# The Origin of Geometry in India

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## A Study in the *Śulbasūtras*

By

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*For*

**Biswanath Basu**

**An old friend and associate**

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**Part 2**

# PREFACE

This monograph seeks to offer an overview of the texts that contain information concerning the origin of geometry in India and highlight some issues that do not seem to have received adequate attention from scholars who have studied the *Śulbasūtra*s, a source book not only of geometry but also of mensuration. Emphasis has been laid on the unity of head and hand, the actual work of craftsmen, masons, and carpenters on the one hand and the Vedic priests who incorporated their experience in the performance of *yajña*s (ritual sacrifices) and codified them in the books on rituals.

The work is divided into two parts. Part 1 gives an outline of the kind of geometry that is to be found in the *Śulba* texts; Part 2 deals with some inherent problems that a student encounters in studying this special brand of geometry that arose out of the ritual practices of the Vedic people.

The *Śulba* texts are concerned with altars and bricks of various shapes and sizes; there is nothing essentially religious in them. Yet, there is no denying that these works are a part of the *Śrautasūtra*s which are nothing but books on rituals, containing long and detailed instructions of various Vedic sacrifices. It should be remembered that the study of phonetics too developed out of the muttering of *mantra*s (magic spells), that accompany all sacrificial rituals. Nevertheless, like the science of grammar, the geometrical content of the *Śulba* texts ultimately assumed a secular character and therefore can be studied without any reference to the particular ritual called *somayāga*. Editors and translators of the *Śulbasūtra*s have successfully worked out the scientific content quite independent of all rituals.

What needs to be noted is that geometry is not something purely western in origin. It arose in different parts of the world at different times out of the actual needs and their solutions. Euclidean geometry is not the only kind of geometry in the world. Special care has been taken in the initial chapters to drive this point home.

One caveat however is to be given at the very outset. Although I have referred to the *Rāmāyaṇa* and the *Mahābhārata* in the course of the study, the information they provide should always be taken with a pinch of salt. In an illuminating article, Professor Ganesh Umakant Thite has convincingly shown that the authors/redactors of the two works were not well conversant with Vedic ritual practices (2014).

References to the *Śulba* texts in this monograph, unless otherwise mentioned, are as follows: for the *Āpastamba Śulbasūtra*, Prakash and Sharma (1968), for the *Baudhāyana Śulbasūtra*, Prakash and Sharma (1968), for the *Kātyāyana Śulbasūtra*, Nene (1936), and for the *Mānava Śulbasūtra*, van Gelder (1959-63).

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Some of the chapters in Part Two appeared in scholarly journals on Indology and the History of Science. I would like to thank the editors and publishers as well.

The responsibility for all errors and shortcomings, however, is mine and mine alone.

# List of Abbreviations

| | |
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| *ĀŚrs* | *Āpastamba Śrautasūtra* |
| *ĀŚus* | *Āpastamba Śulbasūtra* |
| *BŚrs* | *Baudhāyana Śrautasūtra* |
| *BŚus* | *Baudhāyana Śulbasūtra* |
| *KŚrs* | *Kātyāyana Śrautasūtra* |
| *KŚus* | *Kātyāyana Śulbasūtra* |
| *MŚus* | *Mānava Śulbasūtra* |
| *Mbh* | *Mahābhārata* |
| *MBr* | *Maitrāyaṇīya Brāhamaṇa* |
| *Rām* | *Rāmāyaṇa* |
| *ŚBr* | *Śatapatha Brāhamaṇa* |
| *Śrs* | *Śrautasūtra* |
| *Śus* | *Śulbasūtra* |
| *TS* | *Taittirīya Saṃhitā* |
| *VS* | *Vājasaneyi Saṃhitā* |
| *YV* | *Yajurveda* |

# PART 1

# CHAPTER ONE

# BEGINNINGS OF GEOMETRY

Before learning anything about the beginnings of geometry in ancient India, we have to unlearn a few things first.

The first thing to unlearn is that there can be one and only one kind of geometry in the world and that is Greek (Euclidean) geometry. If it were so, we would have to believe that even though the Egyptians, the Babylonians and the Indians erected the most imposing monuments and built planned cities, they had no geometry whatsoever, whereas the Greeks, because of some miracle, conceived and gave birth to geometry out of nothing.

Even the best minds of the west have been under the spell of this notion. It has been claimed time and again by eminent European scientists and historians of science that one of the greatest factors for the development of western science has been "the invention of the formal logical system (in Euclidean geometry) by the Greek philosophers." Thus said Einstein.[1] To this Joseph Needham, the acknowledged authority on science and civilization in China, remarked:

> Einstein himself would have been the first to admit that he knew almost nothing concrete about the development of the science in the Chinese, Sanskrit and Arabic cultures except that *modern* science did not develop in them, and his great reputation should not be brought forward as a witness in this court. I find myself in complete disagreement with all these valuations....[2]

Geometry in India definitely does not resemble its counterpart in Greece. However, we should keep in mind that (a) long before the birth of *modern* science there was science in the ancient and medieval times both in Asia and Europe, and (b) science in the eastern cultures has been practice-oriented and less theoretical, but that does not make it less

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[1] Albert Einstein wrote this in a letter to J.E. Switzer of San Matco, California, 1953 as quoted in Joseph Needham 1979, 43.

[2] Needham 1979, 43-44 (italics in the original).

scientific. On the other hand, what Needham said about the over-estimation of the value of Euclidean geometry is worth pondering:

> Although historians of science are never tired of hymning the services of Euclidean deductive geometry to the western world, I vividly remember a conversation with Dr. Paul Lorenzen of Bonn in 1949 in which he expressed the view that Europe had had more geometry than was good for it. Of course geometry was an essential basis for modern science, but it did have the bad effect of inducing too ready a belief in abstract timeless axiomatic propositions of all sorts of supposedly self-evident, and too willing an acceptance of rigid logical and theological formulations.[3]

Hence we prefer to start with the supposition that ancient Egypt, Babylonia and India too had their own kinds of geometry, derived from the actual practice of the masons and carpenters.

Why masons and carpenters? Didn't geometry originate from measurement of land in Egypt? The Greek word *geometria* itself suggests so. *Ge* (short form of *gea*) means earth or land, *metria* means measurement. It was from this that the word *geometria* acquired a second meaning: "the branch of mathematics concerned with the properties and relations of points, lines, surfaces and solids, and higher dimensional analogues" (as the *Concise Oxford English Dictionary*, 2011 defines it).

Herodotus, the Greek historian of the fifth century BCE, says that it was Egypt that provided the Greeks with the knowledge of geometry. Sesostris (Rameses II, c. 1347 BCE), king of Egypt, is said to have divided the country among all the Egyptians by giving each an equal square parcel of land, and made this his source of revenue. He appointed the payment of a yearly tax on this basis. Any man who was robbed by the river Nile of a part of his land would come to the king and declare what had happened to the parcel of his land. The king would then send his men to measure the space by which the land was diminished, so that it should be in proportion to the tax originally imposed. "From this," says Herodotus, "to my thinking, the Greeks learnt the art of measuring land (*geometria*)."[4]

The passage has very often been referred to and quoted at length. It at least shows that the Greeks themselves did not claim geometry to be their own invention. They admitted that it was imported from Egypt. However, we should also notice the expression, "to my thinking,"/"Perhaps" in

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[3] Needham 1979, 288 n2.

[4] Herodotus, [*Histories*], Book II. 109. Trans. A.D. Codley 1966, 399. In another translation the last sentence of the passage runs as follows: "Perhaps this was the way in which geometry was invented, and passed afterwards into Greece" (*The Histories*, trans. Aubrey de Selincourt 1960, 142).

Herodotus' statement.[5] He was a historian, interested in war and politics, not in science. His conjecture may not be altogether reliable.

However, Proclus (410–485 CE) tells the same story. He wrote a commentary on Euclid's *Elements* (300 BCE), so he knew what he was speaking of. The lateness of Proclus should not stand in the way of our accepting his view. He is said to have learnt this from Eudemus' history of mathematics written before the fourth century BCE. Unfortunately, Eudemus' work has not survived.[6]

The legend of Egypt being the original home of land measurement, and hence of geometry, is also corroborated by Diogenes Laertius' (c. 300 CE) biographical sketch of a Presocratic philosopher, Democritus (c. 400 BCE). This early exponent of the atomic theory is allegedly travelled into Egypt to learn geometry from the priests; he also went into Persia to visit the Chaldeans as well as to the Red Sea. Diogenes adds: "Some say that Democritus associated with the gymnosophists in India and went to Aethiopia."[7] (By "gymnosophists", which literally means 'naked philosophers', Diogenes might have meant the Digambara Jains or some such sect).

Democritus was a great thinker but by no means can he be described as a modest person. In a surviving fragment (no complete work of his has come down to us), we find him declaring:

> I have travelled most extensively of all men of my time, making the most distant inquiries, and have seen the most climes and lands, and have heard the greatest number of learned men: and none has surpassed me in the composition of *treatises with proofs*. Not even the so-called Arpedonaptae of Egypt; with them I passed five years on foreign soil.[8] (Emphasis added)

"*Arpedonaptae*" (in Greek *harpedonaptai*, literally "rope-stretchers") were the surveyors of ancient Egypt. The geometers were perhaps also called by the same name. So the connection between land measurement and this particular branch of mathematics seems to be well attested, both

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[5] Clement of Alexandria (150–214 CE) also said that the Egyptians were "the inventors of geometry". *Stromata* 1979, I.XVI, vol. II, 317, column 1.

[6] "Geometry", says Proclus, "had its origin in Egypt, taking its rises from the perpetual necessity of resurveying the land after the Nile floods had removed the boundaries. This and every other science naturally have their origin in practical needs. Arithmetic similarly arose among the Phoenicians out of the requirements of commerce and contracts. Thales was the first to fetch the study out of Egypt to Greece". Farrington 1966, 210.

[7] Diogenes Laertius 1925, IX.35, 445. See also Kirk and Raven 1966, 404n.

[8] Fragment 299 (Diels). Trans. Freeman 1962, 119.

by linguistic and historical evidence. The logical arrangement of the geometrical propositions, it appears from Democritus' own words, had already been made by the Egyptians long before Euclid (300 BCE) or even his predecessors like Hippocrates of Chios (c. 430 BCE) attempted to do so.[9] The legend of the Greek miracle is thus refuted by the works of the Greeks themselves.

Whatever that may be, the story of the birth of geometry from land measurement seems to be endorsed by all the authorities we have referred to above. Why then do we speak of the masons and carpenters instead of the land surveyors? We shall come to that now.

V. Gordon Childe (1892–1957), the famous archaeologist, first challenged the prevalent view about the origin of geometry. He reviewed the extant Babylonian and Egyptian documents and came to the following conclusion in 1936: "The theory that exact geometry arose out of land-surveying in Egypt or Babylonia is not supported by the evidence at our disposal."[10]

What made Childe say so? He does not deny that the extant Babylonian and Egyptian documents exhibit the knowledge of basic geometry, particularly in relation to the area of fields, for "estimations of the seed required for sowing them and the rent or tax that might be exacted in respect of them." However, he points out, "for such estimates and assessments absolute accuracy was unnecessary; the bailiff only wanted to know roughly how much grain to allow for each field; the tax collector needed a general idea of the yield to be expected."[11]

Who would then require exact ideas and absolute accuracy? The architects and engineers, answers Childe. It was they who "often required more exact calculations to fulfill the tasks imposed on them. The accuracy of the pyramid was a matter of vital ritual significance. And so the Egyptian scribes had discovered and used the correct formula for the volume of a truncated pyramid."[12] Childe also quotes a sum found in the Moscow Papyrus in support of his view.

W.W. Sawyer in his delightful book, *Mathematician's Delight* (1943), comes to the same conclusion; presumably independent of Childe (in any case, Sawyer does not refer to him). "The first mathematicians, then, were practical men, carpenters and builders," he asserts.[13] He also draws attention to some interesting linguistic evidence from the English

[9] For the predecessors of Euclid, see Singer 1964, 63-64.
[10] Childe 1956, 207.
[11] Childe 1956, 205.
[12] Childe 1956, 207.
[13] Sawyer 1976, 13.

language. The fact that the carpenters and builders had something to do with mathematics, he says, 'has left its mark on the very words used in the subject. What is a "straight line"? If you look up straight in the dictionary you will find that it comes from the Old English word for "stretched", while "line" is the same word as "linen", or "linen thread". A straight line, then, is a stretched linen thread – as anyone who is digging potatoes or laying bricks knows.'[14]

Thus, in addition to the architects and engineers mentioned by Childe, we have the carpenters and builders, who too contributed to the birth of geometry.

J.D. Bernal (1901–1971), an eminent physicist and author of *Science in History* (first published in 1954, last revised edition 1969) fully endorses the views of Childe and Sawyer without mentioning them. Bernal said:

> The operation of building itself also contributed, probably even before land survey, to the foundation of *geometry*. Originally, town buildings were simply village huts made of wood or reeds. In the restricted space of a city, with the added danger of fire, mud was a great improvement on reeds. The next step was to have even greater consequences: the invention of the standard moulded block of dried mud – the brick. The brick may not be an original invention, but a copy, is the only material available in the valley country, of the stone slabs that came naturally to hand for dry walling in the hills. Bricks cannot conveniently be piled unless they are rectangular, and their use led necessarily to the idea of the right angle and the use of *straight line* – originally the stretched line of the cord-maker or weaver.[15]

Bernal also provides an illustration showing the Egyptian techniques of rope-making, cabinet-making, brick-making and building, etc. of around 1470 BCE.

The historical evidence of Egypt, one of the original homes of mathematics, conclusively proves that geometry was necessitated by the needs of the masons, not so much by those of the bailiffs. Craftsmen required exactitude in measurement, with no intention of cheating and defrauding anybody.

Why speak of defrauding? We may cite an example from the recent past to show how poor peasants were exploited by the land surveyors during the Mughal rule in India. In the seventeenth century, ropes made of hemp were usually employed for measuring and assessing land. Now, the hemp rope would shrink when wet and lengthen when dry. The

[14] Sawyer 1976, 13.

[15] Bernal 1954, 81. This passage will also be found in a slightly modified form in the illustrated four-volume edition, 1969, vol. I, 131.

government officials used to keep the rope wet on all sorts of pretexts. Abdu-l Qadir Bada'uni, a historian, quotes a verse in his *Muntakhabu-l Tawarikh*: "In the wary-laden eyes of the cheated man, the double-headed snake is better than the measuring rope." Later on the hemp rope was replaced by the more accurate bamboo rod with iron rings.[16]

On the basis of what has been said above, we may safely dispense with the notion that the etymology of the Greek word *geometria* tells the *whole* story of the origin of this branch of mathematics.

Benjamin Farrington in his *Greek Science* points out that every branch of science has its origin in the practical needs of the people, and geometry too is no exception.[17] The way Euclid systematized the knowledge of the masons and carpenters, by dissociating it from their practice, does not reflect the actual chronological sequence of its development. On the other hand, the textbooks of plane geometry (all following the model of Euclid's *Elements*) provide us only with the end-product of what the Greeks made of the achievements of the Babylonian and Egyptian manual workers – brick-makers, masons, etc. Gordon Childe has cogently pointed out that the Egyptians and Babylonians had obtained geometrical rules which are perfectly correct but 'they certainly had not been deduced *a priori* from the properties of abstract space as they purport to be in Euclid's geometry. For such a "pure geometry", there is no evidence at all.'[18]

Sawyer also complains against the way geometry is taught as a purely logical system, beginning with definitions of points, lines and figures, moving to the axioms and postulates, and then to the theorems and problems. In his opinion, "It is quite unfair to expect children to start studying geometry in the form that Euclid gave it."[19] Geometry started with the art of pyramid-building in the fourth millennium BCE. There is thus a gap of no less than three thousand and six hundred years between the building of the Great Pyramid in Egypt (3900 BCE) and Euclid's *Elements* (300 BCE). "One cannot leap 3,600 years of human effort so lightly!" he exclaims.

"The best way to learn geometry," says Sawyer, "is to follow the road which the human race originally followed: *Do* things, *make* things, *notice* things, *arrange* things and only then – *reason* about things." This is indeed the way geometry evolved. Greek geometry is not the only geometry in the world. And the idea of "abstract space" presupposes some concrete experiments with slabs of stones, bricks, etc. Therefore credit should be

[16] Habib 1963, 214 and n74.
[17] Farrington 1966, 210.
[18] Childe 1956, 209.
[19] Sawyer 1976, 17. The quotations that follow are from the same page.

given first to the manual workers and then to the scholars who made the appearance of geometry possible.

## *Geometry in Ancient India: Harappa*

At the beginning of the chapter we said that before learning anything about the origin of geometry, we must unlearn a few things. Now we come to the second thing to unlearn. Presupposition is an enemy of learning. It is necessary to overcome the prejudice against manual workers and to get rid of the feeling of over-respect for scholars.

Another thing to unlearn is that the history of India begins with the Vedas and the people who considered them to be "revealed texts". Long, long before the appearance of the Vedic settlements, there were advanced centres of urban civilization in India. We mean the Harappans.[20] We use the word 'Harappan' as convenient shorthand for designating the men who built the Indus Civilization (again, a shorthand to suggest the ancient settlements (c. 2500 BCE) spread far and wide in present-day Afghanistan, Pakistan and India. Though we often pronounce the names of Mohenjo daro and Harappa at the same breath, it is necessary to remember that they are nearly 370 miles (595.33 km) apart from each other. Similarly, the straight-line distance between Lothal, north of the Gulf of Cambay in Gujarat and Ropar at the foothills of the Himalayas is approximately 720 miles (1158.48 km).[21]

In spite of so much of excavations and startling discoveries, all the sites remain dumb witnesses. The Indus script is yet to be satisfactorily deciphered: unanimity is still a far cry (not that the available specimens on the seals could throw much light on everything that we would like to know, particularly about the Harappan people's knowledge of geometry). For our purpose, however, we have the relics of diverse instruments that have been unearthed. They tell us a very interesting but loosely connected story. We have only to reconstruct it.

Excavations in Lothal have provided us with a number of mason's tools.[22] The first is the plumb-bob made of terracotta. There are two types

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[20] As Dales 1979, 144 n1, says, "The name 'Harappan' is merely a modern convenience derived from the present-day name of the site in the Punjab where the Indus Civilization was first recognized. We know not a single word of their language, much less what they called themselves."

[21] See Possehl 1979, 'Introduction', xi. He says, "The actual area encompassed by this civilization is difficult to compute since precise boundaries are not known, but even conservatively the size is three or four times the area of ancient Sumer."

[22] In what follows I have depended almost exclusively on Rao 1979, 73-75.

of this instrument – one has a vertical hole for suspension by passing a string through it, and the other is suspended from a string passing through a horizontal perforation in the knob at the top. A perforated terracotta cylinder was placed over the knob.

Similarly, a hollow cylindrical object of shell has been found which is identified as a compass. It has eight slits, four in the upper range and four in the lower. It has been supposed that the instrument served the purpose of modern cross-staff. It can be used for producing angles of 45˚, 90˚, 180˚ and so on up to 360˚. The straight line obtained by joining the pinpoints seen through opposite slits cut one another at 45˚.

A measuring scale made of ivory has also been found (5 × 0.6 in.). There are twenty-seven lines covering 1.81 in., giving an average of 0.0689 (0.67) in. per division. Twenty such divisions approximate the distance between two circles marked in the Mohenjo daro scale of shell.

The Mohenjo daro scale too is worth noting well. The distance between the five divisions is 1.32 in. – almost similar to the distance between twenty divisions, 1.338 (1.34) in. on the Lothal scale. Ten divisions in the Mohenjo daro scale give 2.64 in. which might have been the basic measuring unit in the Indus Valley. The first ten divisions on the Lothal scale give a distance of 0.689 in. If we add mean error, the distance measured is 0.699 inch, roughly 0.7 in. The traditional measures of later times, as given, for example, in the *Arthaśāstra* (2.20.6-7) by Kauṭilya, is *aṅgula*, fingerbreadth.[23] Interestingly enough, this measure amounts to the same, that is, 0.7 in. Thus the Lothal scale may be said to be nearer the traditional metrology of India than the Mohenjo daro scale. It is also possible that two different linear measures were used in Lothal: one for smaller objects like the seals, and the other for measuring buildings.

Even earlier (1934) E. J. H. Mackay was surprised to find that "an instrument was actually used" for the purpose of drawing a number of four-petalled devices in continuous designs, without as well as with the original circles, and also enclosed in square compartments, all the specimens dating back to 2500 BCE.[24] One has to think of both the compass and scale to explain the exactitude.

Mud-brick altars have been found in the private houses in Lothal. The altars are rectangular in plan and measure. There are also circular pits.

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[23] "...the maximum width of the middle (part) of the middle finger of a middling man is an aṅgula." *The Kauṭilīya Arthaśāstra*, Part I (text), 1969, 71; Part II (English translation), 1972, 188. There are some other measures before *aṅgula*, but they are purely "metaphysical" and are not actually measurable without modern sophisticated instruments.

[24] Mackay 1934, 222.

Whether or not the objects found in the pits in Lothal can be "satisfactorily accounted for in terms of Vedic sacrifices"[25] (as S. R. Rao believed) or not, the fact remains that the people there were accustomed to such geometrical shapes as the perfect circle and the rectangle.

More important, of course, are the bricks found in all the urban centres of this civilization. Not just the mud bricks, but burnt bricks. K. N. Dikshit observed:

> The bricks used for the building of houses in Mohenjodaro and Harappa are well burnt and of excellent proportions, which have excited the admiration of modern engineers in Sind. The most usual size of burnt bricks is 11´´ by $5^1/_4$´´ or $5^1/_2$´´ with a thickness of $2^1/_4$´´ to $2^3/_4$´´. At no other period has the Indian builder ever struck upon the most business-like size of bricks and it is remarkable that the evolution of bricks in the historical period from Asoka commences with bricks of about double the length and breadth of the Indus Valley brick. It gradually diminishes in the Kushana, Gupta and mediaeval periods, but never attains the true proportion of length, breadth and thickness as $1: {}^1/_2: {}^1/_4$, which makes for an excellent bond.[26]

Dikshit is not the only one to wax eloquent on the bricks found in Mohenjo daro and Harappa. There are other archaeologists too who praise the proportion of the bricks in the same way. John Marshall said:

> The walls are generally built of solid brick masonry in mud mortar the size of bricks in common use being 11´´ ×$5^1/_2$´´ ×$2^3/_4$´´ the ratio of the length, breadth and thickness thus being 4:2:1, which is admirably suited for the purpose of bonding. Two well baked bricks of larger dimensions measuring respectively 17´´ ×$8^1/_2$´´ ×3´´ and 16´´ ×8´´ ×$2^1/_4$´´ were met with in the course of digging, but they cannot be definitely assigned to any of the existing buildings. … The floors of several rooms were paved with brick tiles, the pavement being often sub-divided by brick-on-edge partitions.[27]

Dikshit makes another interesting observation:

> That the ideal proportion [of bricks] was not entirely forgotten is shown by the fact that a later text (Kaśyapa Saṃhitā) prescribes a proportion of 10 fingers of length to 5 fingers of width and half of the latter for thickness; but it is doubtful whether in actual practice the masons ever followed this in historical period. Any way, it is clear that the burnt brick of Indus

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[25] Rao 1979, 218. See also Rao 1973, 139-141.
[26] Dikshit 1967, 15-16.
[27] Marshall, in: Possehl 1979, 184.

> Civilization has been unexcelled in India and is not comparable with any attempts made in ancient Sumer, Egypt and other countries, till we come down to the Roman times.[28]

Thus the two basic ingredients for cultivating geometry – mason's tools and brick-making – were already there in India as early as 2500 BCE.

How and why the Indus Valley civilization ceased to continue is still a matter of dispute. Several hypotheses have been proposed and each one of them has been challenged. We are not concerned with them here. What we would like to note, however, is that archaeological findings do exhibit signs of decadence. The last reconstructions of the Harappan cities are no longer so well planned; old bricks were being re-used for building new houses. With the decay of the urban centres, the knowledge of the arts and crafts was also lost.[29]

However, after a considerable gap in time, we have *written* records of the rebirth of geometry in India.[30] They are the *Śulba* (or Śulva)-*sūtra*s, literally, "the collection of aphorisms relating to the rope". The works may be dated at 600 BCE (according to Ghatage and others), although the actual knowledge must have been much older, at least not later than 1000 BCE, the time of the composition of the Brāhmaṇa-s that were compiled after the Saṃhitās but before the *Śrautasūtra*s, to which the *Śulbasūtra*s belong.

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[28] Dikshit 1967, 16. Mackay, however, noticed some similarities with the masonry works of Egypt, Ur and Warka, vol. I, 1938, 428, 649.

[29] See Dales, in: Possehl 1979, 311.

[30] That is, written down later. The terse, brief aphoristic form suggests that the *Śulba* texts were meant to be memorized, not read. Afterwards some scholars wrote long commentaries on the texts but for which some aphorisms would not be comprehensible at all. Even then, some aphorisms cannot be understood properly, for the commentators themselves were not sure what the aphorisms actually suggested. The redactors of the *Śulba* texts too, it would appear, were not sure about the shapes of the circular altars. Speaking of the Droṇa *citi*, *BŚus* 1968, 6.1 first says that it has to be piled. But there are two kinds of the same *citi*: square and circular (6.2-3). However, there is nothing specifically said about which kind of Droṇa *citi* is to be piled. Hence, both are presented (6.4). This formula is repeated in relation to the Kūrma *citi* (*BŚus* 1968, 9.1-3).The time lag between the actual performance of the fire sacrifices and the redaction of the texts is quite apparent.

# CHAPTER TWO

# THREE-DIMENSIONAL FIRE ALTARS

All branches of science, including mathematics, came into being in order to satisfy some basic material and intellectual needs of a community. One cannot think of metallurgy prevailing among the stone-age people. Similarly, such branches of mathematics as geometry and algebra call for a more developed form of society. No science can originate in a vacuum.

We know almost nothing about the *theoretical* sides of the learning of the Harappans. However, there are reasons to believe that, as in ancient Babylon and Egypt, so in India, the masons and carpenters had their own kind of geometry, derived from the actual practice of making and building things. The two basic ingredients for geometry – brick-making and the availability of necessary instruments for drawing rectilinear and curvilinear figures (mainly polygons and circles) – were already there in north-western India as early as 2500 BCE.

The next phase begins with the Vedic people whose arrival may be roughly dated as 1500 BCE. However, so far as geometry is concerned, we seem to notice a new beginning. Instead of any readymade instruments like the scale or the compass, we find the more primitive bamboo (*veṇu*), gnomon (*śaṅku*), and a piece of rope (*śulba*, also spelt *śulva*; the more oft-used term in the texts of the *Śulbasūtra*s is *rajju*).

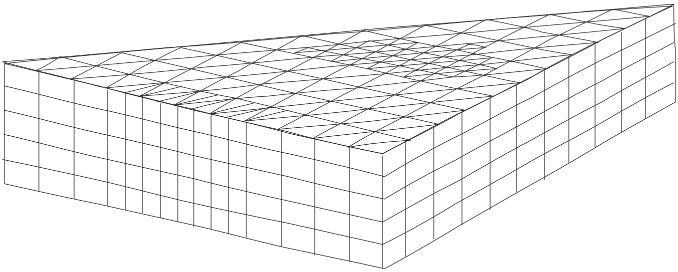
The first evidence of the geometrical knowledge of the Vedic people is found in the Brāhmaṇas, prose works ancillary to the Vedas. They deal with various sacrificial rites (*yajña*s). It was in connection with a ritual called Soma-yāga that special altars (*citi*s) were to be piled. The ritual chants (*mantra*s) would be sung or recited while the priests performed rites in front of or standing or sitting on the sacrificial fire-altars (*vedī*s). These altars were made of bricks of different shapes and sizes.

This is rather enigmatic, because the Vedic people used to live in villages; their houses were made of clay and wood, not of kiln-burnt bricks. Yet bricks are essential for the piling up (*cayana*) of the fire-altar (*agni*). It has been justly presumed that, in spite of many a break, the use of bricks shows a continuity of the pre-Vedic and Vedic cultures. The

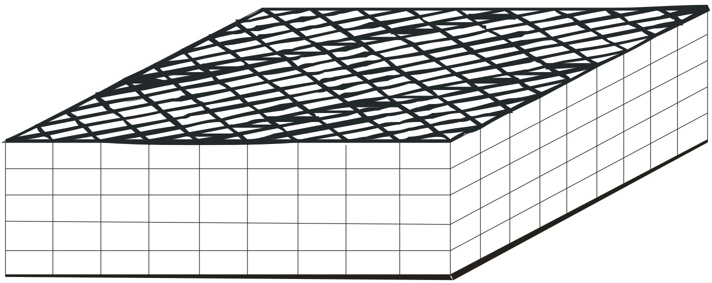
Harappan priests might have introduced the custom of using bricks in the performance of this particular kind of Vedic ritual.[1]

The *citi*s, too, were of various shapes. Some of the names show the rudiments of geometrical concepts. For example,

(1) Praüga, "The isosceles-triangle altar". The word Praüga originally means "the fore part of the shafts of a chariot" which was triangular in shape;

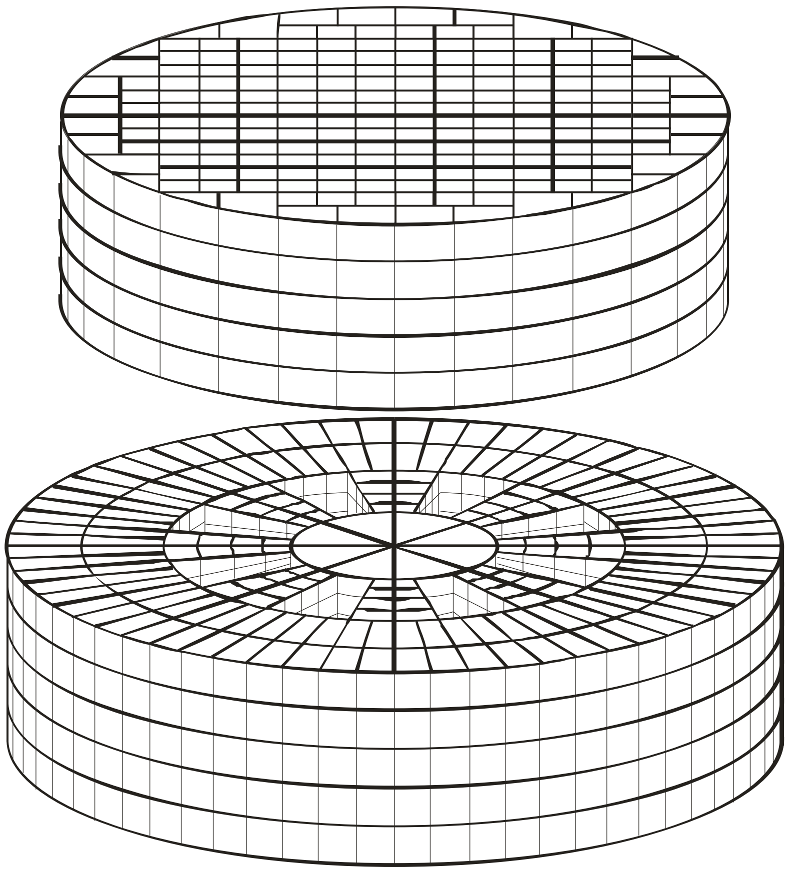


(2) Ubhayataḥ Praüga, "altar triangular on both sides", i.e., rhombus-shaped;

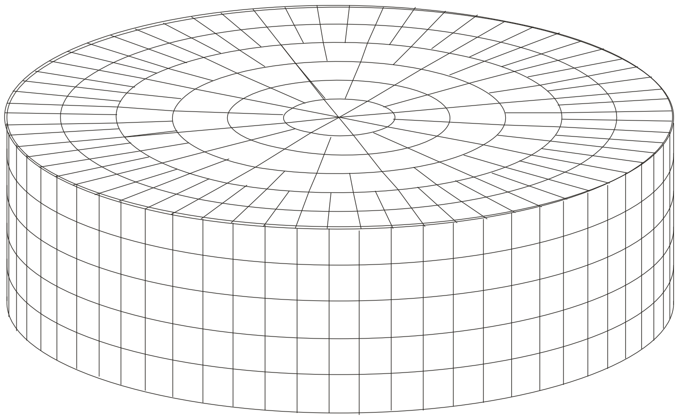


[1] See Chattopadhyaya 1984, iv-xvi.

(3) Rathacakra, “chariot (or cart) wheel altar”, i.e., a circle with or without spokes:

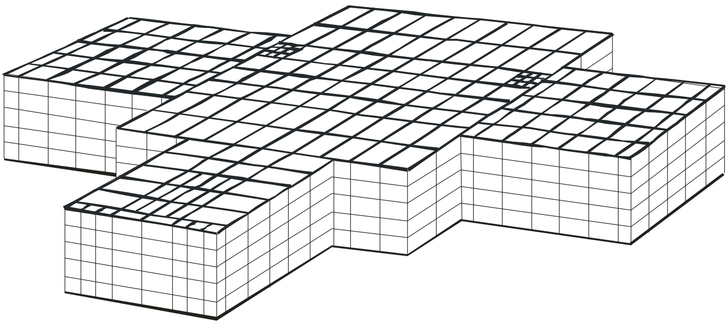


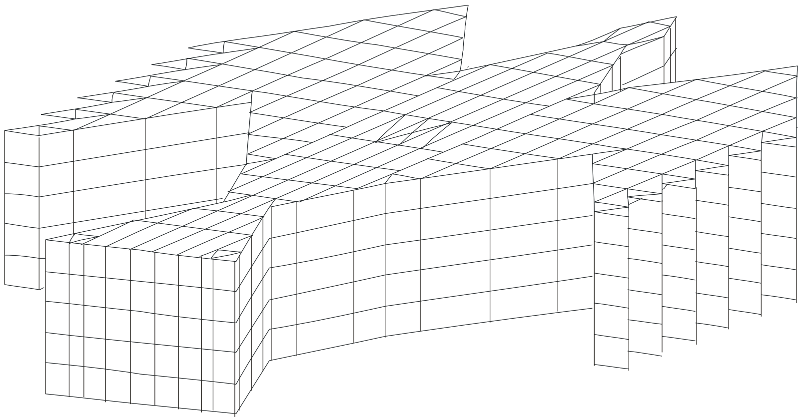
(4) Upacāyya / Paricāyya, “circle altar”:



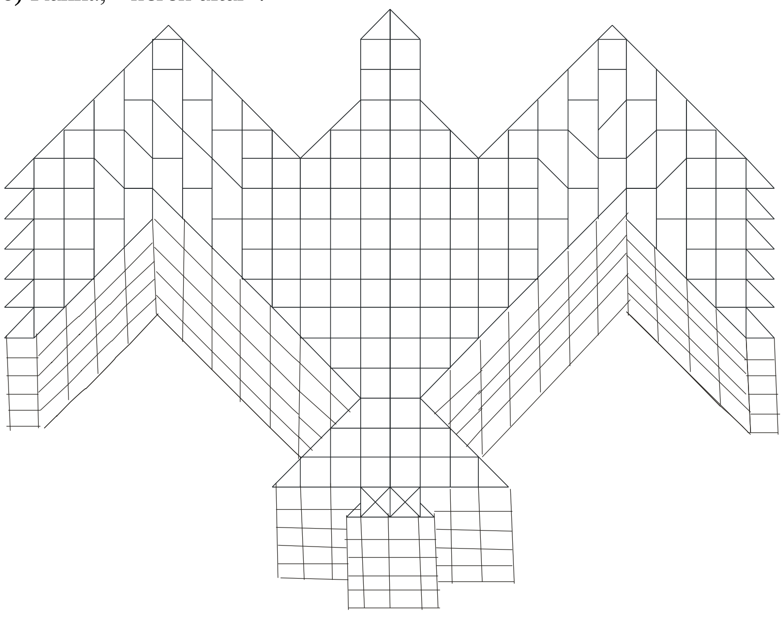
There are also some more interesting forms like the different types of

(5) Śyena, “hawk altar”:

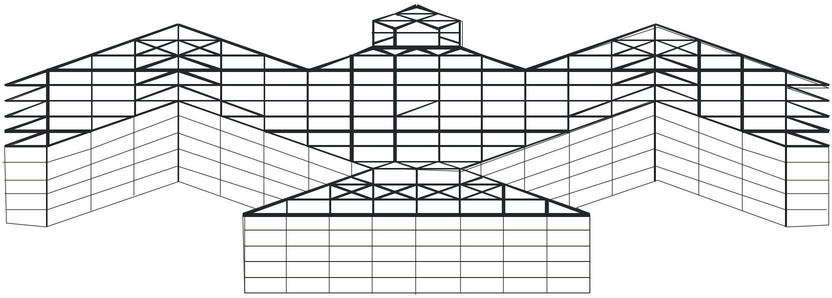




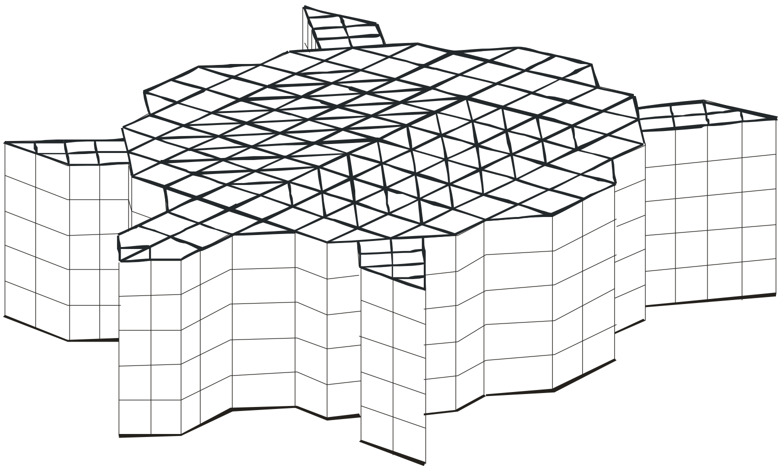
(6) Kaṅka, “ heron altar”:



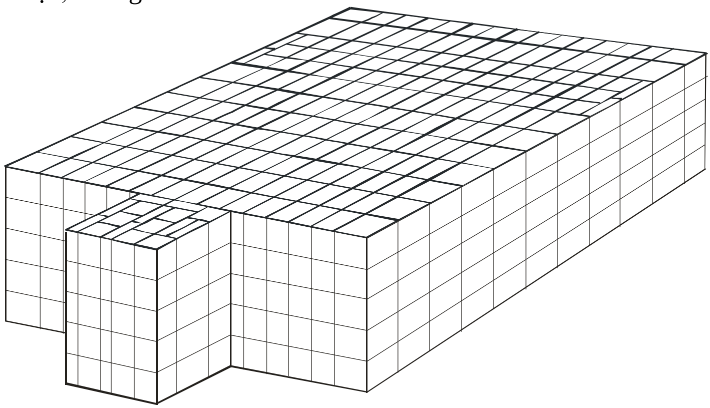
(7) Alaja



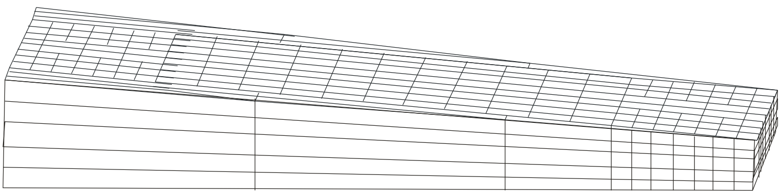
(8) Kūrma, “tortoise altar”:



(9) Droṇa, “trough altar”:

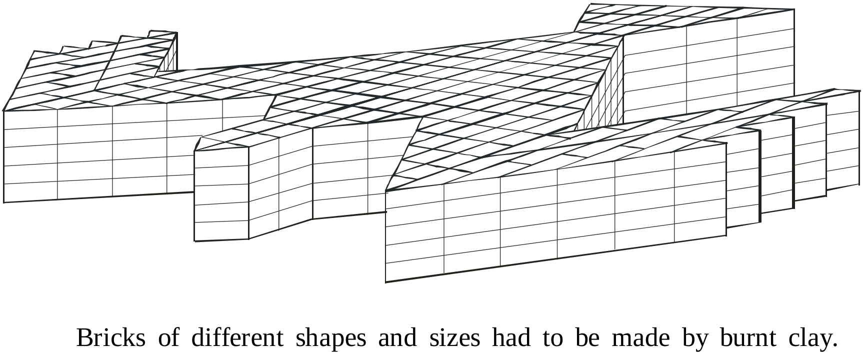


(10) Śmaśāna, “funeral pyre altar”:



There is another *citi* called

(11) Suparṇa, “the mythical bird Garuḍa”, mentioned only in the *Mānava Śulbasūtra*:



Bricks of different shapes and sizes had to be made by burnt clay. Perfect symmetry was achieved by the fixed number of bricks of a variety of shapes and sizes. The altars are also aesthetically satisfying.