Exact Sciences in Greek Antiquity

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Edited by

Stephanos A. Paipetis
and Vassilis Kostopoulos

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PREFACE

The present volume includes a selection of eighteen papers on Ancient Greek Science and Technology presented at an international conference "Ancient Greece and the Contemporary World," Ancient Olympia, Greece, August 28–31, 2016, covering such areas as mathematics, physics, engineering analysis, astronomical methods and instruments, along with some papers of general interest.

The Conference was strongly interdisciplinary covering, besides philosophy and associated themes, mathematics, physics, engineering analysis and astronomical methods and instruments, and constituted the first attempt at a holistic approach to that great civilization and its lasting influence on the contemporary world. The conference was organized by the University of Patras, members of which have developed a long tradition of research on matters of Ancient Greek civilization and technology, including four international conferences within the last twenty years, all in Ancient Olympia, and was put under the auspices of the President of the Hellenic Republic.

The present editors, as the chair of the conference and member of the Organizing Committee respectively, found unique satisfaction in attending the common efforts of a great variety of distinguished scientists from many countries, working in many different disciplines, and seeing philosophers discussing themes of common interest with engineers and scientists. It was also a proof that ancient civilizations, especially the Greek civilization, cannot be investigated by the various sciences separately, but only by a holistic approach which can lead to reliable results.

The book consists of six parts, as follows:

1. General papers: four papers dealing with the origins of mechanical design and structural engineering as detected in Greek Antiquity and medical care in the Ancient Olympiads.
2. Mathematics: three papers dealing with mathematical concepts in Plato, the concept of the rate of change in the various areas of Mathematics, and the concept of symmetry in Ancient Greece.
3. Physics: seven papers dealing with Aristotle's Physics on freefalling bodies, world-structure formation and matter according to the Presocratics, Aristotelian aether and modern physics, life and death

from the standpoint of modern physics and ancient Greek philosophy, the informational properties of water, and archaeoacoustics.

4. Engineering analysis: four papers dealing with important structural achievements, such as the Trojan Horse and ancient chariots, offensive and defensive weaponry in Homer, and telecommunications in Ancient Greece.
5. Astronomical methods and instruments: four papers dealing with the Antikythera Mechanism, and prehistoric calendars and relevant models.

Certainly, a great number of people gave the best of their efforts for the successful outcome and deserve sincere thanks, but the editors wishe to express their grateful thanks to H. E. the President of the Hellenic Republic Professor Prokopios Pavlopoulos for putting the conference under his auspices and speaking at the opening ceremony, delivering a truly magnificent speech. Many thanks go to Mrs. Aikaterini Panagopoulos, national ambassador of Greece to the Council of Europe for Sports, Tolerance and Fair Play, and president of the International Centre for Sciences and Hellenic Values, whose constant inspiration and endless support made this important project a reality.

Finally, grateful thanks are due to the University of Patras, and in particular to Professor Venetsana Kyriazopoulou, MD, rector and Professor Demosthenes Polyzos, deputy rector for research and development, for full support to the conference through all possible means and also for the financial support to the present publication.

S. A. Paipetis and V. Kostopoulos
Editors

1. General Papers

1.1

THE GREEK LEGACY FOR THE DEVELOPMENT AND PROMOTION OF MECHANICAL ENGINEERING

PROFESSOR MARCO CECCARELLI
UNIVERSITY OF CASSINO AND SOUTH LATIUM, ITALY

Abstract

Greek Mechanics are still present in modern mechanical engineering, both in concepts and processes and mechanism design. This paper illustrates what is old in what is considered new by highlighting the Greek legacy persisting in the modern developments and promotion of mechanical-engineering activities. The latter are recognizable mainly in the principles of the analysis of machine operation, machine design with inspired applications (like automata for theater plays or religious rituals), and the hope that mechanical technology and its applications will provide society with benefits and improvements.

1. Introduction

'Nothing is new except what has been forgotten.' This was written by Julius Sextus Frontinus, a Roman Engineer, in the first century BC. Are all modern mechanical systems really new? How many of the fundamentals and concepts in mechanism design have their origin in Ancient Greek mechanics?

This paper presents a personal perspective on what can be still recognized in modern mechanical engineering, mainly in mechanism design, as coming from the ingenuity of Ancient Greek achievements.

In general, the fundamental contribution of Ancient Greek culture (Philosophy and particularly mechanics) is well recognized in the historical evolution of science and technology within a huge literature. A considerable amount is referred to such a historical view from different viewpoints. However, specific considerations on the developments of machine technology with an Ancient Greek character very seldom appears, as pointed out by Dimarogonas (1993).

The history of mechanical engineering is also usually attached to timeline contributions, often forgetting the background of fundamentals, like those in Bautista et al. (2010) and Rossi and Pagano (2011), for example.

However, attention is focused on searching for principles guiding machine developments over time (Roth 2004). Specific studies and investigations were also carried out, keeping records on and interpreting the development from the past to modern times, as reported in the Proceedings of HMM Symposia (2000–16) and the dictionary series on the legacy of past figures (Ceccarelli 2007; 2010; 2015).

In this paper, a brief survey is presented with examples that show that the Ancient Greek ideas still persist in modern engineering, as a tribute to their capability to transfer scientific results to practical applications, in spite of the general belief that the philosophers/scientists were not interested in such applications.

The discussion is focused on three main aspects: the development of theories with algorithm formulation for designing and testing, the identification and formation of professionals in machine design, and multidisciplinary integration. Examples show how much is old in what is considered new today, and includes the Greek legacy.

2. The Greek legacy in modern times

It is known that science and philosophy were considered activities of high significance in Antiquity, while technology was considered as a means not strictly related to scientific knowledge but rather an activity of manual labor, and therefore of minor significance for culture (Ceccarelli and De Paolis 2008). Nevertheless, the Greek philosophers were always looking for the applications of scientific results and even within technological advances. It is also recognized that the contribution of technology to science was important (Ceccarelli 2012).

Thus, the interaction and mutual influence of science and technology were established since their beginnings, and the Greek approaches somehow still persist today as a cultural background.

In the following, the main effects of machine technology are discussed with a few emblematic examples for further consideration by the reader.

3. Theory and experimental activity in machine design

The Theory of MMS is nowadays considered as related to abstraction and algorithm deduction, which were invented by Greek philosophers.

But the meaning of the word “theory” needs further explanation. The Greek word for theory comes from the corresponding verb, whose main semantic meaning is related with the examination and observation of phenomena. Even in its classical meaning, the word “theory” includes practical aspects of observation in experiencing the reality of phenomena, so that theory also means the application of analytical results.

In fact, the latter meaning is what was included in the discipline of modern TMM (today MMS), as Monge established it at the École Polytechnique at the beginning of the nineteenth century (see, for example, Lanz and Betancourt [1808], including synthesis procedures). In conclusion, the modern meaning of MMS is a discipline that treats both the analysis and synthesis of mechanisms and machines. In fact, the IFToMM Terminology, published in mechanism and machine theory in 1991 and 2003, states:

> Machine: mechanical system performing a specific task, such as forming of material, and transference and transformation of motion and force.
>
> Mechanism: system of bodies designed to convert motions of, and forces on, one or several bodies into constrained motions of, and forces on, other bodies.

As an example, let’s refer to the mechanics of levers that was and remains somehow the basis of mechanics of mechanisms. The ideas come from Archimedes who, after abstracting all machines with elementary parts (an important basis for modern mechanism design and rationalization), formulated its functioning with mechanical principles (being the first to state the equilibrium of momentum [Ceccarelli 2014]). Fig. 1 below summarizes the above from the work of Galilei, as pointed out in Ceccarelli (2006), by considering the real system (a) with its early kinematic diagram (b) and interpreted model, (c) from the first academic approach on the analysis and design of machines in 1593–98. This is an example through which to consider modern classifications of machines via machine elements and elementary machines originating from the Greek mechanics and its speculations. The operation analysis of machines is a result of observation and abstraction with a deduction process that was invented by the Greek philosophers, which we still use in machine analysis and design. The mathematical formulation of the mechanical principles is a modern means, but its reasoning can be recognized due again to the

Greek philosophers. In addition, it is worth noting that the “theoretical” activity mentioned is based on the experimental observation of machine operation according to Galilei following the Greek tradition.

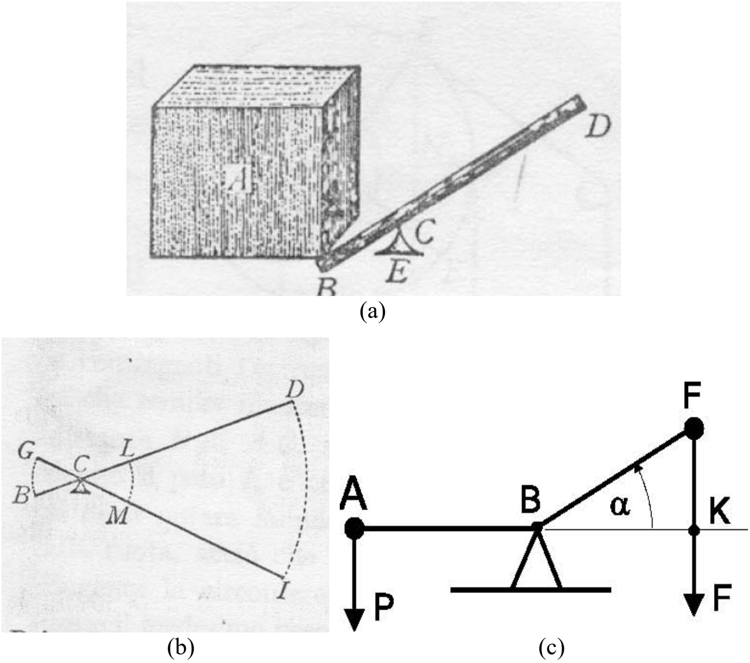


Fig. 1. The Greek concepts in the analysis of machine elements by Archimedes: (a) the mechanical lever; (b) its kinematic diagram; (c) its interpreted mechanical model

4. Professionals with specific dedication in machine technology

The scientific activity in mechanical engineering and MMS, both in research and teaching, today aims at creating a new generation of engineers and scientists who, while developing engineering science and transferring its results to applications, will improve the quality of life of humankind.

The figure of the professional in machine technology was determined at the time of the Roman Empire as a person devoted to the construction and operation of machines through practical expertise. The modern figure of the engineer in machine technology can be considered as becoming established during the Renaissance (Ceccarelli 2008), but full identity and fame were achieved during the Industrial Revolution in the nineteenth century.

In Greek antiquity it is possible to recognize such figures in the *μηχανικοί* [engineers] and philosophers with an interest in applications. Therefore, since Greek antiquity it has been understood and witnessed that machine technology needs professionals with full dedication and links to scientific developments. Actually, today it is also more and more evident that science achievements are strongly linked to technology via figures with expertise in both areas. This is the case of MMS scientists who work for theoretical developments and apply those results in novel designs within the same activity frames.

The need for dedicated professionals requires specific formation and community aggregation. The Greek culture developed schools of philosophers and the Alexandria School with more evident links to technology, as emblematic frames inspiring modern academic systems and research centers with the formation of engineers. This happened in many disciplines, but was particularly significant in machinery and machine technology.

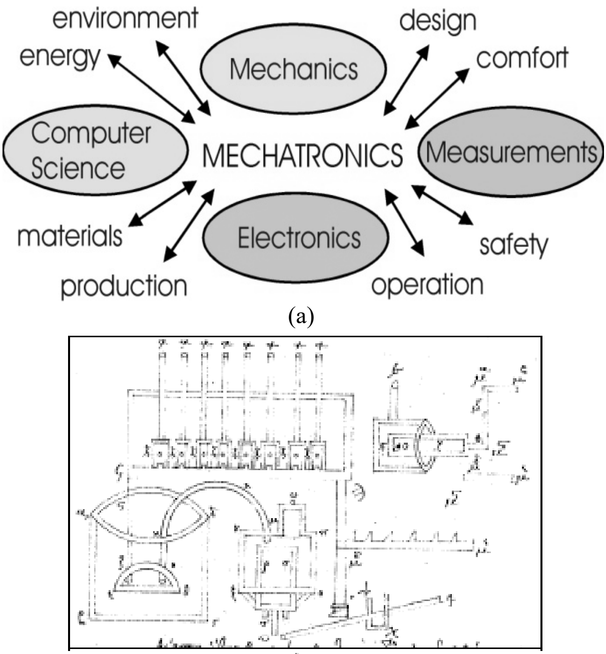
The community aggregation can be recognized in the philosophy schools in Ancient Greece and nowadays in the engineering professional unions, but even more in associations and societies with the key roles of individuals. Of relevant significance in mechanical engineering and MMS is the community of the international Federation for the promotion of MMMS (IFToMM) (www.iftomm.net), with those Greek legacy characteristics of collaboration and sharing knowledge for the dissemination and improvement of mankind without any political or geographical barriers (see Fig. 2 below).

Fig. 2. A historical moment: the foundation of IFToMM, the International Federation for the Theory of Machines and Mechanisms in Zakopane (Poland) on September 27, 1969 (courtesy of the IFToMM Archive), in which one can see: (1) Prof. Ivan Ivanovic Artobolevskii (USSR); (2) Prof. Adam Morecki (Poland); (3) Prof. Kurt Luck (Germany); (4) Mikael Konstantinov (Bulgaria); (5) Prof. Nicolae I. Manolescu (Romania); (6) Prof. Erskine F. Crossley (USA); (7) Prof. Giovanni Bianchi (Italy); (8) Prof. Aron E. Kobrinskii (USSR); (9) Prof. Werner Thomas (Germany); (10) Prof. Jan Oderfeld (Poland).

5. Vision with multidiscipline integration

Today, the modern systems are developed by using the mechatronic concept of multidisciplinary integration as fundamental for the design and operation of efficient systems (see Fig. 3a below). Mechatronics is considered an engineering technology that developed over the last few decades, but its conception can be found in the Greek design of automata since the first solutions for theater equipment.

Mechatronics is usually considered as a recent achievement of modern engineering, by which modern systems are designed and operated because of the integration of several components of various natures with a multi-disciplinary engineering approach. Although engineer formation was and still is achieved by teaching courses on specific discipline subjects separately, machines have nevertheless always been treated by looking at the integration of different aspects. Of course, nowadays the multitude and sophistication of those multidisciplinary aspects require emphasis on the multidisciplinary characters, requiring expertise in specific fields, but in a wide context. The technical integration of different engineering aspects was also considered in the past (Ceccarelli 2007). In fact, one can find early mechatronic designs in Greek machine solutions, like in the example of Fig. 3b below in which a complex machine by Heron of Alexandria (second century BC) is reproduced in a Renaissance-era drawing to show a so-called hydraulic organ with a combination/integration of mechanisms, hydraulic actuators, and regulation devices.



(a)

(b)

Fig. 3. Multidisciplinary integrated design of machines: (a) the modern mechatronic concept; (b) hydraulic organ designed in the second century BC by Heron of Alexandria, as redrawn in the fifteenth century.

6. Conclusions

This paper presents the author’s understanding of the Greek legacy in modern MMS with characteristics still persistent in modern activities, and which is worth disseminating for future developments. The main points of the paper refer to the relevant Greek legacy in mechanical engineering, specifically in MMS, within modern engineering analysis and design, formation activities with dissemination purposes, and visionary multidiscipline solutions with modern features.

References

Bautista, Paz E., M. Ceccarelli, J. Echavarri Otero, and J. J. Munoz Sanz. *A Brief Illustrated History of Machines and Mechanisms*. Dordrecht: Springer, 2010.

Ceccarelli, M. "On the Meaning of TMM Over Time." *Bulletin IFToMM Newsletter* 8, no. 1 (1999).

—. "Early TMM in Le Mecaniche by Galileo Galilei in 1593." *Mechanisms and Machine Theory* 41, no. 12 (2006): 1401–6.

—. "What is Old in what is New in MMS Research." Special Lecture, Proceedings of 13th Symposium on MMS, Japan Council of IFToMM, Tokyo, 2007, 1–11.

—. "An Outline of the History of Mechanism Design in Servicing Science." In Physics, Astronomy and Engineering: Critical Problems in the History of Science and Society—Proceedings of SISFA 2012, The Scientia Socialis Press Siauliai, 1–10.

—. *Proceedings of HMM 2008—the Third IFToMM International Symposium on the History of Machines and Mechanisms*. Dordrecht: Springer, 2008.

—. (ed.). *International Symposium on the History of Machines and Mechanisms—Proceedings of HMM2000*. Dordrecht: Kluwer, 2000.

—. *International Symposium on the History of Machines and Mechanisms—Proceedings of HMM 2004*. Dordrecht: Kluwer, 2004.

—. *Distinguished Figures in Mechanism and Machine Science: their Contributions and Legacies, Part 1. Book series on the History of Machines and Machine Science*, Vol. 1. Dordrecht: Springer, 2007.

—. *Distinguished Figures in Mechanism and Machine Science: their Contributions and Legacies, Part 2. Book series on the History of Machines and Machine Science*, Vol. 1. Dordrecht: Springer, 2010.

—. *Distinguished Figures in Mechanism and Machine Science: their Contributions and Legacies, Part 3. Book series on the History of Machines and Machine Science*, Vol. 1. Dordrecht: Springer, 2014.

—. "Contributions of Archimedes on Mechanics and Design of Mechanisms." *Mechanism and Machine Theory* 72 (2014): 86–93.

Dimarogonas, A. D. "The Origins of the Theory of Machines and Mechanisms." In *Modern Kinematics—Developments in the Last Forty*

Years, edited by A. G. Erdman, 3–18. New York: Wiley, 1993.

IFToMM. “Commission A. Standard for Terminology.” *Mechanism and Machine Theory* 26, no. 5 (1991).

—. “Standardization and Terminology.” *Mechanism and Machine Theory* 38, no. 7–10 (2003).

Koetsier, T., and M. Ceccarelli (eds.) *Explorations in the History of Machines and Mechanisms—Proceedings of HMM 2012*. Dordrecht: Springer, 2012.

Lanz, J. M., and A. Betancourt. *Essai sur la composition des machines*. Paris, 1808.

Lopez-Cajùn, C., and M. Ceccarelli (eds.) *Explorations in the History of Machines and Mechanisms—Proceedings of HMM 2016*. Dordrecht: Springer, 2016

Rossi, C., and S. Pagano. “A Study on Possible Motors for Siege Towers.” *Journal of Mechanical Design* 133 no. 7 (2011).

Roth B. “The Search for the Fundamental Principles of Mechanism Design.” In *Proceedings of HMM 2000—the First IFToMM International Symposium on the History of Machines and Mechanisms*, 187–95. Dordrecht: Kluwer, 2000.

Yan, H. S., and M. Ceccarelli (eds.). *International Symposium on History of Machines and Mechanisms—Proceedings of HMM 2008*. Dordrecht: Springer, 2008.

1.2

The Genesis of the Science of Machines: Theory and Practice Combined

Professor Teun Koetsier
VU University, Amsterdam, Netherlands

Abstract

Presumably, Archytas of Tarentum wrote the first book on mechanics. In the Hellenistic period, mechanics developed into an accepted subdiscipline of mathematics, consisting of a theoretical and a practical part. In the present paper, we will make some remarks about the genesis of the discipline with special attention to the role played by mathematicians.

1. Introduction

Geminus (first century BC) distinguishes two kinds of mathematics: mathematics dealing with intelligibles (arithmetic and geometry) and that attending to sensibles (mechanics, astronomy, optics, geodesy, harmonics, calculation) [20], pp. 31–2. Compared to the traditional Pythagorean quadrivium, there are four new subjects. This paper is about one of them: mechanics, or the science of machines. We will attempt to sketch the genesis of mechanics before Geminus and concentrate on the role of mathematicians in the process.

What do we know? Vitruvius (first century BC) in *De Architectura* mentions twelve authors of works on mechanics[1] whose works he appears to have seen. With the exception of Archytas, Archimedes, Ctesibius, and Philon, however, we do not know anything about these men and their

[1] [27] Book VII, Introduction, Paragraph 14. They are: Diades, Archytas, Archimedes, Ctesibius, Nymphodorus, Philon of Byzantium, Diphilus, Democles, Charias, Polyidus, Pyrrus, and Agesistratus.

work. Elsewhere, Vitruvius mentions Archimedes and Archytas again, and this time together with Aristarchus of Samos, Philolaus of Tarentum, Apollonius of Perga, Erathostenes of Cyrene, and a Syracusan called Scopinas as men who left to posterity "many things connected to mechanics and sundials."[2] We know these men, but as for their work in mechanics we do not know much with certainty. Many manuscripts were lost. Actually, all we have in more-or-less complete form are the following texts directly dealing with machines: *Mechanical Problems* from the Aristotelian corpus (fourth or third centuries BC), the artillery manual of Philon of Byzantium, the *Belopoeica* (probably third century BC), Biton's text on war machines and artillery, *Construction of War Engines and Catapults* (probably third century BC), Vitruvius's book (first century BC), and several of Heron's works (first century AD), in particular the *Belopoeica* on artillery, the *Cheiroballistra* (on the hand-ballista), the *Mechanica*[3], and the *Automata*.

Because of the lack of sources our reconstruction is necessarily hypothetical.

2. The Athenian Period

In his description of the life of Marcellus, Plutarch writes: "For the art of mechanics, now so celebrated and admired, was first originated by Eudoxus and Archytas." He describes how the mathematicians Archytas of Tarentum and his pupil Eudoxus, in solving the geometrical problem of finding two mean proportional lines, had recourse to mechanical arrangements. Plutarch adds:

> But Plato was incensed at this, and inveighed against them as corrupters and destroyers of the pure excellence of geometry, which thus turned her back upon the incorporeal things of abstract thought and descended to the things of sense, making use, moreover, of objects which required much mean and manual labor. For this reason mechanics was made entirely distinct from geometry, and being for a long time ignored by philosophers, came to be regarded as one of the military arts. [19] Chapter 14.

In Plutarch's story, Plato defends the purity of mathematics and criticizes Archytas and Eudoxus. It seems that Plato's criticism did not have much effect. Archytas, for example, was genuinely interested in machines.

[2] [27] Book I, Chapter I, Paragraph 16.
[3] The fact that Heron's mechanics only survived in an Arabic translation illustrates how easily major works could get lost.

According to Aristotle, he designed a rattle for children and probably an automaton in the shape of a wooden dove. The dove may have been connected to a pulley and a counterweight in order to "fly" upwards in the twilight (to make the strings invisible). Moreover, Archytas was not only interested in practical mechanics, but in its theory as well. Diogenes Laertius writes that Archytas wrote a systematic treatise on mechanics based on mathematical principles [6]. Vitruvius also mentions Archytas as the author of a text on mechanics. The oldest extant book about mechanics is *Mechanical Problems* and it is often assumed that it was written by a pupil of Aristotle in the time of Strato, who was a contemporary of Euclid. Yet, Krafft has argued that the text was probably written by the young Aristotle, and he traces part of its contents back to Archytas [12].[4] I assume that Archytas's treatise will at least have contained in some germinal fashion the results that we find in the *Mechanical Problems*.[5] The basic idea of the *Mechanical Problems* is that the functioning of many tools can be understood by means of the law of the balance, which is related to circular motion. The effect of a weight is viewed as proportional to the distance covered when we rotate the balance. *Mechanical Problems* is a book on theoretical mechanics aimed at understanding, not at design. A clever Pythagorean like Archytas, keen on discovering regularity in terms of numbers in the world, will have appreciated the law of the balance.

The problem that Eudoxus and Archytas attempted to solve by mean of "mechanical" methods was that of finding two mean proportional lines: given two straight-line segments A and B, find by means of a construction two other straight-line segments X and Y, such that A:X=X:Y=Y:B. This is a problem from pure mathematics. The well-known problem of the doubling of the cube (given the edge of a cube, find the edge of a cube that has a volume that is twice as big) is a special case. When we have the two

[4] Recently, Thomas Nelson Winter has given an argument that identifies Archytas as the most likely author [28].

[5] According to Humphrey et al., the steelyard (balance with unequal arms) replaced the Bronze Age balance pans for weighing sometime in the Hellenistic period [7], p. 50. At the time *Mechanical Problems* was written, the steelyard was generally used. Mark Schiefsky has correctly pointed out that in Aristophanes's *Peace* (421 BC), 3d Act, 1st Scene, the main character Trygaeus suggests an arms dealer to transform a trumpet into a steelyard for weighing figs: "Well, here's another idea. Pour in lead as I said, add here a dish hung on strings, and you will have a balance for weighing the figs which you give your slaves in the fields." Source: http://classics.mit.edu/Aristophanes/peace.html. Conclusion: the steelyard was known much earlier than the Hellenistic period.

mean proportional lines X and Y of A=1 and B=2 we have $X^3=2$. This means that we double a cube with edge length 1.

Plato imagined a solution based exclusively on the use of compass and ruler. We do not know which "mechanical" solution Plato must have referred to in the case of Eudoxus. Archytas's very ingenious construction takes place in space and requires several rotations. For a recent discussion see [16]. It is based on a curve that is being generated by rotating a semicircle about one of its tangents and intersecting this semicircle during its motion with a cylinder. The generation of the cylinder requires a rotation as well. Moreover, the curve that we get in this way is intersected with the surface of a cone, obtained by rotating a triangle about a straight line. According to the story, for Plato all these motions made the solution mechanical and unacceptable. It is hardly a practical solution, but it is a wonderful example of visual thinking, of the kind that mechanical engineers are good at [3].

At the end of the years of Athenian glory, dramatic events radically changed the world. King Philip of Macedonia prepared the ground and his son Alexander (356–323 BC) took the dynamic of the Macedonian conquest to unprecedented lengths. Alexander's father was one of the first to use torsion catapults. It is possible that the Greek engineers had drawn the conclusion that of the materials in the composite bow, sinew, wood and horn, the major contribution to the power came from the sinew. The next question is: How can we better use the sinew? Their answered was: By twisting a sinew bundle. The basic idea is that one can plait sinew into cords and wrap the cords around two parallel beams. By twisting one of the beams, the bundle of cords can be stretched considerably and a huge tension builds up. A lever pushed through the middle of such a stretched bundle can exert an enormous force if pulled out of its position. The torsion catapult was based on two such bundles (see Fig. 1 below). In Alexandria, mathematicians would get involved in the design of such machines. We saw above that Plutarch hinted at the military origin of mechanics.

3. The Alexandrian Period: Erathostenes

The successors of Alexander in Egypt, the Ptolemies, turned Alexandria into the powerhouse of Greek culture. They founded the famous museum with its library in which they collected men with very different backgrounds and abilities—not only theoreticians but engineers as well.

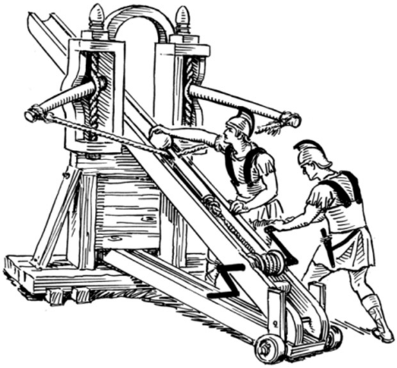


Fig. 1. Sketch of a torsion catapult[6]

In Alexandria, the engineers were highly respected individuals. In the anonymous *Laterculi Alexandrini* from probably the second century BC, which contains some sort of "hall of fame,"[7] the engineer Abdaraxus is mentioned as "he who constructed the machines in Alexandria" [5], p. 429. The engineers impressed the kings with remarkable machines. We have a description by Kallixeinos of Rhodes of a Grand Procession that took place in Alexandria in the early third century BC. In Kallixeinos's description we read: "a four-wheeled cart was led along by sixty men … twelve feet wide, on which there was a seated statue of Nysa twelve feet tall, wearing a yellow chiton woven with gold thread, and wrapped in a Laconian himation. This statue stood up mechanically without anyone laying hand on it, and it sat back down again after pouring a libation of milk from a golden phiale."[8]

Understandably, the Alexandrian engineers spent considerable time on the engines that were used in warfare. According to Philon, the methods used to design catapults that would throw a specific weight over a specific distance were discovered after experimentation and investigation and discovered at "Alexandria through much association with the craftsmen engaged in such matters and through intercourse with many craftsmen in Rhodes, from whom we understood that the most efficient engines more or less conformed to the method we are about to describe" [15], p. 109.

Heron wrote:

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[6] Source: Person Scott Foresman, https://commons.wikimedia.org.

[7] In the words of Lucio Russo [21], p. 96.

[8] See [22], pp. 10–13. We discussed this automaton in [11].

> When one efficient engine has been completed, it is possible to calculate others from it. Let the diameter of the engine be AB, and let it be required that we construct from it another engine throwing, let us suppose, a missile treble the size of the one mentioned. Now, since the spring is the cause of the discharge of the stone, the engine to be calculated will need a spring treble the size of the one whose diameter is AB, and not with just any sort of hole, but with the spring's height proportionate to the hole, so that the cylinders formed by the springs are similar. [15], p. 41.

This immediately leads to the calibrating formula for stone-throwers. Let us suppose the Alexandrians experimented with an engine with a spring diameter of 11 dactyls (21 cm), hurling weights of 10 minae (4,366 grams) over a distance of several hundred meters. Merging three of such machines gives an engine that can throw 30 minae. This means we triple the volume of all parts. More generally, if we want to throw a weight of $\lambda.10$ minae we need a sinew cylinder with a volume equal to λ times the volume of the original cylinder. For the diameter this means that we have to multiply it with the cube root of λ, which immediately leads to the calibrating formula:

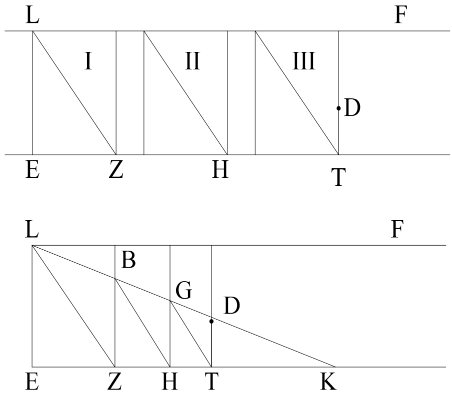
$$diameter = 11.\sqrt[3]{\frac{weight}{10}} = 1.1.\sqrt[3]{100.weight}$$

Here, 11 is the diameter of the engine that can throw 10 minae over several hundred meters. This formula is given by both Heron and Philon, although in words [15], p. 41.

Mathematicians were involved. Erathostenes of Cyrene (third century BC) served under King Ptolemy III Evergetes, the third ruler of the Ptolemaic dynasty in Alexandria. In order to thank Ptolemy, Erathostenes erected a monument consisting of a column with an epigram inscribed on it:

> If you purpose, o good sir, to build from a small one a double cube, or any solid nature into another well to transform, this is possible for you … but the hardly contrived works of Archytas' cylinders and the cone-sectioning of Menaechmean triads seek you not, neither seek to trace out some such curvilinear form of the god-like Eudoxus; for in these very plates you could easily build ten thousand means-tracers, beginning out of a slight base … anyone seeing this monument, may he say, this is [the gift] of the Cyrenean Erathostenes. [9], p. 150.

The text refers to an instrument that Erathostenes had devised to determine the mean proportional of two arbitrary given lines. Just below the crown of the column the instrument made of bronze was fastened, and below that a short proof of its functioning correctly together with a figure [26], pp. 294–5.



Figs. 2 and 3: Erathostenes's instrument.

Consider Fig. 2. The three rectangular plates, I, II and III, are congruent. The one in the middle is fixed, while the other two can slide between the two parallel lines LF and ET. Rectangular plate number III slides to the left under the one in the middle, and rectangular plate number I slides to the right above the one in the middle. While the rectangular plates are sliding, G is the point of intersection of the diagonal of III and the right edge of II. Point B is the point of intersection of the diagonal of II and the right side of I. The point D is a point that we have marked on the right edge of III. The goal is to find the two mean proportionals of LE and TD. Fig. 3 shows that we can slide the rectangular plates in such a way that the points L, B, G, and D are collinear. This is realized by means of a ruler LK. We have

LE:BZ=BZ:GH=GH:DT

Erathostenes's instrument was designed to also be used by the builders of torsion catapults. Eutocius wrote about it: "and this conception will be useful also for those wishing to increase artillery and stone throwing devices; for all these must be increased relative to both the thicknesses and the sizes, and the apertures and the washers and the inserted cords, if also the shot is to be increased proportionately and these cannot be done without the finding of the means" [9], p. 148. Independent of whether Erathostenes's instrument was really useful, the interaction between Alexandrian scholars and craftsmen was considerable.

The catapults did not radically change the way in which wars were fought, yet they could inflict considerable damage. Flavius Josephus (first century AD) wrote: "The force of the spear-throwers and catapults was such that a single projectile ran through a row of men, and the momentum of the stones hurled by the engine carried away battlements and knocked the corners off tower" [8]. Modern research confirms that the machines must have worked quite well [23].

4. The Alexandrian Period: Archimedes (Ca. 287–ca. 212)

Archimedes is considered to be the greatest mathematician of classical antiquity. It seems probable that his father, the astronomer Pheidias, taught him the fundamentals of mathematics and afterwards sent him to Alexandria. There, he will have met Erathostenes and Conon of Samos and maybe Dositheus of Pelusium, later Archimedes's main correspondent in Alexandria.

Archimedes may even have met the old Euclid in Alexandria. It is interesting that there is an Arabic manuscript of a text called *Euclid's Book about the Balance*, in which an axiomatic deduction of the law of the balance is given. This work is theoretical mechanics, like the *Mechanical Problems*, although the approach is different and not based on the properties of circular motion.

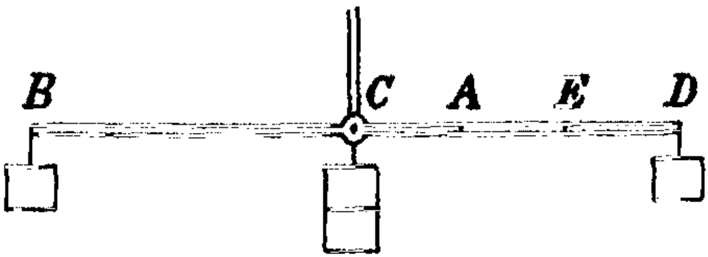


Fig. 4. Euclid's proof of the law of the balance[9]

[9] Source of figure and proof: [2], p. 36.

Duhem summarized the idea of the proof as follows. Equal weights *W* are suspended at B and D with CD=CB. WE have equilibrium. CA=AE=ED are each one third of CD. One of the axioms now says that, if in a situation of equilibrium, we move on one arm a weight over a certain distance *d* towards the center, and at the same time on the same arm an equal weight *d* over an equal distance away from the center equilibrium is maintained. We apply the axiom twice and move the weight *W* from D to A in two steps. The result is equilibrium with B still hanging from B and now *3W* hanging from A.

Note that the axiom is rather natural—it concerns changes on one arm symmetrical with respect to its middle. The text shows how mathematicians attempted to give the approach to machines that we find in *Mechanical Problems* a more rigorous foundation.

When Archimedes returned to Syracuse he had absorbed everything there was to know about mathematics and mechanics. In a way he is the typical Alexandrian mathematician, interested in both pure mathematics and its applications. His geometrical work shows strong influence from mechanics—not only did he create statics and hydrostatics as pure mathematical disciplines, he also used mechanical arguments to solve difficult problems concerning ratios of areas and volumes of geometrical figures. His work on statics is obviously related to *Euclid's Book about the Balance*. Although the Archimedean statics and hydrostatics are highly theoretical and references to practice are absent, they were definitely seen as concerning mechanics. For example, Heron in his mechanics refers to Archimedes's work for the proof of the law of the balance.

Archimedes was actively involved in the design of machines. Unfortunately, we do not know the details, but his reputation in antiquity was such that this conclusion is inevitable. He wrote a book on mechanics that is lost, although some of it can be reconstructed on the basis of Heron's *Mechanics*. Elsewhere, I have argued that he indeed may have invented the screw and the screw pump, and that his work on spirals may have helped him there [10]. It seems reasonable to also assume that the theory of simple machines that Heron describes to us in his *Mechanics* was born in this period.

Well known is the text in which Plutarch writes that Archimedes was not at all inclined to apply his geometrical knowledge and only designed the engines that helped to defend Syracuse after King Hiero had begged him to and "at last persuade him to turn his art somewhat from abstract notions to material things" [19], chapter fourteen. This cannot be true. It is out of the question that only after being urged by the king did Archimedes turn to mechanics, and suddenly started designing fantastic machines out of the blue. Yet, Plutarch's remark reflects an attitude that was quite common among the elite in classical antiquity—manual labor was viewed as inferior.

5. Apollonius and Hipparchus

Many pure mathematicians must in one way or another have been involved in mechanics. Apollonius of Perga (ca. 262–ca. 190) is famous for writing a brilliant book on conic sections. Yet he also wrote a lost work called *On the Cylindrical Helix*. We know this from Proclus, who adds that Apollonius did prove that the cylindrical helix can slide along itself by means of a screw motion—it can move while it goes on coinciding with itself. This is precisely the property that makes it useful in bolts and nuts. In Arabic, a manuscript survives which is called *The Construction of the Machine of the Flute Player*, of which Apollonius the carpenter and geometer is mentioned as author. Lewis has convincingly argued that this must be the same Apollonius as the one who authored the *Conics* [13]. One of his arguments is that Vitruvius associates Apollonius with mechanical work. In the ninth century, the Banu Musa read the Greek manuscript, developed the idea, and left us with a description of a mechanical flute player. Pins on a rotating drum open via levers holes on a flute. The wind is generated by water that fills a reservoir and forces the air out.

Some of the key words that characterize Hellenistic mechanics are measurements, experiments, application of mathematical knowledge, and instruments. We find exactly the same attitude in the astronomer Hipparchus (ca. 190–ca. 120 BC). The early Greek astronomers came up with the first kinematical models of the universe.

Eudoxus attempted to describe the motion of the sun, the moon, and the planets by means of a model in which spheres were rotating with uniform velocities inside the rotating sphere of the fixed stars. Such models could generate the retrograde motion of the planets but they were only qualitatively correct. It seems to have been Apollonius who suggested the possibility of using planar kinematical models based on uniform circular motion. Hipparchus, however, turned Greek astronomy into a true empirical science and succeeded in producing kinematical models that were in wonderful accordance with the observations. His very accurate model for the motion of the Sun is famous. There is evidence that the idea of the astrolabe goes back to Hipparchus [17], p.124. Because the astrolabe is based on a stereographic projection of the spherical universe, it is quite possible that the armillary sphere came first, and it is tempting to