The Change of Paradigm in Nuclear Fuel Optimization

The Change of Paradigm in Nuclear Fuel Optimization:

From Conservatism to Synergy

By

Sergey Pelykh

Cambridge Scholars Publishing

The Change of Paradigm in Nuclear Fuel Optimization:
From Conservatism to Synergy

By Sergey Pelykh

This book first published 2019

Cambridge Scholars Publishing

Lady Stephenson Library, Newcastle upon Tyne, NE6 2PA, UK

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

ISBN (10): 1-5275-3095-7
ISBN (13): 978-1-5275-3095-9

This monograph is devoted to the 100th anniversary of Odessa National Polytechnic University (1918–2018)

CONTENTS

ACRONYMS

$A(\tau)$ – specific dispersion energy (SDE), J/m^3;
A-algorithm – advanced power control algorithm;
AO – axial offset,%;
AS – axial segment;
B – nuclear fuel burnup, MWd/kg;
CET – creep energy theory;
CF – capacity factor;
CFR – complex frequency response;
CM – criterion model;
DF – determining factor;
EPRI – corrosion model developed at the Electric Power Research Institute (USA);
FA – fuel assembly;
FE – fuel element;
FEMAXI – LWR fuel analysis code;
FGR – fission gas release;
FP – fission product;
GSR – general regulations on ensuring the safety of nuclear power plants OPB-88/97;
IAEA – International Atomic Energy Agency;
KhNPP – Khmelnitsky NPP (Ukraine);
LHR – linear heat rate, W/cm;
LOCA – loss-of-coolant accident;
LWR – light water reactor;
M1, M2, M3 – power maneuvering methods;
MATPRO-A – corrosion model;
MCS – Monte Carlo sampling;
MSC – main steam collector;
N – reactor's thermal power, MW
NNEGC – National Nuclear Energy Generating Company;
NPP – nuclear power plant;
NSR – nuclear safety regulations for NPP reactor plants NP-082-07;
NTAI – neutron-thermoacoustic instability;
PCMI – pellet-cladding mechanical interaction;
PDF – probability density function;
PM – probabilistic model;
PWR – pressurized-water reactor;
RA – FA rearrangement algorithm;
RCS – reactor control and protection system;

RG – regulating group;
RLR – radioactive leakage rate, Ci/s;
RS – reactor simulator code;
SBA – surface boiling area;
SCC – stress corrosion cracking;
SDE – specific dispersion energy, J/m^3;
SF – safety factor;
TAI – thermoacoustic instability;
TAO – thermoacoustic oscillations;
VVER – a type of PWR;
ZNPP – Zaporizhzhya NPP (Ukraine);
$\omega(\tau)$ – FE cladding damage parameter.

INTRODUCTION

The first nuclear power plant (NPP) with 5 MW of electric power was connected to the power network in Obninsk, a small Russian city situated 100 kilometers southwest of Moscow, on June 27, 1954. Thus, the history of NPPs is over 60 years old. Nuclear power advantages include low-level emissions of carbon dioxide, and stable and large-scale electricity generation. However, at the same time, the operation of nuclear reactors yields highly radioactive nuclear wastes, while the construction of each new NPP requires an enormous outlay of capital.

Presently, there are nearly 450 nuclear power units being operated; moreover, nearly 60 units are under construction throughout the world. Nuclear power reactors in 30 countries currently provide about 11% of the world's electricity.

The problem of nuclear energy safety and efficiency is tightly connected to the problem of safety and efficiency for nuclear fuel operation because fuel cladding is the key safety barrier when operating nuclear reactors. As the operational safety and efficiency of NPPs is determined mainly by the safety and efficiency of nuclear fuel operation, nuclear fuel is the key element of nuclear reactors. Considering the current state of the nuclear fuel optimization problem, it can be concluded that there are continuing pressures to improve the fuel cycle safety and economics in increasingly challenging operating environments.

The nuclear fuel optimization problem is very close to the problem of development of adequate and clear methods for forecasting fuel element cladding integrity. In order to ensure that a good quality of electricity is maintained, NPPs should have the ability to follow load on a regular basis, including daily variations in the power demand. As the exact cause of cladding failures in nuclear reactors is still not always known for certain, in order to guarantee fuel operation safety and efficiency complex methods for controlling the cladding failure probability must be developed, considering different physical mechanisms leading to cladding failures, including damage accumulation.

When developing a complex method for controlling nuclear fuel properties, besides some practical benefits we come to a new philosophy of nuclear fuel optimization. In other words, aiming to minimize the radioactive leakage through fuel claddings into a reactor circuit simultaneously with optimization of fuel operation parameters, in order to ensure the fuel operation safety-efficiency balance, a need for a new paradigm in nuclear fuel optimization becomes obvious.

To underline the key idea in the foundation of this new approach to nuclear fuel optimization, allowing us to resolve the safety-efficiency contradiction when designing and operating nuclear reactors, I call this new paradigm the "synergic paradigm". This monograph is a complete explanation of principles and some applications of the synergic approach to nuclear fuel optimization. Unsolved problems still remaining in synergic nuclear fuel optimization are also discussed.

CHAPTER ONE
THE SYNERGIC PARADIGM OF NUCLEAR FUEL OPTIMIZATION

Wishing to find out if there is a real need for a new paradigm in nuclear fuel optimization, it is reasonable to consider the nature of several of the most hazardous nuclear accidents in history. Then we will look at the current paradigm in nuclear fuel optimization, which has been traditionally used since the start of the nuclear power epoch.

To ensure safe handling of nuclear fuel, reduce the uncertainty in estimations of fuel cladding failure conditions, and improve the safety-efficiency balance for nuclear fuel operation under increasingly challenging core conditions, the nuclear fuel optimization paradigm should be in conformity with the nature of nuclear accidents. Consequently, the paradigm of nuclear fuel optimization should be formulated and grounded.

1.1. The nature of the most hazardous nuclear accidents

According to the International Nuclear and Radiological Event Scale developed by the International Atomic Energy Agency (IAEA), only two nuclear accidents in history have been assigned the highest 7th level of radiation hazard: the Chernobyl (1986) and Fukushima (2011) disasters. The chronologies of both severe nuclear accidents have been described minutely in numerous publications and will not be repeated here. Having set aside the managerial causes, aiming to find out the fundamental philosophical aspects of these disasters relating to nuclear fuel, let's consider only physical causes of these nuclear accidents.

The physical cause of the Chernobyl disaster is determined very simply: the chain reaction of fission of heavy nuclei (first of all, U-235 and Pu-239) contained in the nuclear fuel forming the reactor core (active core) had become uncontrollable. Mathematically, we have the equation in the prompt criticality case (accounting only for prompt neutrons):

$$k_{ef} \cdot (1-\beta) = 1, \tag{1.1}$$

where k_{ef} is the effective multiplication factor; and β is the delayed neutron fraction.

Having made elementary transformations, the prompt criticality condition for reactivity ρ is obtained from equation (1.1):

$$\rho = \beta\,. \tag{1.2}$$

As prompt criticality means that the chain fission reaction is uncontrollable because of a very small reactor period, the reactor control condition follows from equation (1.2):

$$\rho < \beta . \tag{1.3}$$

In the general case, the reactor uncontrol condition, which is also true for the Chernobyl disaster, is written as:

$$\rho \geq \beta . \tag{1.4}$$

There have been several key factors influencing the reactivity value of the Chernobyl reactor (e.g., positive power and steam coefficients of reactivity at the fuel campaign end, the so-called “end effect” of reactivity for control elements moving into the core, etc.), and all those factors have been discussed many times. The most important fact for our analysis is that all these factors influencing the reactivity are not critical if they are acting separately. On the contrary, as soon as these factors start to act jointly the synergic effect appears in the form of an unpermitted increase in reactivity yielding $\rho \geq \beta$ and, correspondingly, a very fast, explosive increase in the reactor power, resulting in the Chernobyl disaster.

The physical cause of the Fukushima disaster is also determined very simply: the phenomenon of nuclear fuel afterheat due to the radioactive decay of fission products. The rate of this afterheat differs greatly depending on many factors affecting the nuclear fuel composition; first among them are the reactor type (thermal, fast or intermediate neutron reactor), the duration of reactor operation and the exposure time for fission products, and the reactor power history. For instance, if a thermal reactor is operated at the level of power N_0 for 1 day, then the nuclear fuel afterheat at time τ is described as follows: $N(\tau)/N_0 = 1$, 0.1 and 0.01% at $\tau = 5$ min, 1 day and 12 days, respectively.

So, the afterheat must be removed for a sufficient time until the reactor power reaches a safe level. Like the Chernobyl disaster, at Fukushima there have been several key factors influencing the nuclear fuel state, specifically the afterheat removal rate (earthquakes, tsunami waves, the lack of reliable and efficient systems for passive core cooling, etc.).

But the most significant feature of the Fukushima disaster is that all the factors influencing the afterheat removal rate are not critical if they act separately. As soon as these factors begin to operate jointly, the synergic effect appears in the form of an unpermitted increase in the fuel temperature and, in consequence, the zirconium-steam reaction leading to a fast increase in the hydrogen concentration starts, which in turn generates a considerable amount of oxyhydrogen gas, and Fukushima reactor blocks are destructed by its powerful explosions.

1.2. The physics of thermoacoustic instability

As we have seen above, two major catastrophes in the history of civil nuclear power, the Chernobyl and Fukushima accidents, have been characterized by the synergic effect that appeared in the form of an unpermitted change in some physical parameter which had a critical impact on the safety of nuclear fuel operation, due to the combination of joint action of several decisive factors influencing this safety-significant parameter. That is, the nature of the most hazardous accidents related to nuclear fuel has been synergic.

From the point of view of stability theory, both the Chernobyl and Fukushima accidents have been so-called "aperiodic instabilities". Since the safety of nuclear fuel operation can also be greatly influenced by oscillatory instabilities occurring in the reactor core, it may be helpful to analyze the nature of processes leading to self-organization in the form of periodic self-oscillations of reactor core parameters.

Let us first consider the physics of so-called "thermoacoustic instability" in non-nuclear channels. In the beginning, when the heat flux density q_s (W/m^2) increases gradually from zero, there is no boiling in the channel and pressure transducers register turbulent noises only. However, with increasing q_s, when keeping a significant underheating of water below the saturation temperature T_s, pressure transducers register spontaneous high-frequency pressure oscillations at some point in time. Thermoacoustic oscillations (TAO) have the following features (Gerliga and Skalozubov 1990, 372–75):

—the amplitude A_p of pressure oscillations increases with growing q_s, then A_p decreases;

—regular high-frequency pressure oscillations are usually absent before the heat exchange crisis, while pressure transducers register random low-frequency noises only;

—the amplitude of pressure oscillations is highest when heated lengths are relatively small and underheatings of water below T_s are significant;

—TAO may destroy tube channel walls if pressure pulsations with high amplitudes proceed for several hours.

The physics of a thermoacoustic instability (TAI) in non-nuclear channels are described as follows (Khabensky and Gerliga 1994). A hydraulic channel is an oscillating system in terms of acoustics. This is determined by the coolant compressibility, and it's expressed through flow acoustic equations. Natural pressure oscillations in a hydraulic channel are similar to oscillations of a string tied down at both ends.

Turbulent and boiling-induced deviations initiate low-intensity pressure oscillations in the boiling channel, which are close to natural harmonic components. These harmonic components will grow further in amplitude if there exists a mechanism by which heat or mass is supplied to the coolant flow at the moment of pressure increase, while heat or mass is removed from the flow at the moment of pressure decrease. Actually, it is the Rayleigh condition which states that if mass or heat supply rate deviations and pressure deviations are in phase this promotes excitation of acoustic oscillations in the working substance of a hydraulic channel.

So, back actions energizing the oscillatory system and contributing to the development of TAI can be described as follows. Let a bubble be near the antinode of a harmonic of pressure oscillations, so that the pressure deviation gradient does not influence the bubble. Thus, the bubble volume and heat transfer surface area decrease when the channel pressure increases. Hence, if the bubble is in water underheated to the saturation temperature, then the intensity of steam condensation decreases when the pressure increases; that is, the steam outflow from the bubble decreases compared to that of the unperturbed state of the bubble.

If the unperturbed state of the bubble is assumed to be neutral, as the intensity of steam condensation decreases when the pressure increases and the bubble surface area decreases, then a steam mass is supplied to the bubble at the moment of pressure increase. Conversely, the bubble surface area increases and a steam mass is removed from the bubble at the moment of pressure decrease.

Thus, compared to the unperturbed state of the bubble, a steam supply to the bubble at the moment of pressure increase promotes a further increase in the pressure, while a steam outflow from the bubble at the moment of pressure decrease promotes a further decrease in the pressure. According to the Rayleigh condition, the described pressure deviation feedback leads to self-excitation of oscillations (Khabensky and Gerliga 1994).

1.3. A calculation method to analyze TAI

Considering the bubble flow in a non-nuclear steam-generating channel, the calculation method for analysis of TAI borders has the following features (Gerliga and Skalozubov 1990, 386–403):

—the bubble flow is considered as a heterogeneous mixture;

—bubbles that are in a certain place, at different times have different parameters (diameter, velocity, heat flow to the surrounding fluid, etc.)

depending on full histories of their development including generation conditions;

—the system of closing equations is expressed through a range of internal flow characteristics (density of steam-generation centers, bubble detachment frequency, bubble diameter, etc.) which are physically measured microparameters of any two-phase flow;

—bidimensional parametrization is applicable in the wall-adjacent zone only.[1]

When calculating TAI borders in a non-nuclear steam-generating channel, the mathematical model of an underheated water flow includes the following equations (Gerliga and Skalozubov 1990, 387–88):

—conservation law equations describing the distribution of cross-section-averaged parameters (pressure, flow rate, enthalpy) of a steam-water mixture;

—conservation law equations for each group of bubbles distinguished over the channel length;

—initial and boundary conditions for the flow and each group of bubbles;

—equations connecting macro and micro parameters of the flow (flow-averaged and bubble group-averaged parameters).

The initial system of flow motion equations is written in the deviation form, then it is linearized and, assuming zero initial conditions, the Laplace transform is finally used. The characteristic equation is obtained as a result of solving the boundary problem for a system of homogeneous differential equations written with regard to deviations in independent parameters. Consequently, having analyzed the hodograph behavior, TAI borders may be found (Gerliga and Skalozubov 1990, 403–7).

1.4. The physics of neutron-thermoacoustic instability

On the assumption that thermoacoustic oscillations may appear, under certain conditions, in non-nuclear heated channels with subcooled nucleate boiling flows, a more complicated instability, so-called “neutron-thermoacoustic instability” characterized by joint oscillations of neutron flux density, coolant pressure and flow, can occur in reactor steam-generating channels with surface boiling (Pelykh 1997).

---

[1] Simplifying assumptions like neglecting pulsation and rotary motions of bubbles, and derivatives of turbulent and viscous stresses are also used.

Characteristic frequencies of pressure oscillations

Let us consider a bubble boiling flow in the thermo-hydraulic channel of a shrouded fuel assembly (FA), which is placed in the active core of a VVER-1000 reactor operated under normal conditions.[2] Taking into account the fundamental harmonic of pressure oscillations only, one half of the pressure oscillation period is expressed as

$$0.5 \cdot T = H / a, \tag{1.5}$$

where T is the pressure oscillation period, s; H is channel length, H = 3.5 m; a is sound velocity, m/s.

Thus, the cyclic frequency of pressure oscillations is

$$\omega = 2 \cdot \pi / T = \pi \cdot a / H. \tag{1.6}$$

The sound velocity in water is determined as (Kuzmichev 1989, 372):

$$a = (\kappa \cdot \rho)^{-1/2} \approx 1700 \text{ m/s}, \tag{1.7}$$

where κ is the compressibility factor for water, $\kappa \approx 0.47 \cdot 10^{-9}$ Pa^{-1}; ρ is the water density, $\rho \approx 713$ kg/m^3.

So, if the channel is fully filled with water then the cyclic frequency of pressure oscillations is $\omega_{\max} \approx 1500$ rad/s, which is the superior limit value for the characteristic frequency of pressure oscillations in the simulated thermo-hydraulic channel of a shrouded FA. If one half of the channel length is occupied by the steam-water mixture, while the other half is filled with pure water, the average volumetric steam content in the surface boiling area is near 5%, and the sound velocity is near 100 m/s, while the average sound velocity is (Pelykh 1996a):

$$a_{\min} \approx (1700 + 100)/2 = 900 \text{ m/s}.$$

Thus, under the assumption that one half of the channel is occupied by the steam-water mixture, while the other half is filled with water, the inferior limit value for the characteristic frequency is obtained by substituting $a_{\min}$ in equation (1.6): $\omega_{\min} \approx 800$ rad/s. Hence, the characteristic frequency of pressure oscillations in the simulated thermo-hydraulic channel lies in the range 800−1500 rad/s. For example, according to equation (1.6), if $\omega = 1000$ rad/s then the characteristic period of pressure oscillations is $T_{\mathrm{TAO}} = 2 \cdot \pi / 1000 \approx 6$ ms.

[2] An FA cover is necessary to simulate a thermo-hydraulic channel with boundaries that do not allow the acoustic energy to dissipate in the radial direction.

The neutron flux deviation feedback

As explained earlier, thermoacoustic oscillations in a non-nuclear steam-generating channel can be excited by deviations in energy transfer through the bubble surface, according to the Rayleigh condition, due to the dependence of bubble-water heat exchange conditions on pressure deviations. Now let us consider neutron-thermoacoustic instability (NTAI). A pressure wave brings a local pressure deviation; for example, a local pressure growth compressing bubbles and decreasing the local volumetric steam content φ. A local decrease in φ leads to a local increase in moderating power ξ of the steam-water mixture.

Typical neutron moderation and diffusion times in a VVER-1000 reactor are approximately $T_{\text{mod}} \approx 6.7\,\mu\text{s}$ and $T_{\text{dif}} \approx 0.2\,\text{ms}$, respectively (Bartolomey et al. 1989, 162), while, as obtained above, the period of thermoacoustic pressure oscillations is $T_{\text{TAO}} \approx 6\,\text{ms}$; thus,

$$(T_{\text{mod}} + T_{\text{dif}}) \ll T_{\text{TAO}}.$$

Hence, it is arguable that a local deviation in channel pressure causes an almost instantaneous change in the local value of the neutron flux density Φ. Then a deviation in neutron flux density $\delta\Phi$ can influence the value of the coolant temperature in two ways:

1) $\delta\Phi$ leads to a change in fuel temperature δt_f, leading to a change in the fuel element (FE) cladding temperature δt_{clad} and, as a result of heat transfer from cladding to water, changes in coolant temperature δt_w and bubble-water heat flow density δq_b:

$$\delta\Phi \to \delta t_f \to \delta t_{clad} \to \delta t_w \to \delta q_b. \tag{1.8}$$

2) $\delta\Phi$ leads to a deviation in the rate of direct energy release in the coolant $\delta q_{v,w}$, which occurs due to moderation of neutrons and absorption of fission gamma-ray quantums in water. This change in direct energy release in the coolant immediately influences the bubble-water heat exchange conditions:

$$\delta\Phi \to \delta q_{v,w} \to \delta t_w \to \delta q_b. \tag{1.9}$$

As the fuel element is a macroscopic object with considerable inertia, the first way in which $\delta\Phi$ influences coolant temperature t_w, described by equation (1.8), is rather slow in comparison to the second way defined by equation (1.9). For example, the time constant of heat transfer from FE cladding to water is 160 ms, as compared to the characteristic period of thermoacoustic pressure oscillations $T_{\text{TAO}} \approx 6\,\text{ms}$ (Pelykh 1996a).

Hence, when studying the onset of neutron-thermoacoustic instability in a thermo-hydraulic channel with subcooled boiling flow, which exists in the active core of a VVER-type reactor operated under normal conditions, it is wrong to include water temperature deviations (caused by fuel temperature deviations) and pressure deviations in one set of parameters.

The second way in which $\delta\Phi$ can influence coolant temperature is essential because nearly 6% of the reactor's thermal power is released directly in the core coolant, at the expense of moderation of neutrons and absorption of fission gamma-ray quanta in water (Bartolomey et al. 1989, 70). If the volumetric steam content decreases locally, then a thermal neutron flux density deviation occurs, and the local direct energy released in the coolant also changes practically immediately. So, these local deviations can be considered as interrelated variables: an increase in pressure $\rightarrow$ a compression of bubbles and a decrease in φ $\rightarrow$ an increase in the mode-rating power ξ of the steam-water mixture $\rightarrow$ an increase in thermal neutron flux density $\rightarrow$ an increase in direct energy release in the coolant $\rightarrow$ an increase in interphase heat flow density $\rightarrow$ an increase in pressure, that is:[3]

$$\delta p > 0 \rightarrow \delta\varphi < 0 \rightarrow \delta\xi > 0 \rightarrow \delta\Phi > 0 \rightarrow \delta q_{v,w} > 0 \rightarrow \delta q_b > 0 \rightarrow \delta p > 0.$$

Thus, we have obtained the Rayleigh condition leading to self-excitation of pressure and neutron flux oscillations (Pelykh 1997).

A mathematical interpretation of neutron-thermoacoustic instability

The destabilizing effect of the neutron-heat-radiating feedback on thermoacoustic instability is interpreted mathematically as follows. When a local pressure growth occurs, steam bubbles are compressed and the local volumetric steam content decreases. This leads to a momentary increase in thermal neutron flux density and increasing direct energy release in the coolant and, consequently, increasing interphase heat flow density. Assuming zero initial conditions and applying the Laplace transform to a deviation in bubble-water heat flow density δq_b, we obtain:

$$\delta q_{b,\mathrm{L}} = \delta p_{\mathrm{L}} \cdot [\mathrm{W}(\delta p_{\mathrm{L}}; \delta q_{b,\mathrm{L}}) + \mathrm{W}(\delta p_{\mathrm{L}}; \delta\varphi_{\mathrm{L}}) \cdot \mathrm{W}(\delta\varphi_{\mathrm{L}}; \delta q_{b,\mathrm{L}}) + \\ + \mathrm{W}(\delta p_{\mathrm{L}}; \delta\Phi_{\mathrm{L}}) \cdot \mathrm{W}(\delta\Phi_{\mathrm{L}}; \delta q_{b,\mathrm{L}})], \qquad (1.10)$$

where $\mathrm{W}(\delta p_{\mathrm{L}}; \delta q_{b,\mathrm{L}})$, $\mathrm{W}(\delta p_{\mathrm{L}}; \delta\varphi_{\mathrm{L}})$ and $\mathrm{W}(\delta p_{\mathrm{L}}; \delta\Phi_{\mathrm{L}})$ are transfer

[3] Here the sign of interphase energy transfer for heat flowing from water to a bubble is positive.

functions from a deviation in pressure to deviations in bubble-water heat flow density, volumetric steam content and neutron flux density, respectively; $W(\delta\varphi_L; \delta q_{b,L})$ and $W(\delta\Phi_L; \delta q_{b,L})$ are transfer functions from deviations in volumetric steam content and neutron flux density, respectively, to a deviation in bubble-water heat flow density.

It should be noted that the following conditions are satisfied:

$$\text{If } \delta p > 0, \text{then } \delta\varphi < 0, \delta\Phi > 0. \tag{1.11}$$

Moreover, the signs of partial derivatives of bubble-water heat flow density are (Pelykh 1996b):

$$\frac{\partial q_b}{\partial\varphi} < 0; \frac{\partial q_b}{\partial p} < 0; \frac{\partial q_b}{\partial\Phi} > 0. \tag{1.12}$$

Taking into account that

$$\frac{\partial q_b}{\partial\varphi} = \frac{\delta q_b}{\delta\varphi}; \frac{\partial q_b}{\partial p} = \frac{\delta q_b}{\delta p}; \frac{\partial q_b}{\partial\Phi} = \frac{\delta q_b}{\delta\Phi} \tag{1.13}$$

for noninertial processes, it follows from equations (1.12) and (1.13) that:

$$W(\delta\varphi_L; \delta q_{b,L}) < 0; W(\delta p_L; \delta q_{b,L}) < 0; W(\delta\Phi_L; \delta q_{b,L}) > 0. \tag{1.14}$$

According to the Rayleigh condition, if a mass or heat is supplied to a bubble at the moment of pressure increase, this leads to self-excitation of bubble volume oscillations. Thus, the approximate condition of self-excitation of thermoacoustic instability is written in the form:

$$\delta q_b > 0 \ \text{ when } \ \delta p > 0. \tag{1.15}$$

Hence, based on equations (1.10) and (1.15), taking into account the neutron-heat-radiating feedback, the condition of self-excitation of neutron-thermoacoustic instability is given as

$$\begin{gathered} W(\delta p_L; \delta q_{b,L}) + W(\delta p_L; \delta\varphi_L) \cdot W(\delta\varphi_L; \delta q_{b,L}) + \\ + W(\delta p_L; \delta\Phi_L) \cdot W(\delta\Phi_L; \delta q_{b,L}) > 0 \end{gathered} \tag{1.16}$$

Using equation (1.11), we have:

$$W(\delta p_L; \delta\varphi_L) < 0; W(\delta p_L; \delta\Phi_L) > 0. \tag{1.17}$$

And, having used equations (1.14) and (1.17), it is concluded that

$$\begin{cases} W(\delta p_L; \delta q_{b,L}) < 0; \\ W(\delta p_L; \delta\varphi_L) \cdot W(\delta\varphi_L; \delta q_{b,L}) > 0; \\ W(\delta p_L; \delta\Phi_L) \cdot W(\delta\Phi_L; \delta q_{b,L}) > 0. \end{cases} \tag{1.18}$$

So, according to equations (1.16) and (1.18):

—the direct influence of pressure deviations on deviations in bubble-water heat flow density, at the expense of changing thermodynamic properties of steam and water in the boundary layer, has a restricting or stabilizing effect on TAI.

—the indirect influence of pressure deviations on deviations in bubble-water heat flow density, through deviations in volumetric steam content and neutron flux density, has a promoting or destabilizing effect on TAI.

To summarize, the neutron-heat-radiating feedback can have a destabilizing effect on thermoacoustic instability in channels of VVER-1000 type reactors, and neutron-thermoacoustic instability borders can be found by calculating complex frequency characteristics based on the corresponding characteristic equation.

1.5. A mathematical model of NTAI

When considering reactor control problems, the one-group neutron diffusion model is usually applied to VVER-1000 type thermal power reactors (Philipchuk, Potapenko, and Postnikov 1981, 7–9), so this neutron diffusion model may be used in neutron-thermoacoustic instability studies also. The following key processes should be modelled to describe the development of neutron-thermoacoustic instability (Pelykh 1997):

—diffusion of neutrons;

—heat conductivity of fuel elements;

—two-phase flow movement in a heat-exchange channel;

—movement of bubbles in water after their lift-off.

Using a one-dimensional model of neutron-thermoacoustic instability, the position of any point is specified by axial coordinate z only. Thus, the Laplacian of neutron flux density $\Delta\Phi$ is written as

$$\Delta\Phi = \frac{d}{dz}[\frac{d\Phi}{dz}], \tag{1.19}$$

where

$$\frac{d\Phi}{dz} = \frac{I}{D}, \tag{1.20}$$

where I is neutron current modulus; D is diffusion coefficient.

In order to describe underheated boiling flow in a nuclear reactor channel, the following equations should be considered when modelling neutron-thermoacoustic instability:

—neutron flux equations;

—FE thermal conductivity equation;

—flow continuity equation;

—flow momentum conservation equation;

—flow energy equation;

—equations connecting averaged “macroparameters” and “microparameters” of flow, written in the form:

$$\varphi = \int_{z_k}^{z} N_b \cdot V_b \cdot d\xi ; \tag{1.21}$$

$$G_s = \int_{z_k}^{z} \rho_s \cdot w_b \cdot N_b \cdot V_b \cdot F_c \cdot d\xi ; \tag{1.22}$$

where $N_b(z,\xi,\tau)$ is the number of bubbles in a cubic unit, at cross-section z in moment τ, which are born on a length unit of the channel, at cross-section ξ;
$V_b(z,\xi,\tau)$ and $w_b(z,\xi,\tau)$ are the same for bubble volume and velocity, respectively;
G_s and ρ_s are steam rate and density, respectively;
F_c is the cross-sectional area of the equivalent channel corresponding to the triangular grid of fuel elements;

—continuity equation for the number of bubbles;

—balance equation for bubble interface forces including added mass, resistance and pressure gradient forces;

—bubble-water heat-and-mass transfer equation.

The microstructure of bubble flow may be described using the following assumptions (Gerliga and Skalozubov 1990, 293–300):

— $\Delta z = \text{const}$ is the length of a conditional piece of the channel;

—$n(z)$ is the number of a conditional piece of the channel corresponding to coordinate z;

— $N_{b,i}(z)$ is the concentration of bubbles at cross-section z in moment τ, which are born on a unit of the channel length, at the i-th conditional piece of the channel;

—$V_{b,i}(z)$ and $w_{b,i}(z)$ are the same for bubble volume and velocity, respectively.

Then the initial system of equations is written in the deviation form and linearized. The following remark should be made now. The specificity of neutron flux stabilization allows using equations linearized with respect to small deviations (perturbations) from stationary parameter values. This is due to the fact that deviations in the neutron field are restricted by automatic regulators. Accordingly, if neutron field deviations are not sufficiently small they are already too large from the safety point of view, and thus they activate the reactor protection system (Philipchuk, Potapenko, and Postnikov 1981, 27).

Assuming zero initial conditions, the Laplace transform is applied to the linearized system of equations. Denoting the Laplace variable as *s*, the following system of equations for deviations in integral parameters of the core (neutron current modulus, neutron flux density, flow rate, pressure, etc.) and microstructural parameters of flow (volume, velocity and concentration of bubbles) is obtained:

$$\frac{d\vec{Y}}{dz} = \mathbf{P} \cdot \vec{Y} + [\vec{L} \cdot (\mathbf{M} \cdot \vec{Y}) + \vec{N} \cdot (\mathbf{T} \cdot \vec{Y})] \cdot \Delta z, \tag{1.23}$$

where $\vec{Y}(z, s)$ is the vector:

$$\vec{Y} = [\delta I, \delta\Phi, \delta G, \delta P, \delta i_w, \delta N_{b,1} .. \delta N_{b,n(H)}, \delta w_{b,1} .. \delta w_{b,n(H)}, \delta V_{b,1} .. \delta V_{b,n(H)}]_{\mathrm{L}}^{\mathrm{T}},$$

the vector length is $[3 \cdot n(H) + 5]$;

δG, δP and δi_w are deviations in steam-water mixture flow rate, channel pressure and water enthalpy, respectively;

$\mathbf{P}(z,s), \mathbf{M}(z)$ and $\mathbf{T}(z)$ are matrices, and their order is $[3 \cdot n(H) + 5]^2$;

H is channel length, and it equals the core height;

$\vec{L}(z)$ and $\vec{N}(z)$ are vectors, and their length is $[3 \cdot n(H) + 5]$.[4]

Lastly, having performed operations on the matrices, the simplified form of equation (1.23) is obtained:

$$\frac{d\vec{Y}}{dz} = \mathbf{S} \cdot \vec{Y}, \tag{1.24}$$

where $\mathbf{S}(z,s)$ is a matrix, the order of which is $[3 \cdot n(H) + 5]^2$, having the following elements constructed of corresponding elements of $\mathbf{P}(z,s)$, $\mathbf{M}(z)$, $\mathbf{T}(z)$, $\vec{L}(z)$ and $\vec{N}(z)$:

$$\mathbf{S}(z,s) = [s_{k,j}],\ s_{k,j} = p_{k,j} + (l_k \cdot m_{k,j} + n_k \cdot t_{k,j}) \cdot \Delta z.$$

The solution of equation (1.24) may be found as

$$\vec{Y}(z) = \mathbf{\Phi}(z,s) \cdot \vec{Y}(z_k), \tag{1.25}$$

where $\mathbf{\Phi}$ is a fundamental matrix; its order is $[3 \cdot n(H) + 5]^2$, normalized at the point of intensive vaporization start $z = z_k$. Its elements are determined from the equations (Pelykh 1996c):

---

[4] The elements of $\mathbf{P}(z,s), \mathbf{M}(z)$ and $\mathbf{T}(z)$, as well as $\vec{L}(z)$ and $\vec{N}(z)$, for the case $n(H) = 10$ are given by Pelykh (1996a).

$$\frac{d\phi_{k,i}}{dz} = \sum_{j=1}^{3\cdot n(H)+5} s_{k,j} \cdot \phi_{j,i}\,, \tag{1.26}$$

where $\phi_{kj}(z_k) = 1$ if $k = i$; $\phi_{kj}(z_k) = 0$ if $k \neq i$.

Now using equation (1.25), we obtain:

$$\vec{Y}(H) = \mathbf{\Phi}(H,s) \cdot \vec{Y}(z_k). \tag{1.27}$$

Like the method of Leppik and Shevelyov (1984), it is assumed that deviations in integral parameters at the core inlet are given as

$$\delta I_{in} \neq 0;\ \delta G_{in} \neq 0;\ \delta\Phi_{in} = \delta P_{in} = \delta i_{w,in} = 0. \tag{1.28}$$

Meanwhile, the following conditions exist at the core outlet:

$$\delta\Phi(H) = \delta P(H) = 0. \tag{1.29}$$

It can be assumed that deviations in flow microparameters at the point of intensive vaporization start z_k are zero (Leppik and Shevelyov 1984):

$$\delta N_b(z_k) = \delta w_b(z_k) = \delta V_b(z_k) = 0. \tag{1.30}$$

If $\mathbf{\Phi_0}$ denotes the fundamental matrix for the one-phase section, then

$$\delta I_{\mathrm{L}}(z_k) = \delta I_{in,\mathrm{L}} \cdot \phi_{0,11}(z_k) + \delta G_{in,\mathrm{L}} \cdot \phi_{0,13}(z_k). \tag{1.31}$$

$$\delta\Phi_{\mathrm{L}}(z_k) = \delta I_{in,\mathrm{L}} \cdot \phi_{0,21}(z_k) + \delta G_{in,\mathrm{L}} \cdot \phi_{0,23}(z_k). \tag{1.32}$$

$$\delta G_{\mathrm{L}}(z_k) = \delta I_{in,\mathrm{L}} \cdot \phi_{0,31}(z_k) + \delta G_{in,\mathrm{L}} \cdot \phi_{0,33}(z_k). \tag{1.33}$$

$$\delta P_{\mathrm{L}}(z_k) = \delta I_{in,\mathrm{L}} \cdot \phi_{0,41}(z_k) + \delta G_{in,\mathrm{L}} \cdot \phi_{0,43}(z_k). \tag{1.34}$$

$$\delta i_{w,\mathrm{L}}(z_k) = \delta I_{in,\mathrm{L}} \cdot \phi_{0,51}(z_k) + \delta G_{in,\mathrm{L}} \cdot \phi_{0,53}(z_k). \tag{1.35}$$

The elements of $\mathbf{\Phi_0}$ are determined as

$$\frac{d\phi_{0,k,i}}{dz} = \sum_{j=1}^{5} p_{k,j} \cdot \phi_{0,j,i}\,. \tag{1.36}$$

Based on equations (1.25) and (1.29), the following is obtained:

$$\begin{aligned}\delta P_{\mathrm{L}}(H) &= \delta I_{\mathrm{L}}(z_k)\cdot\phi_{41}(H) + \delta\Phi_{\mathrm{L}}(z_k)\cdot\phi_{42}(H) + \delta G_{\mathrm{L}}(z_k)\cdot\phi_{43}(H) + \\ &+ \delta P_{\mathrm{L}}(z_k)\cdot\phi_{44}(H) + \delta i_{w,\mathrm{L}}(z_k)\cdot\phi_{45}(H).\end{aligned} \tag{1.37}$$

$$\begin{aligned}\delta\Phi_{\mathrm{L}}(H) &= \delta I_{\mathrm{L}}(z_k)\cdot\phi_{21}(H) + \delta\Phi_{\mathrm{L}}(z_k)\cdot\phi_{22}(H) + \delta G_{\mathrm{L}}(z_k)\cdot\phi_{23}(H) + \\ &+ \delta P_{\mathrm{L}}(z_k)\cdot\phi_{24}(H) + \delta i_{w,\mathrm{L}}(z_k)\cdot\phi_{25}(H).\end{aligned} \tag{1.38}$$

Having substituted equations (1.31)−(1.35) in (1.37) and (1.38), taking into account equation (1.29), the following matrix equation is written:

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{pmatrix} \delta I_{in,\mathrm{L}} \\ \delta G_{in,\mathrm{L}} \end{pmatrix} = 0, \tag{1.39}$$

where the matrix elements h_{11}, h_{12}, h_{21} and h_{22} are presented in Pelykh (1996a).

So, the characteristic equation of NTAI is obtained from equation (1.39) as

$$h_{11} \cdot h_{22} - h_{21} \cdot h_{12} = 0. \tag{1.40}$$

The simultaneous inclusion of deviations in both integral core parameters, especially neutron current modulus and neutron flux density, and flow microparameters in one vector of variables, when solving the boundary-value problem, has been a development of the theory of bubble boiling flows. This is because this novation allows us to consider the influence of the neutron flux deviation feedback on the propagation of thermoacoustic instability in nuclear steam-generating channels of a VVER-type reactor (Pelykh 1997).

It should be added that, in order to take into account the neutronic aspects of thermoacoustic instability in nuclear channels, a deviation in bubble-water heat flow density δq_b can be calculated using the following equation (Pelykh 1996a):

$$\delta q_b = \delta\varphi \cdot \frac{\partial q_b}{\partial \varphi} + \delta p \cdot \frac{\partial q_b}{\partial p} + \delta\Phi \cdot \frac{\partial q_b}{\partial \Phi}, \tag{1.41}$$

where $\partial q_b / \partial\varphi \approx -8.8 \cdot 10^5$; $\partial q_b / \partial p \approx 0$;

$$\frac{\partial q_b}{\partial \Phi} = \frac{\partial q_b}{\partial q_{v,w}} \cdot \frac{\partial q_{v,w}}{\partial q_v} \cdot \frac{\partial q_v}{\partial \Phi}, \tag{1.42}$$

where, compared to heat conduction, heat radiation makes a major contribution to the heat transfer at microdistances (less than 1 mm) from heat release microcenters which emerge due to moderation of fission neutrons and absorption of fission gamma-ray quantums in water.

Thus, considering heat radiation as the main mechanism of heat transfer at microdistances in water, a partial derivative of bubble-water heat flow density q_b with respect to water volume energy release density $q_{v,w}$ may be found. This partial derivative $\partial q_b / \partial q_{v,w}$, which is necessary for calculating neutron-thermoacoustic instability borders, may also be written as a deviation in bubble-water heat flow density δq_b divided by a deviation in water volume energy release density $\delta q_{v,w}$ and, for bubble diameters $d_b = 10^{-4}$–10^{-3} m, it has been estimated as (Pelykh 1996b):

$$\partial q_b / \partial q_{v,w} = \delta q_b / \delta q_{v,w} \approx 4 \cdot 10^{-3};\ \partial q_{v,w} / \partial q_v \approx 0.03;\ \ \partial q_v / \partial \Phi \approx 2 \cdot 10^{-10}.$$

1.6. Neutron-thermoacoustic instability features

In compliance with equation (1.40), the characteristic equation of neutron-thermoacoustic instability can be represented as a function of complex variable s:

$$F(s) = 0, \tag{1.43}$$

where the solution of the characteristic equation is found in the form:

$$s = j \cdot \omega, \tag{1.44}$$

where ω is cyclic frequency of neutron flux and pressure oscillations.

Like the method of Gerliga and Skalozubov (1990, 405), if we find the value of ω making condition (1.43) true, when the frequency characteristic $F(j \cdot \omega)$ passes through the point $(0; j \cdot 0)$ on the complex plane we obtain the cyclic frequency ω at which the stability boundary is crossed by any variable of the set:

$$(\delta I, \delta\Phi, \delta G, \delta P, \delta i_w, \delta N_{b,1} .. \delta N_{b,n(H)}, \delta w_{b,1} .. \delta w_{b,n(H)}, \delta V_{b,1} .. \delta V_{b,n(H)}).$$

In order to determine an instability area, first points belonging to an area of evident stability are marked on the complex frequency response plane. For example, the mode of one-phase flow is evidently stable in the case of neutron-thermoacoustic instability.

Dependence of the NTAI lower boundary on the steam coefficient of reactivity

The following conditions of a shrouded FA, which is placed in a VVER-1000 reactor operated under normal conditions, with maximum permitted linear heat rates and steam contents, are considered:

—the water temperature at the surface boiling area (SBA) inlet is constant: $t_{w,b,in} = 335\ ^{\circ}\mathrm{C}$;

—the axial coordinate of the SBA inlet is constant: $z_{b,in} = 2.6$ m;

—the SBA length is variable: L_b = 10; 20; 40; 50 cm.

—at first the steam reactivity coefficient is zero: $\partial k / \partial \varphi = 0$.

By increasing the heat flux density in the SBA, the lower instability boundary is found for different L_b values. As the lower instability boundary at L_b = 50 cm and $\partial k / \partial \varphi = 0$ is 113% of the nominal $< q_s >$ value, the lower boundary cannot be achieved at $L_b \geq 50$ cm and $< q_s > = 100\%$ —see Table 1.1.

Table 1.1.
Lower instability boundaries at $\partial k/\partial\varphi = 0$

L_b, cm	$<q_{s,lb}>$, MW/m^2	$<q_{s,lb}>$,%	$<\varphi_{lb}>$,%
10	0.19	18	0.2
20	0.23	23	0.3
40	0.61	70	1.6
50	0.92	113	3.2

Conversely, the lower instability boundary is achieved at L_b = 10−40 cm – the complex frequency response (CFR) for the case of L_b = 10 cm is shown in Fig. 1.1.[5]

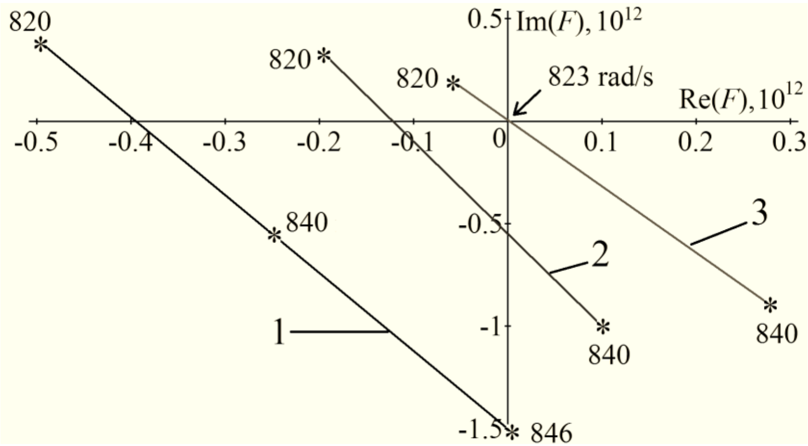

Fig. 1.1. CFR at L_b = 10 cm and $\partial k/\partial\varphi = 0$: (1), (2) and (3) $<q_s> = 0.11, 0.16$ and 0.19 MW/m^2, respectively

As shown in subsection 1.4, the sign of $\partial k/\partial\varphi$ plays a key role in the physics of the neutron flux deviation feedback. If the steam coefficient of

[5] Here and below, the CFR segments may be conditionally shown as straight lines. The calculated frequency of pressure oscillations is 823 rad/s (Fig. 1.1).