purposeful search for such planar and spatial configurations, and it is no coincidence that these searches are called "hunting for statically determinate configurations" [14, 15].

The statically determinate truss structures in their pure form are rarely used, with hinges most often replaced by rigid or semi-

[1] The dimensions of the bridge are impressive, with spans of 521 m, rod (pipe) diameters of up to 3.6 m, and support heights of 100 m.

rigid joints, and additional rods simply turning an original statically determinate truss into a statically indeterminate one to ensure rigidity. During flexibility analysis, a statically determinate truss can be considered a model of a real structure, or, the main system in the force method.

Statically determinate trusses are also of interest for their simplicity of calculation, which in most cases is performed numerically. Modern symbolic mathematics software tools, e.g. Maple, Mathematica, Derive, Reduce, etc., allow performing the same calculations analytically as well. When entering the data it is enough to merely replace the numbers with symbols, thereby obtaining the final formula for the deflection or force in a rod, depending on the load, dimensions and elastic characteristics of the material of construction. With this formula, it is possible to change the size of the truss, the magnitude of the load, and the elastic moduli of the rods, while taking into account the strength and stiffness. Although, the scope of application of such a formula is still limited to a specific design.

Fig. 3. Road bridge in Seattle, WA, USA

Far more interesting formulas can be applied to a whole class of designs and can be determined for regular systems with a certain symmetry and periodicity of the structure. The element of periodicity can be a truss panel or a group of rods. In more complex cases, several independent periodicity groups can be distinguished in a truss, for example, in the frames of the panel group along the span length and panel height. Then, induction is carried out on two (trusses 4.10, p. 156, 4.14, p. 167) or more (trusses 4.12, p. 162, 4.13, p. 165) natural numbers, thereby widening the scope of application of the formulas.

In the works of the author and his students, the method of induction was used to obtain analytical solutions of the deflection problem for a number of planar and spatial trusses in the form of finite formulas. Some of these solutions are included in this handbook.

The trusses are divided into five chapters in the handbook, but this division is conditional, with some trusses appearing in more than one chapter. Trusses with rather complicated lattice work, assigned to simple girder trusses, may be mentioned in the second chapter which describes lattices with two supports (girder lattices), and some arches and frames featuring several supports correspond quite well to the trusses from the third chapter which focuses on external statically indeterminate trusses. Formulas for the deflection of trusses are obtained for the case of equal stiffness of all the rods of a truss.

Although the author tried to avoid complex solutions, the formulas in the handbook are sometimes cumbersome. Using them involves manual typing, which concedes errors and typos, thus negating the advantages of beautiful analytical solutions. To avoid this, the author converted all the solutions from the Maple software into text files, and by simply copying, all the formulas can be extracted. These texts are free to download from: http://vuz.exponenta.ru/Trusstxt.rar.

In the handbook, the notation Δ is introduced for deflection (vertical displacement of the hinge); δ_A is notation for the horizontal displacement of the movable support.

Formulas are found only for some forces that, as a rule, are critical with respect to loss of stability or strength. In the rods the notations are standard in structural mechanics. For upper belt the notation is O, lower belt U, the forces in the bracings are marked D, and in the uprights, V. The rods have stiffness EF; where E is the modulus of elasticity, F is the section area.

The author accepts all comments and suggestions at c216@ya.ru.

Chapter 1

Beam trusses with parallel belts

Truss 1.1

Truss with a height of h (Fig. 4) containing $2n$ panels in the lower zone, consists of $8n-1$ rods.

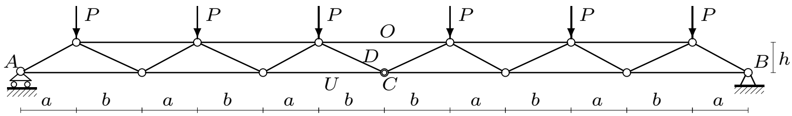

Fig. 4. Truss, $n=3$

The deflection (vertical displacement of the middle node C in the lower zone) has the form

$$\Delta = Pn(C_1a^3+C_2b^3+C_3c^3+C_4d^3+C_5a^2b+C_6ab^2)/(2h^2EF), \quad (1.1)$$

where $c=\sqrt{a^2+h^2}$, $d=\sqrt{b^2+h^2}$.

1.1.1. Load on the upper belt Fig. 4

Deflection. The coefficients in (1.1):

$C_1=(5n^3+4n^2+n+2)/6$, $C_2=(5n^3-4n^2+n-2)/6$,

$C_3=n+1$, $C_4=n-1$, $C_5=(15n^3+4n^2+3n+2)/6$,

$C_6=(15n^3-4n^2+3n-2)/6$,

$\Delta_{b=a}=Pn^2(2a^3(5n^2+1)+3c^3)/(3h^2EF)$.

Support offset:

$\delta_A=Pn(a(n+1)(2n+1)+b(2n-1)(n-1))(a+b)/(3hEF)$,

$\delta_{A,b=a}=4Pa^2n(2n^2+1)/(3hEF)$.

Forces:

$O = -Pn(a(n+1)+b(n-1))/(2h)$, $U = -O$, $D = 0$.

Supports reactions: $Y_A = Y_B = Pn$, $X_B = 0$.

1.1.2. Load on the lower belt

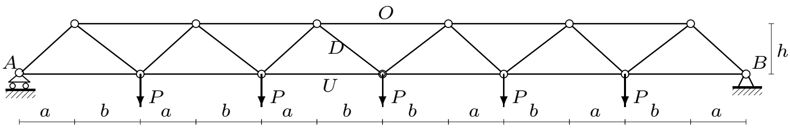

Fig. 5. Truss, $n = 3$

Deflection. The coefficients in (1.1):

$C_1 = C_2 = n(5n^2+1)/6$, $C_5 = C_6 = n(5n^2-1)/2$, $C_3 = C_4 = n$,

$\Delta_{b=a} = Pn^2(a^3(10n^2-1)+3c^3)/(3h^2EF)$.

Support offset:

$\delta_A = Pn(a(4n^2+3n-1)+b(4n^2-3n-1))(a+b)/(6hEF)$,

$\delta_{A,b=a} = 2Pa^2n(4n^2-1)/(3hEF)$.

Forces:

$O = -Pn^2(a+b)/(2h)$,

$U = P(n^2a+(n^2-1)b)/(2h)$, $D = Pd/(2h)$.

Reaction of supports:

$Y_A = Y_B = P(2n-1)/2$, $X_B = 0$.

1.1.3. Focused force in the middle of the span

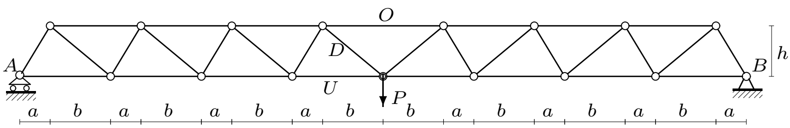

Fig. 6. Truss, $n = 4$, [36]

Deflection. The coefficients in (1.1):

$C_1 = C_2 = (1 + 2n^2)/3,\ C_3 = C_4 = 1,\ C_5 = C_6 = 2n^2,$

$\Delta_{b=a} = Pn(a^3(8n^2 + 1) + 3c^3)/(3h^2EF).$

Support offset:

$\delta_A = Pn(a + b)(a(n + 1) + b(n - 1))/(2hEF),$

$\delta_{A,b=a} = 2Pn^2a^2/(hEF).$

Forces:

$O = -Pn(a+b)/(2h),\ U = P(na+(n-1)b)/(2h),\ D = Pd/(2h).$

Reaction of supports:

$Y_A = Y_B = P/2,\ X_B = 0.$

Truss 1.2

Truss with a height of h (Fig. 7) containing n panels in the lower zone, consists of $8n + 1$ rods [43, 50].

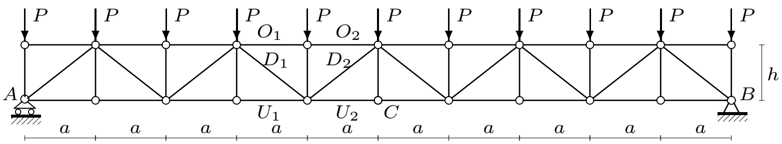

Fig. 7. Truss, $n = 5$

The deflection (vertical displacement of the middle node C in the lower zone) has the form

$$\Delta = P(C_1a^3 + C_2c^3 + C_3h^3)/(2h^2EF), \tag{1.2}$$

where $c = \sqrt{a^2 + h^2}$.

1.2.1. Load on the upper belt Fig. 7

Deflection. The coefficients in (1.2):

$C_1 = n^2(5n^2 + 1)/6,\ C_2 = n^2,\ C_3 = 0.$

Support offset:

$\delta_A = Pa^2n(2n^2 + 1)/(3hEF).$

Forces ($n > 1$):

$O_1 = -Pa(2n^2 - 3(-1)^n - 5)/(4h)$,

$O_2 = -Pa(2n^2 + (-1)^n - 1)/(4h)$,

$D_1 = -3Pc(-1)^n/(2h)$, $D_2 = Pc(-1)^n/(2h)$,

$U_1 = Pa(2n^2+3(-1)^n-5)/(4h)$, $U_2 = Pa(2n^2-(-1)^n-1)/(4h)$.

Supports reactions:

$Y_A = Y_B = P(2n + 1)/2$.

1.2.2. Loading the lower belt

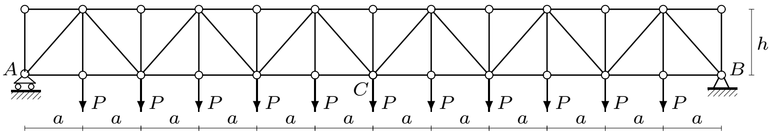

Fig. 8. Truss, $n = 6$

Deflection. The coefficients in (1.2):

$C_1 = n^2(5n^2 + 1)/6$, $C_2 = n^2$, $C_3 = 1 - (-1)^n$.

Support offset:

$\delta_A = Pa^2n(2n^2 + 1)/(3hEF)$.

Forces ($n > 1$):

$O_1 = -Pa(2n^2 - 3(-1)^n - 5)/(4h)$,

$O_2 = -Pa(2n^2 + (-1)^n - 1)/(4h)$,

$D_1 = -3Pc(-1)^n/(2h)$, $D_2 = Pc(-1)^n/(2h)$,

$U_1 = Pa(2n^2 + 3(-1)^n - 5)/(4h)$,

$U_2 = Pa(2n^2 - (-1)^n - 1)/(4h)$.

Supports reactions:

$Y_A = Y_B = P(2n - 1)/2$, $X_B = 0$.

1.2.3. Concentrated force in the mid-span

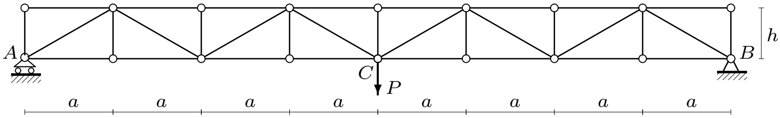

Fig. 9. Truss, $n = 4$

Deflection. The coefficients in (1.2):

$C_1 = n(2n^2 + 1)/3$, $C_2 = n$, $C_3 = 1 - (-1)^n$.

Support offset:

$\delta_A = Pa^2(2n^2 - (-1)^n + 1)/(4hEF)$.

Forces ($n > 1$):

$O_1 = -Pa(2n-(-1)^n-3)/(4h)$, $O_2 = -Pa(2n+(-1)^n-1)/(4h)$,

$D_1 = -Pc(-1)^n/(2h)$, $D_2 = Pc(-1)^n/(2h)$,

$U_1 = Pa(2n + (-1)^n - 3)/(4h)$, $U_2 = Pa(2n - (-1)^n - 1)/(4h)$.

Supports reactions: $Y_A = Y_B = P/2$, $X_B = 0$.

Truss 1.3

Truss with a height of h (Fig. 10) containing $2n$ panels in the lower zone, consists of $8n + 1$ rods.

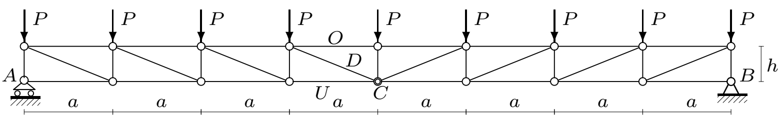

Fig. 10. Truss, $n = 4$

The deflection (vertical displacement of the middle node C in the lower zone) has the form

$$\Delta = P(C_1a^3 + C_2c^3 + C_3h^3)/(2h^2EF), \quad (1.3)$$

where $c = \sqrt{a^2 + h^2}$.

1.3.1. Load on the upper belt Fig. 10

Deflection. The coefficients in (1.3):

$C_1 = n^2(1 + 5n^2)/6$, $C_2 = n^2$, $C_3 = n^2 + 2n$.

Support offset:

$$\delta_A = Pa^2n(4n + 1)(n - 1)/(6hEF). \qquad (1.4)$$

Forces:

$$O = -Pan^2/(2h, \; U = Pa(n^2 - 1)/(2h), \; D = Pc/(2h). \qquad (1.5)$$

Supports reactions:

$Y_A = Y_B = P(2n + 1)/2$, $X_B = 0$.

1.3.2. Loading the lower belt

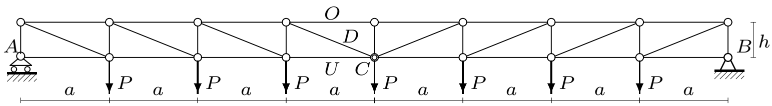

Fig. 11. Truss, $n = 4$

Deflection. The coefficients in (1.3):

$C_1 = n^2(1 + 5n^2)/6$, $C_2 = C_3 = n^2$.

Support offset: (1.4)

Forces: (1.5)

Supports reactions:

$Y_A = Y_B = P(2n - 1)/2$, $X_B = 0$.

1.3.3. Concentrated force in the mid-span

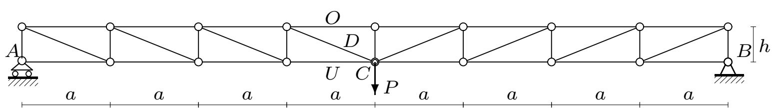

Fig. 12. Truss, $n = 4$

Deflection. The coefficients in (1.3):

$C_1 = n(1 + 2n^2)/3$, $C_2 = C_3 = n$.

Support offset:

$\delta_A = Pn(n-1)a^2/(2hEF)$.

Forces:

$O = -Pan/(2h)$, $U = Pa(n-1)/(2h)$, $D = Pc/(2h)$.

Supports reactions:

$Y_A = Y_B = P/2$, $X_B = 0$.

Truss 1.4

Truss with a height of h (Fig. 13) containing $2n$ panels in the lower zone, consists of $12n + 1$ rods.

The deflection (vertical displacement of the middle node C in the lower zone) has the form

$$\Delta = P(C_1a^3 + C_2c^3 + C_3h^3)/(2h^2EF), \tag{1.6}$$

where $c = \sqrt{a^2 + h^2}$.

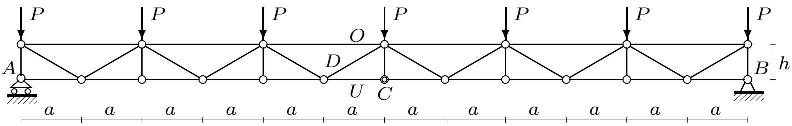

Fig. 13. Truss, $n = 3$

1.4.1. Load on the upper belt Fig. 13

Deflection. The coefficients in (1.6):

$C_1 = 2n^2(10n^2 - 1)/3$, $C_2 = 2n^2$, $C_3 = 2n + 1$.

Support offset:

$\delta_A = 2Pa^2n(4n^2 - 1)/(3hEF)$.

Forces:

$O = -Pa(2n^2 - 1)/(2h)$, $U = Pan^2/h$,

$D = -Pc/(2h)$.

Supports reactions:

$Y_A = Y_B = P(2n+1)/2$, $X_B = 0$.

1.4.2. Loading the lower belt

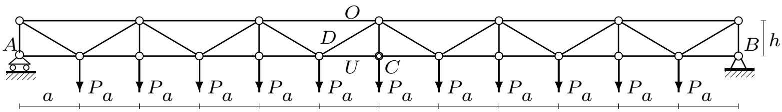

Fig. 14. Truss, $n = 3$

Deflection. The coefficients in (1.6):

$C_1 = 2n^2(20n^2+1)/3$, $C_2 = 4n^2$, $C_3 = 4n+1$.

Support offset:

$\delta_A = 4Pn(4n^2-1)a^2/(3hEF)$.

Forces:

$O = -Pa(4n^2-1)/(2h)$, $U = 2aPn^2/h$,

$D = -Pc/(2h)$.

Supports reactions:

$Y_A = Y_B = P(4n-1)/2$, $X_B = 0$.

1.4.3. Concentrated force in the mid-span

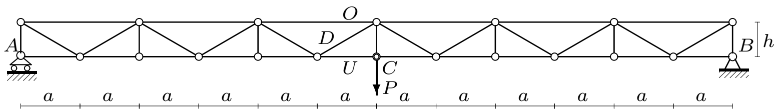

Fig. 15. Truss, $n = 3$

Deflection. The coefficients in (1.6):

$C_1 = 2n(1+8n^2)/3$, $C_2 = 2n$, $C_3 = 3$.

Support offset:

$\delta_A = 2Pn^2a^2/(hEF)$.

Forces:

$O = -Pa(2n-1)/(2h)$, $U = Pan/h$, $D = -Pc/(2h)$.

Supports reactions:

$Y_A = Y_B = P/2$, $X_B = 0$.

Truss 1.5

Truss with a height of h (Fig. 16) containing $2n$ panels in the lower zone, consists of $6n-1$ rods.

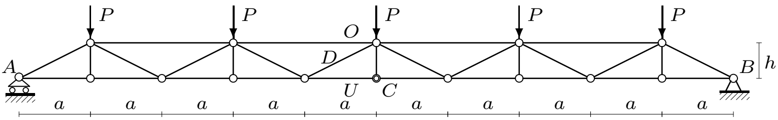

Fig. 16. Truss, $n = 5$

The deflection (vertical displacement of the middle node C in the lower zone) has the form

$$\Delta = P(C_1a^3 + C_2c^3 + C_3h^3)/(2h^2EF), \tag{1.7}$$

where $c = \sqrt{a^2+h^2}$.

1.5.1. Load on the upper belt Fig. 16

Deflection. The coefficients in (1.7):

$C_1 = (5n^4 + 4n^2 + 3(1-(-1)^n)/2)/12$,

$C_2 = (2n^2 + 1 - (-1)^n)/4$, $C_3 = 0$.

Support offset:

$\delta_A = Pn(2+n^2)a^2/(3hEF)$.

Forces:

$O = -Pa(2n^2 + (-1)^n - 1)/(8h)$, $D = -Pc(1-(-1)^n)/(4h)$,

$U = Pa(2n^2 - (-1)^n + 1)/(8h)$.

Supports reactions: $Y_A = Y_B = Pn/2$, $X_B = 0$.

1.5.2. Loading the lower belt

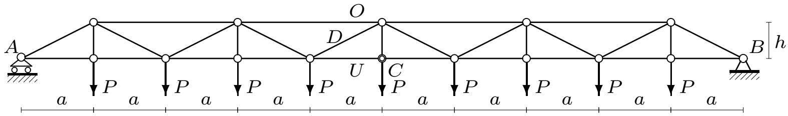

Fig. 17. Truss, $n = 5$

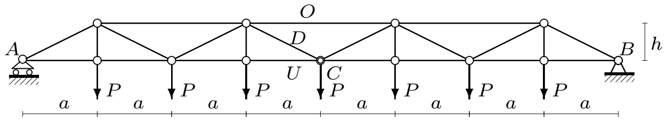

Fig. 18. Truss, $n = 4$

Deflection. The coefficients in (1.7):

$C_1 = n^2(1 + 5n^2)/6$, $C_2 = n^2$, $C_3 = 1 - (-1)^n$.

Support offset:

$\delta_A = Pn(2n^2 + 1)a^2/(3hEF)$.

Forces:

$O = Pa(1 - 2n^2 - (-1)^n)/(4h)$, $D = Pc(-1)^n/(2h)$,

$U = Pa(2n^2 - (-1)^n - 1)/(4h)$.

Supports reactions:

$Y_A = Y_B = P(2n - 1)/2$, $X_B = 0$.

1.5.3. Concentrated force in the mid-span

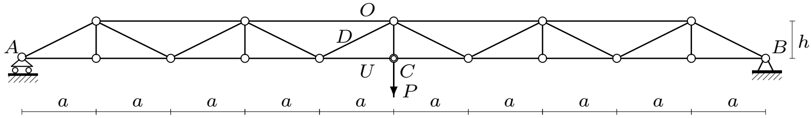

Fig. 19. Truss, $n = 5$

Deflection. The coefficients in (1.7):

$C_1 = n(2n^2+1)/3$, $C_2 = n$, $C_3 = 1-(-1)^n$.

Support offset:

$\delta_A = Pa^2(2n^2+1-(-1)^n)/(4hEF)$.

Forces:

$O = Pa(1-(-1)^n-2n)/(4h)$, $D = Pc(-1)^n/(2h)$,

$U = Pa(2n-(-1)^n-1)/(4h)$.

Supports reactions:

$Y_A = Y_B = P/2$, $X_B = 0$.

Truss 1.6

Truss with a height of h (Fig. 20) containing $2n$ panels in the lower zone, consists of $8n-1$ rods.

The deflection (vertical displacement of the middle node C in the upper zone) has the form

$$\Delta = P(C_1a^3 + C_2c^3)/(2h^2EF), \quad (1.8)$$

where $c = \sqrt{a^2+h^2}$.

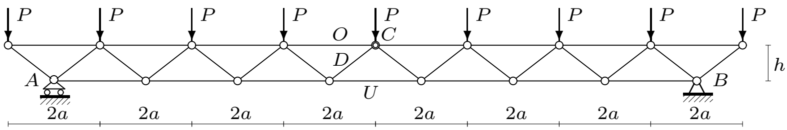

Fig. 20. Truss, $n = 4$

1.6.1. Load on the upper belt Fig. 20

Deflection. The coefficients in (1.8):

$C_1 = (20n^4-40n^3+10n^2+10n-3)/3$, $C_2 = 2n^2-2n+1$.

Support offset:

$\delta_A = Pa^2(8n^3-12n^2-2n+3)/(3hEF)$.

Forces:

$O = -Pa(n^2-n-1)/h$,

$U = Pa(2n^2-2n-1)/(2h)$, $D = -Pc/(2h)$.

Supports reactions:

$Y_A = Y_B = P(2n+1)/2,\ X_B = 0.$

1.6.2. Load on consoles

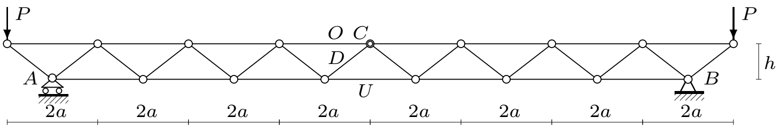

Fig. 21. Truss, $n = 4$

Deflection. The coefficients in (1.8):

$C_1 = -2(2n-1)^2,\ C_2 = 0.$

Support offset:

$\delta_A = 2Pa^2(2n-1)/(hEF).$

Forces:

$O = Pa/h,\ U = -Pa/h,\ D = 0.$

Supports reactions:

$Y_A = Y_B = P,\ X_B = 0.$

1.6.3. Loading the lower belt

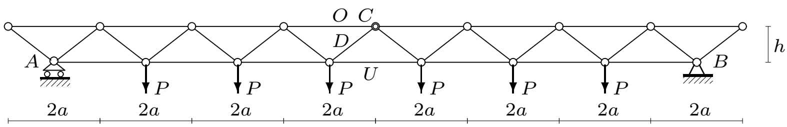

Fig. 22. Truss, $n = 4$

Deflection. The coefficients in (1.8):

$C_1 = 4n(n-1)(5n^2-5n+2)/3,\ C_2 = 2n(n-1).$

Support offset:

$\delta_A = 4Pna^2(2n^2-3n+1)/(3hEF).$

Forces:

$O = -Pn(n-1)a/h$, $D = 0$, $U = -O$.

Supports reactions:

$Y_A = Y_B = P(n-1)$, $X_B = 0$.

1.6.4. Concentrated force in the mid-span

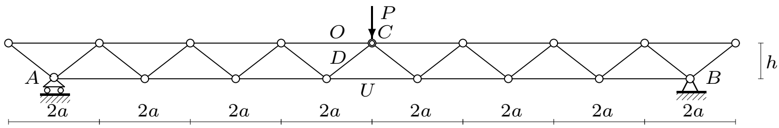

Fig. 23. Truss, $n = 4$

Deflection. The coefficients in (1.8)

$C_1 = (16n^3 - 24n^2 + 14n - 3)/3$, $C_2 = 2n - 1$.

Support offset:

$\delta_A = Pa^2(2n^2 - 2n + 1)/(hEF)$.

Forces:

$O = -Pa(n-1)/h$, $D = -Pc/(2h)$, $U = Pa(2n-1)/(2h)$.

Supports reactions:

$Y_A = Y_B = P/2$, $X_B = 0$.

Truss 1.7

Truss with a height of $4h$ (Fig. 24) containing $2n$ panels in the upper zone, consists of $64n - 3$ rods.

The deflection (vertical displacement of the middle node C in the lower zone) has the form

$$\Delta = P(C_1a^3 + C_2c^3 + C_3h^3)/(h^2EF), \qquad (1.9)$$

where $c = \sqrt{a^2 + h^2}$.

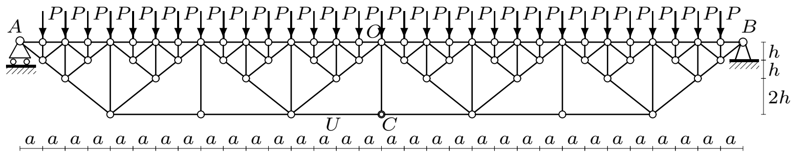

Fig. 24. Truss, $n = 2$

1.7.1. Load on the upper belt Fig. 24

Deflection. The coefficients in (1.9):

$C_1 = 4n^2(80n^2 + 13)/3$, $C_2 = 32n^2$, $C_3 = 0$.

Support offset:

$\delta_A = 8Pa^2n(32n^2 + 13)/(3hEF)$.

Forces:

$O = -Pa(16n^2 - 1)/(2h)$, $U = 8Pan^2/h$.

Supports reactions:

$Y_A = Y_B = P(16n - 1)/2$, $X_B = 0$.

1.7.2. Loading the lower belt

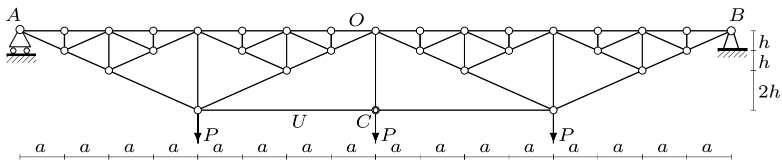

Fig. 25. Truss, $n = 1$

Deflection. The coefficients in (1.9):

$C_1 = 4n^2(20n^2 + 1)/3$, $C_2 = 8n^2$, $C_3 = 4$.

Support offset:

$\delta_A = 8Pa^2n(8n^2 + 1)/(3hEF)$.

Forces:

$O = -Pa(4n^2 - 1)/(2h)$, $U = 2Pan^2/h$.

Supports reactions:

$Y_A = Y_B = P(4n-1)/2,\ X_B = 0.$

1.7.3. Concentrated force in the mid-span

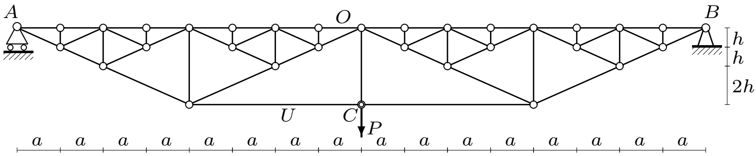

Fig. 26. Truss, $n = 1$

Deflection. The coefficients in (1.9):

$C_1 = 4n(8n^2+1)/3,\ C_2 = 4n,\ C_3 = 4.$

Support offset:

$\delta_A = 8Pa^2n^2/(hEF).$

Forces:

$O = -Pa(2n-1)/(2h),\ U = Pna/h.$

Supports reactions:

$Y_A = Y_B = P/2,\ X_B = 0.$

Truss 1.8

Truss with a height of $3h$ (Fig. 27) containing $2n$ panels in the lower zone, consists of $24n+1$ rods.

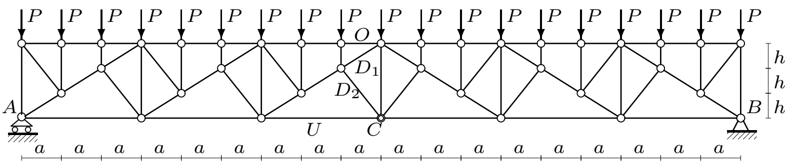

Fig. 27. Truss, $n = 3$

The deflection (vertical displacement of the middle node C in the lower zone) has the form

$$\Delta = P(C_1 a^3 + C_2 c^3 + C_3 h^3)/(2h^2 EF), \qquad (1.10)$$

where $c = \sqrt{a^2 + h^2}$.

1.8.1. Load on the upper belt Fig. 27

Deflection. The coefficients in (1.10):

$C_1 = n^2(15n^2 + 7)/2$, $C_2 = 9n^2$, $C_3 = 9n^2 - 12n + 11$.

Support offset:

$\delta_A = Pna^2(12n^2 + 9n + 1)/(2hEF)$.

Forces:

$O = -Pa(9n^2 - 7)/(6h)$, $D_1 = -7Pc/(6h)$, $D_2 = -Pf/(3h)$,

where $f = \sqrt{a^2 + 4h^2}$.

$U = Pa(9n^2 + 2)/(6h)$.

Supports reactions:

$Y_A = Y_B = P(6n + 1)/2$, $X_B = 0$.

1.8.2. Loading the lower belt

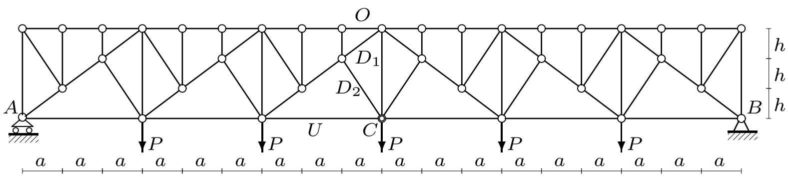

Fig. 28. Truss, $n = 3$

Deflection. The coefficients in (1.10):

$C_1 = n^2(5n^2 + 1)/2$, $C_2 = 3n^2$, $C_3 = 3(n^2 + 1)$.

Support offset:

$\delta_A = Pna^2(4n^2 + 3n - 1)/(2hEF)$.

Forces:

$O = -Pa(n^2 - 1)/(2h)$, $D_1 = -Pc/(2h)$, $D_2 = 0$,

$U = Pan^2/(2h)$.

Supports reactions: $Y_A = Y_B = P(2n - 1)/2$, $X_B = 0$.

1.8.3. Concentrated force in the mid-span

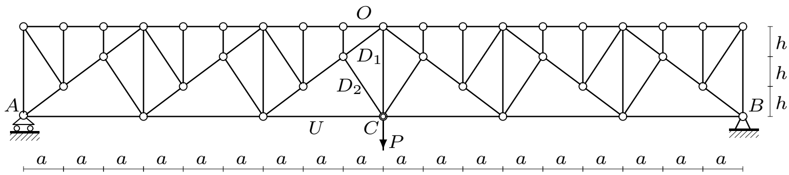

Fig. 29. Truss, $n = 8$

Deflection. The coefficients in (1.10):

$C_1 = n(2n^2 + 1)$, $C_2 = 3n$, $C_3 = 3(n + 1)$.

Support offset:

$\delta_A = 3Pna^2(n + 1)/(2hEF)$.

Forces:

$O = -Pa(n - 1)/(2h)$, $D_1 = -Pc/(2h)$, $D_2 = 0$, $U = Pan/(2h)$.

Supports reactions: $Y_A = Y_B = P/2$, $X_B = 0$.

Truss 1.9

Truss with a height of $2h$ (Fig. 30) containing $n = 2k + 1$ panels in the upper zone, consists of $10n - 1$ rods.

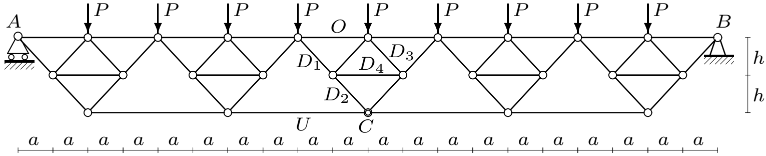

Fig. 30. Truss, $k = 2$, $n = 2k + 1 = 5$

The deflection (vertical displacement of the middle node C in the lower zone) has the form

$$\Delta = P(C_1a^3 + C_2c^3)/(2h^2EF), \tag{1.11}$$

where $c = \sqrt{a^2 + h^2}$.

1.9.1. Load on the upper belt Fig. 30

Deflection. The coefficients in (1.11):

$C_1 = 2(40k^4 + 80k^3 + 62k^2 + 22k + 3)/3,\ C_2 = 8k^2 + 8k + 1.$

Support offset:

$\delta_A = 2Pa^2(2k + 1)(8k^2 + 8k + 3)/(3hEF).$

Forces:

$O = -Pn^2a/(2h),\ D_1 = Pc/(2h),\ D_2 = 0,$

$D_3 = -D_1,\ D_4 = Pa/h,\ U = 2Pa(k + 1)k/h.$

Supports reactions:

$Y_A = Y_B = P(2n - 1)/2,\ X_B = 0.$

1.9.2. Loading the lower belt

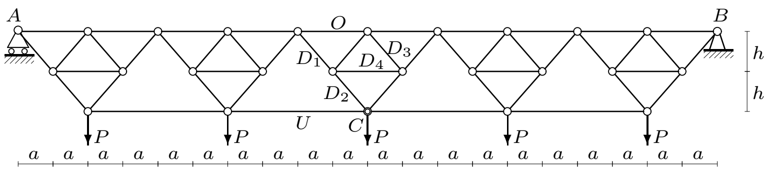

Fig. 31. Truss, $k = 2$, $n = 2k + 1 = 5$

Deflection. The coefficients in (1.11):

$C_1 = 2(20k^4 + 40k^3 + 34k^2 + 14k + 3)/3,\ C_2 = 2(2k^2 + 2k + 1).$

Support offset:

$\delta_A = 2Pa^2(2k + 1)(4k^2 + 4k + 3)/(3hEF).$

Forces:

$O = -Pa(1 + 2k + 2k^2)/(2h),\ D_1 = D_2 = Pc/(2h),$

$D_3 = D_4 = 0$, $U = Pa(k+1)k/h$.

Supports reactions:

$Y_A = Y_B = Pn/2$, $X_B = 0$.

1.9.3. Concentrated force in the mid-span

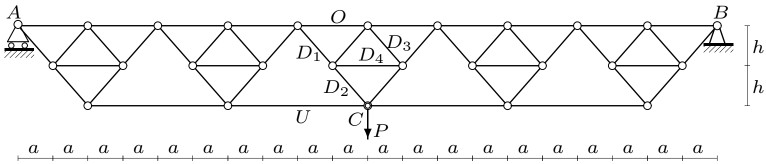

Fig. 32. Truss, $k = 2$, $n = 2k + 1 = 5$

Deflection. The coefficients in (1.11):

$C_1 = 2(2k+1)(8k^2+8k+3)/3$, $C_2 = 2(2k+1)$.

Support offset:

$\delta_A = 2Pa^2(2k^2+2k+1)/(hEF)$.

Forces:

$O = -Pan/(2h)$, $D_1 = D_2 = Pc/(2h)$,

$D_3 = D_4 = 0$, $U = Pak/h$.

Supports reactions: $Y_A = Y_B = P/2$, $X_B = 0$.

Truss 1.10

Truss with a height of h (Fig. 33) containing $2n$ panels in the lower zone, consists of $16n + 1$ rods.

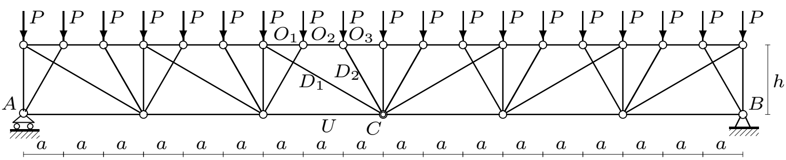

Fig. 33. Truss, $n = 3$