Fundamental Constants

Fundamental Constants:

Evaluating Measurement Uncertainty

By

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Any true is only one facet of the truth
—Anonymous

CONTENTS

PREFACE

The book is an attempt to apply the methods of information theory, similarity theory, modeling theory and experimental design theory to assess the *a priori* model mismatch before the actual experiment or computer calculations.

It contains rich experimental material, confirming the attractiveness of the information-oriented method for experimental and theoretical physics, including measurement of fundamental physical constants.

The focus is on the organic link between the original mathematical terms of information theory, similarity theory, and the theory of planning of experiments. So, the information-oriented approach of modeling physical phenomena is perceived as a system of ideas that have a clear physical meaning.

The book is based on experimental and theoretical investigations carried out by the author over 35 years, as well as development experience and extensive research activities in modeling measurements of the fundamental physical constants.

The introduced method is very simple and easy to digest, so appropriate technical skills are easily acquired. But even the experience of its formal use cannot teach the relevance and reasonable use of it without a stencil or even without direct mistakes occurring. To apply the information method, you must first understand its physical content.

The book is supplemented by a rich bibliography with internet addresses.

It may be useful for scientists, engineers working in the enterprises and organizations of the corresponding profile, and students of universities and colleges. Comments and suggestions about the content of the book should be sent to the following email: meninbm@gmail.com.

ABOUT THE AUTHOR

Boris M. Menin was Director of the Laboratory of Ice Generators and Plate Freezers in St. Petersburg (then Leningrad) from 1977 to 1989, at which time he emigrated from the Soviet Union to Israel. He has since managed at Crytec Ltd. (Beer-Sheba, Israel) on the development, production and marketing of pumpable ice generators, cold energy storage systems, and also high accuracy instrumentation for heat and mass processes among other matters. Dr. Menin is credited with developing an entirely new branch of modeling: an information-oriented approach, by which the lowest achievable absolute and relative uncertainties of measured quantity can be calculated before the realization of experiments or a model's computerization. An earlier book by the same author called *Information approach for modeling physical phenomena and technological processes* (published by Lambert Academic Publishing, Germany, 2017) presents a theoretical explanation and grounding of application of information and similarity theories for the calculation of the threshold discrepancy between a model and researched phenomena. This current book focuses on the methods for experimental data processing of fundamental constants measurements and considers this aspect of physics in greater depth.

INTRODUCTION

The illiterate of the 21st century will not be those who cannot read and write, but those who cannot learn, unlearn, and relearn
—Alvin Toffler

"There is nothing new to be discovered in physics now. All that remains is more and more precise measurement." This worldview statement was by Lord Kelvin in 1900, but it was shattered only five years later when Einstein published his paper on special relativity.

In the 21st century, it can be safely asserted that absolutely all modern achievements in the field of science are based on the successes of the theory of measurements, on the basis of which the practical recommendations useful in physics, engineering, biology, sociology, etc. are extracted. In addition, this is because the application of the principles of the theory of measurements in determining the fundamental constants allows us to verify the consistency and correctness of the basic physical theories. Complementing the above, quantitative predictions of the basic physical theories depend on the numerical values of the constants involved in these theories: each new sign can lead to the discovery of a previously unknown inconsistency or, conversely, can eliminate the existing inconsistency in our description of the physical world. At the same time, scientists came to a clear understanding of the limitations of our efforts to achieve very high measurement accuracy.

The very act of the measurement process already presupposes the existence of the physical-mathematical model describing the phenomenon under investigation. Measurement theory focuses on the process of measuring the experimental determination of the values by using special hardware called measuring instruments [1]. This theory only covers the aspects of data analysis and measurement procedures of the quantity observed or after formulating a mathematical model. Thus, the problem that there is uncertainty *before* experimental or computer simulation and caused by the limited number of quantities recorded in the mathematical model is generally ignored in the measurement theory.

The proposed information approach—to assess the model's noncompliance with the physical phenomenon under study—has introduced an additional measurement accuracy limit that is more stringent than the Heisenberg

Uncertainty Principle. And it turns out that the "fuzziness" of the observed object, strangely enough, depends on the personal philosophical prejudices of scientists, which are based on their experience, acquired knowledge and intuition. In other words, when modeling a physical phenomenon, one group of scientists can choose quantities that will differ fundamentally from the set of quantities that are taken into account by another group of scientists. The fact is that the same data can serve as the basis for radically opposite theories. This situation assumes an equally probable accounting of quantities by a conscious observer when choosing a model. A possible, though controversial, example of such an assertion is the consideration of an electron in the form of a particle or wave, for the description of which various physical models and mathematical equations are used. Indeed, it is not at all obvious that we can describe physical phenomena with the help of one single picture or one single representation in our mind.

This book aims to introduce a fundamentally new method for the characterization of the model uncertainty (threshold discrepancy) that is associated with only a finite number of the registered quantities. Of course, in addition to this uncertainty, the total measurement uncertainty includes *a posteriori* uncertainties related to the internal structure of the model and its subsequent computerization: inaccurate input data, inaccurate physical assumptions, the limited accuracy of the solution of integral-differential equations, etc.

The novel analysis introduced is intended to help physicists and designers to clarify the limits of the achievable accuracy of measurements and to determine the most simple and reliable way to select a model with the optimal number of recorded quantities calculated according to the minimum achievable value of the model uncertainty.

The book contains five chapters. Chapter 1 gives base elements of similarity theory, information theory, theory of planning of experiments, and group theory. It includes a classification of measurement inaccuracy and postulates the theory of measurements. The basic definitions and explanations introduced are needed for further development of the main principles of the information-oriented method.

Chapter 2 contains the analysis of publications related to usage of the concepts of "information quantity" and "entropy" for real applications in physics and engineering, calculating information quantity inherent in the physical-mathematical model, and the formulation of a system of base dimensional quantities (SBQ), from which a modeler chooses a number of quantities in order to describe the researched process. Such a system must meet a certain set of axioms that form an Abelian group. This in turn allows the author to employ the approach for the calculation of the total number of dimensionless criteria in the existing International System of Units (SI). Mathematically, the exact expression for the calculation of the comparative uncertainty of the developed model with a limited number of quantities obtained by counting the amount of information contained in the model is formulated.

Chapter 3 is devoted to applications of the information-oriented approach, including its most attractive application which is the measurement of fundamental physical constants. The data and calculations of the accuracy of the Avogadro number, Boltzmann constant, Planck constant, and gravitational constant are presented. In addition, the puzzle of the Maxwell demon and the amount of information related to ordinary matter are analyzed from the point of view of the information approach.

Chapter 4 is expanded to discuss using comparative uncertainty instead of relative uncertainty in order to compare the measurement results of the main quantity of the recognized phenomenon, including the fundamental physical constants, and to verify their true-target value. Moreover, drawbacks and advantages of the introduced method are carefully analyzed.

Chapter 5 focuses on emphasizing that the information-oriented approach is a living topic. This is extremely important because successfully demonstrating its use has many consequences in the measurement of fundamental physical constants, quantum mechanics and cosmology.

CHAPTER ONE

BASIC KNOWLEDGE ABOUT APPLIED THEORIES

Don't let your ears hear what your eyes didn't see,
and don't let your mouth say what your heart doesn't feel
—Anonymous

1.1. The measurement theory basics

To begin with, the first task of the scientist studying a phenomenon is usually to determine the conditions under which the phenomenon can be repeatedly observed in other laboratories and can be verified and confirmed. For an accurate knowledge of the physical variable, you need to measure it. And for its measurement, a certain device is always required (this presupposes the existence of a physics-mathematical model already formulated) that somehow influences this value, as a result of which it becomes known with some degree of accuracy. In turn, the amount of information obtained by measurement can be calculated by reducing the uncertainty resulting from the measurement. In other words, uncertainty about a particular situation is the total amount of potential information in this situation [2].

For all the instructions below, it is important to indicate the difference between the error and uncertainty. The error is in how much the measurement corresponds to the true value. This error is rarely what interests us. In science, we usually do not know the "true" meaning. Rather, we are interested in the uncertainty of measurements. This is what we need to quantify in any measurement. Uncertainty is the interval around the measurement, in which measurements will be repeated. Uncertainty describes the distance from the measurement result within which the true value is likely to lie.

The introduction of measurable quantities and the creation of their units are the basis of the measurements. However, any measurement is always performed on a specific object, and the general definition of the measured quantity must be formulated taking into account the properties of the object and the purpose of the measurement. Essentially, the true value of the measured quantity is introduced and determined in this way. Unfortunately, this important preparatory stage of measurements is usually not formulated [1].

The idealization necessary for constructing the model generates an inevitable discrepancy between the parameters of the model and the real property of the object. We will call this nonconformity a threshold discrepancy. The uncertainty caused by the threshold discrepancy between the model and the object should be less than the total measurement uncertainty. If, however, this component of the error exceeds the limit of the permissible measurement uncertainty, it is impossible to perform the measurement with the required accuracy. This result shows that the model is inadequate. To continue the experiment, if this is permissible for the measurement target, the model must be redefined. If an object is a natural object, the threshold discrepancy means that the model is not applicable and needs to be reviewed. The preceding logic reduces to the following postulates of measurement theory [3]:

- There is a true value of the measured quantity;
- In every measurement there is one true value;
- The true value of the measured quantity is constant;
- True value cannot be found due to the existence of an inevitable discrepancy between the parameters of the model and the real property of the object, called the threshold discrepancy.

In addition, there are other inevitable limitations to the approximation of the true value of the measured quantity. For example, the accuracy of measuring devices is inevitably limited. For this reason, we can formulate the following statement: the result of any measurement always contains an error. Thus, the accuracy of the measurement is always limited, and in particular, it is limited by the correspondence between the model and the phenomenon. We add that the achievable measurement accuracy is determined by *a priori* information about the measurement object.

The accepted model can be considered as corresponding to the studied physical phenomenon, if the differences between the obtained estimates of the mathematical expectation of the process are much smaller than the permissible measurement error. If, however, these differences are close to or exceed the error, then the model must be redefined, which is most easily done by increasing the observation interval.

It is interesting to note that the definitions of some quantities seem at first sight sufficient for high accuracy of measurements (if the errors of the measuring device are ignored). Examples of these are the parameters of stationary random processes, the parameters of distributions of random variables, and the mean value of a quantity. One would think that to achieve the required accuracy in these cases it is sufficient to increase the number of observations during the measurement. However, in reality, the accuracy of measurements is always limited, and in particular it is limited by the correspondence between the model and the phenomenon, i.e., the threshold discrepancy.

When the true value cannot be determined, measurement is impossible. For example, in the last few years, much has been written about the measurement of random variables. However, these values, as such, are not of true value, and for this reason they cannot be measured [1]. It is important to emphasize that *the present study refers only to variables for which a true value may exist*.

1.2. The similarity theory basics

Usually in textbooks on the theory of similarity, we first introduce the necessary concepts, including “quantity”, “likeness”, “dimensionality”, “homogeneity” and others. Then, Buckingham’s theorem is derived and many examples of the application of this theory are given in mechanics, heat transfer, hydraulics, etc. In contrast to this scheme, the author strives to focus only on those points that are directly related to the formulation of the presented approach. This, in turn, requires the reader to undertake some preliminary preparation and possess knowledge of the fundamental aspects of the theory.

The similarity theory is suitable for several reasons. When studying phenomena occurring in the world around us, it is advisable to consider not individual quantities but their combinations or complexes, which have a certain physical meaning. The methods of the similarity theory, based on the analysis of integral-differential equations and boundary conditions, additionally determine the possibility of identifying these complexes. Furthermore, the transition from dimensional physical quantities to dimensionless quantities reduces the number of counted values. The specified value of the dimensionless complex can be obtained using various combinations of dimensional quantities included in the complex. This means that when we consider problems with new quantities, we consider not an isolated case but a series of different events, united by some common properties. It is important to note that the universality of the similarity transformation is determined by invariant relationships that characterize the structure of all laws of nature, including the laws of relativistic nuclear physics. Moreover, dimensional analysis from the point of view of a mathematical apparatus has a group structure, and the transformation coefficients (similarity complexes) are invariants of groups. The concept of a group is a mathematical representation of the concept of symmetry, which is one of the most fundamental concepts of modern physics [4].

At the same time, it should be noted that the similarity theory does not answer the question of the number of possible combinations of dimensional characteristics included in the description of the dimensionless physical process and the form of these combinations. In addition, it is unclear what criteria for several interacting parts of an object are suitable for describing the physical process and how necessary they are for a given uncertainty in determining the selected base quantity [5].

It is obvious that all the physical dimensional quantities appearing in the mathematical model cannot make an infinite interval of changes in the real world. These values lie in certain intervals, the boundaries of which can be selected based on experience and intuition of the modeler, and an analysis of published scientific, technical, regulatory and technological literature.

The reasons for choosing the allowable intervals for the remaining physical characteristics included in the developed system of equations can be explained. The rules for the transition from differential equations to expressions in the final form are described in detail in [6]. In any case, for any physical phenomena and processes, as well as for any models describing a material object, it is necessary to choose the interval of expected changes in the main observable or measured quantity (criterion).

Bridgman [7] showed that for *all* physical dimensional quantities, a monomial formula satisfies the principle of absolute significance of relative magnitude *only* if it has the power-law form:

$$q \supset C_1^{\tau_1} \cdot C_2^{\tau_2} \cdot \ldots \cdot C_H^{\tau_h}, \qquad (1)$$

where q is the dimensional quantity, $C_1, C_2, \ldots C_H$, are numerical values of base quantities, and exponents $\tau_1, \tau_2, \ldots \tau_h$, are real numbers whose values distinguish one type of derived quantity from another. *All monomial derived quantities have this power-law form*; no other form represents a physical quantity.

If you know the range of variation of the exponents $\tau_1, \tau_2, \ldots \tau_h$ of the base quantities $C_1, C_2, \ldots C_H$ and these exponents take, for example, an integer value, then it is possible to calculate the total number of possible combinations contained in a finite set that includes all dimensional quantities. This statement will be discussed in Chapter 2.2, and necessary calculations with respect to the SI system will be carried out.

The logical continuation of (1) is the question of the possible number of dimensionless complexes that can be built on the basis of the selected base quantities. This question is answered by π-theorem that was proved by Buckingham [8].

Buckingham's π-theorem states that when the total relationship between dimensional physical quantities is expressed in dimensionless form, the number of independent quantities that appear in it decreases from the initial n to n-k, where k is the maximum number of initial n that are independent of dimension. The dimensional analysis reduces the number of values that must be specified to describe the event. This often leads to a huge simplification. At the same time, the π-theorem simply indicates to us the number of dimensionless quantities that affect the value of a particular dimensionless recognized value. It does not tell us about the form of dimensionless quantities. The form should be opened by experiments or theoretically solved problems.

There is no point in adding or subtracting quantities that have different units. You cannot add length to the mass. The point is that all the terms of the equation must have the same dimensions. This is called the dimensional homogeneity requirement.

A homogeneous equation is one in which each independent additive term has the same dimensions. There are functions that are homogeneous in their structure. The homogeneity of these functions does not depend on any additional assumptions on the properties of the transformations. Such functions are properly called unconditionally homogeneous. Only the degree complexes possess the property of absolute homogeneity.

All other operations related to the theory of similarity and dimensional analysis, and containing a choice of argument complexes and construction of parametric criteria, are based on considerations that are not within the scope of this study. Therefore, we have finished the discussion of the similarity theory. Only the above definitions will be used to formulate the proposed approach.

1.3. The information theory basics

The definition of information comes from statistical considerations. In this case, we define information as a result of a choice that always has a positive value. In our approach, we do not consider information as a result, which can be used to make a different choice. In this case, *the human evaluation of information is completely ignored*.

Random events—in our case, the choice of quantities in the model at the desire of the researcher—can be described using the concept of "probability". The probability theory allows us to find (calculate) the probability of one random event or a complex experience, combining a number of independent or unrelated events. If the event is accidental, it means there is a lack of full confidence in its implementation, which in turn creates uncertainty in the results of experiments related to the event. Of course, the degree of uncertainty is different for different situations.

Consider a system that represents P different events, when a particular quantity will be equally probable. When we impose restrictions on the quantities that reduce freedom of choice, these conditions exclude some of the pre-existing features. The new number of events P' with the restrictions should clearly be smaller than the original P. It should be noted that any limitation, additional requirement or condition imposed on the possible freedom of choice leads to a reduction in information. Therefore, we need to get the new value of information $Z' < Z$:

- without limitations: P equally probable outcomes, $Z = K\ln P$;
- with limitations: P' equally probable outcomes at $P' < P$, and

$$Z' = K \cdot \ln P' < Z, \qquad (2)$$

$$\Delta Z = Z - Z' = K \cdot \ln P - K \cdot \ln P' = K \cdot \ln(P/P'), \qquad (3)$$

where ΔZ is a change of information during the experiment, K is a constant, and ln is the natural logarithm.

One can prove [9] that in this case, information Z has a maximum when all events P are equal. The use of the logarithm in (2) is justified by the fact that we wish that the information is additive. For the first time, a logarithmic measure of information is suggested by Hartley [10].

In information theory, the information is usually regarded as a dimensionless quantity, and, therefore, the constant K is an abstract number, which depends on the choice of unit system. The most convenient system is based on binary units, which gives us:

$$K = 1/\ln 2 = \log_2 \mathrm{e} \qquad (4)$$

Another system of units can be introduced, if we compare the information with the thermodynamic entropy and measure both values in the same units. As it is known, the entropy has the dimension of energy divided by the temperature. For the entropy there is the Boltzmann formula that is very similar to (2) and contains the factor:

$$k_b = 1.38 \cdot 10^{-23} m^2 \text{kg}/(s^2 K). \qquad (5)$$

This constant ($\boldsymbol{k}_b$) is known as the Boltzmann constant. When we are interested in physical problems, such a choice of units allows us to compare the information with entropy itself. It should be noted that the ratio of units in equations (4) and (5) is equal to:

$$k_b/K = k_b \cdot \ln 2 \approx 10^{-23}. \qquad (6)$$

This numerical magnitude plays an important role in all applications of information theory [9].

At the same time, entropy is directly related to the “surprise” of the occurrence of the event. From this, it follows its information content: if the event is more predictable, it is less informative. This means that its entropy is lower. It remains an open question about the relationship between the properties of information, entropy properties and the properties of its various estimates. But we are just dealing with the estimates in most cases. All this lends itself to the study of the information content of different indexes of entropy regarding the controlled changes of properties and processes, i.e., in essence, their usefulness to specific applications [11].

Our definition of information is very useful and practical. It corresponds exactly to the task of the scientist, who must retrieve all the information contained in the physical- mathematical model, regardless of the limits to the achievable accuracy of the measuring instruments used for observation of the object. According to the suggested approach, the human evaluation of information is completely ignored. In other words, the set of 100 musical notes played by chimpanzees will have exactly the same amount of information as that of the 100 notes played by Mozart in his Piano Concerto No.21 (Andante movement).

The following explanations are specifically intended for a possible application of information theory to the modeling of physical phenomena and experiments.

Let us start with a simple example. We see the position of the point x on the segment of length S (range of observation) with uncertainty Δx. We introduce the definitions:

absolute uncertainty is Δx, (7)
relative uncertainty is $r_x = \Delta x/x$, (8)

comparative uncertainty is $\varepsilon_x = \Delta x / S$. (9)

The accuracy of the experiment ω can be defined as the value inverse to ε_x:

$$\omega = 1/\,\varepsilon_x = S/\Delta x\,. \quad (10)$$

This definition satisfies the condition that greater accuracy corresponds to lower comparative uncertainty. The absolute and relative uncertainties are familiar to physicists, but not comparative uncertainty because it is seldom mentioned. But the comparative uncertainty value is of great importance in the application of information theory to physics and engineering sciences [9].

If all the events are equiprobable, the amount of information obtained by observing the object ΔZ, according to (2) and (3), is equal to:

$$\Delta Z = k_b \cdot \ln(S/\Delta x) = -k_b \cdot \ln\varepsilon_x = k_b \cdot \ln\omega\,. \quad (11)$$

If the range of observation S is not defined, the information obtained during the observation/measurement cannot be determined, and the entropic price becomes infinitely large [9].

In turn, the efficiency Q of experimental observation, on the assumption that some perturbation is added into the system under study, may be defined as the ratio of the obtained information ΔZ to a value equal to the increase in entropy ΔH accompanying observation:

$$\mathrm{Q} = \Delta Z/\Delta H \quad (12)$$

It follows from all the above that the modeling is an information process in which information about the state and behavior of the observed object is obtained by the developed model. This information is the main subject of interest of modeling theory. During the modeling process, the information increases, while the information entropy decreases due to increased knowledge about the object [12]. The extent of knowledge W of the observed object may be expressed in the form:

$$W = 1 - H/H_{\max}, \quad (13)$$

where H is the information entropy of the object and $H_{\max}$ is its maximum value where the amount of knowledge can become A (0, 1). The impossibility of reaching the boundary values A=0 and A=1 is contained within the modeling theorems. These boundaries express ideal states.

It follows from the above, *a priori* and *a posteriori* information of the object must be known. The amount of the model information Z can be determined from the difference between initial H_1 and residual H_2 entropy:

$$Z = H_1 - H_2. \tag{14}$$

We intend to use all the above for defining a model's uncertainty considered and analyzed from an information measure-based perspective. In this case, entropy is used as a measure of uncertainty, and depends only on the amount and the probability distribution of quantities taken into account by the conscious observer for the development of a model.

1.4. Basics of the theory of modeling the phenomena

The key problem for modeling is one of cognition of physical reality, which is viewed through the prism of a set of physical laws that objectively describe the real world. In this regard, one of the main tasks of modeling is the development of theoretical and methodological aspects and procedures for achieving accurate knowledge of objects and processes in the surrounding world, related to the improvement of measurement accuracy. As a concentrated and most universal form of purposeful experience, modeling makes it possible to verify the reliability of the most general and abstract models of the real world, realizing the principle of observability. Modeling is a method of studying objects of cognition (actually existing) in their models; the construction and study of models of objects and phenomena (physical, chemical, biological, social) to determine or improve their characteristics, rationalize the methods of their construction, management, etc.

The model is a concrete image of the object under study, in which real or perceived properties, structures, etc. are displayed. Therefore, increasing the accuracy of measurements is given particular importance in modeling. In turn, the purpose of measurement is the formation of a certain objective image of reality in the form of a symbolic symbol, namely a number. At the same time, "potential measurement accuracy" does not receive enough attention. The task of this book is to fill, if possible, this gap. In its turn, the purpose of measurement is the formation of some objective image of reality in the form of a representative symbol, namely a number. At the same time, "the potential accuracy of measurements" has been given insufficient attention. The task of this book is to fill, if possible, this gap. We will understand by "the ultimate accuracy of measurements" the accuracy with which a physical quantity can be measured at a given stage in the development of science and technology, i.e., the highest accuracy achieved at the present time. “Potential accuracy of measurements” is understood as the maximum achievable accuracy, which has not yet been realized at the present stage of development of science and technology.

Modeling can be defined as a translation of the physical behavior of phenomena components and collections of components into a mathematical representation [13]. This representation must include descriptions of the individual components, as well as descriptions of how the components interact.

Mathematical modeling of various physical phenomena and technological processes is a challenge for the 2010s and beyond. The study of any physical phenomena or processes begins with the creation of the simplest experimental facts. They can formulate laws governing the analyzed material object, and write them in the form of certain mathematical relationships. The amount of prior knowledge, the purpose of analysis, and the expected completeness and accuracy of the necessary decisions determine the level schematic of the test process.

A model is a physical, mathematical or otherwise logical representation of the real system, entity, phenomenon or process. Simulation is a method for implementing a model over time. The real system, in existence or proposed, is regarded as fundamentally a source of data.

In general, every model of the object does not contain the wording of the causal relationships between the elements of the object in the form of ready-made analytical expressions. In some cases, we have to be satisfied with such bonds (qualitative and quantitative) which characterize the material object only in the most general terms, and express a much smaller amount of knowledge about the internal structure of the test process. In all cases, the model is a user-selectable abstraction in the first place because it was built for an intuitively designated object, and also because of the incomplete or inaccurate knowledge (conscious simplification) of the laws of nature. From the point of view of developers, if the difference between the results of theoretical calculations and the data obtained in the course of experiments is less than the measurement uncertainty achieved, the chosen physical and mathematical model is considered acceptable.

However, comprehensive testing of the model is impossible [14]. Exhaustive checking is realized only upon receipt of all results from a model sweep for all possible variants of the input data. In practice, model validation aims to increase confidence in the accuracy of the model. Estimations produced by the model can be made with different levels of detail, but there is no generally accepted or standard procedure which would establish the minimum quantitative requirements for the design of model testing [15].

Over last two decades, many studies have been conducted to identify which method will demonstrate the most accurate agreement between observation and prediction. Unfortunately, the confirmation is only inherently partial. Complete confirmation is logically precluded by the incomplete access to the material object. At the same time, the general strategies of matching models and a recognized object that have been particularly popular from both a theoretical and applied perspective are *verification* and *validation* (V&V) techniques [16].

In [17] the following definition is proposed: *verification* is the process of determining that a computational model accurately represents the basic mathematical model and its solution; *validation* is the process of determining to what degree a model is an accurate representation of the real world from the perspective of the intended use of the model.

Given the above definition, we can say that the quality validation may be useful in certain scenarios, especially when identifying possible causes of errors in the model. However, at the moment, the validation is not able to provide a quantitative measure of the agreement between the experimental and computer data. This makes it difficult to use in determining at what point the accuracy requirements are met [16]. We refer the reader to [18, 19] for a more detailed discussion of the existing developments in V&V.

However, some scholars suggest that the V&V of numerical models of natural systems is impossible [20]. The authors argue that the models can never fully simulate reality in all conditions and, therefore, cannot be confirmed.

So, the causes of numerous attempts to direct the use of the experimental results are the limited applicability of different applied methods (analytical and numerical), and the difficulties with the use of computers and the methods of computational mathematics because of the lack of qualified researchers. Decisions resulting from the correlation of experimental data in the form of graphs, nomograms and criteria equations allow us to judge the quality and, to a certain extent, the proportion of the observed parameters of the process. Nevertheless, the experimental method cannot explain why the process is in the direction of what is observed in practice, nor accurately substantiate the list of selected process parameters.

Experience in dealing with the problems associated with various applications has shown that a preliminary analysis of a mathematical model using the theory of similarity (the definition of a set of physical criteria, each of which controls a specific behavior of a physical phenomenon) and the subsequent application of numerical methods to implement them on a computer allows us to obtain detailed information that cannot be obtained by analytical methods. However, analytical methods, by contrast to numerical methods, allow the creation of more visual solutions with which the influence of selected factors on the result of the decision can easily be analyzed. In addition, in practice it is considered a good result if it is obtained with an accuracy of up to 10% or even more [21]. Thus, research to consider various processes is basically a synthesis based on analytical and numerical methods.

The modern idea of combining the analytical and numerical methods is in the computational experiment [22]. This experiment consists of several stages. The first step is to compile equations of the problem, expressing in quantitative form a general idea of the physical mechanism of the process. They are based on the analysis of the process as a particular application of the fundamental principles of physics. In most cases they are in the form of differential (integral, integral-differential) equations.

Since the studied process is quite complicated and it cannot be investigated on the basis of only one physical law, there is a need to consider various aspects of the model and also different physical laws. Therefore, the overall process is usually determined by the system of equations.

In addition to the basic equations, there are written boundary conditions: a set of constant parameters characterizing the geometric and physical properties of the system that are essential for the process as well as conditions for uniqueness.

After the mathematical model is made, it is necessary to determine the correctness of its formulation (the existence of a solution, its uniqueness, whether it continuously depends on the boundary conditions). However, in practice, for many applications it is impossible to rigorously prove theorems of existence and uniqueness. So, there are some “illegal” mathematical techniques used that do not have a precise mathematical justification [23].

In the second stage of the computational experiment, the selection of the computation algorithm is realized. In a broad sense, the algorithm refers to the exact prescription that specifies the computational process, starting from an arbitrary initial datum and aiming to obtain results which are completely defined by this initial data [24]. In a narrow sense, computational algorithms are the sequence of arithmetic and logical operations, by which the mathematical problem is solved [22].

A computational algorithm focused on the use of modern computers must meet the following requirements: 1) provide a solution of the original problem with a given accuracy after a finite number of actions; 2) implement the decisions of the problem by taking the least possible computer time; 3) ensure the absence of an emergency stop of computers during the calculations; and 4) be sustainable (in the calculation process, rounding errors should not be accumulated). For more detailed information about this phase of the computational experiment, see [25].

In the third stage, the computer programming of a computational algorithm is organized. A huge amount of work is devoted to this issue. Given the specificity of this study, the greatest work of interest can be found in [26].

The fourth stage involves performing calculations on a computer, and the fifth involves the analysis of the numerical results and the subsequent refinement of the mathematical model.

From the standpoint of saving computer time and the practical value of the information obtained, the organization and planning of the last two stages of the computational experiment are important. So, just before the start of the computational experiment, the question of the scope and methods of processing (convolution) output data should be carefully considered. Obviously, in the study of any process, the experimenter has to accommodate a large number of quantities, and accordingly, the solving of the multi-criteria problem.

It should be noted that to find hidden relationships between quantities in the case of the multi-quantity model is very difficult. So, it is valuable to use the methods of the theory of similarity, which are in accordance with modern ideas and can be called a theory of generalized quantities [6]. Application of this theory is advisable for several reasons mentioned in Chapter 1.1. At the moment, the similarity theory does not answer the question about the number of possible combinations of dimensional characteristics included in the description of the dimensionless physical process, and the form of these combinations. In addition, it is not clear what criteria, for many interacting quantities, are suitable for the description of the physical process and how much they are required for a given uncertainty in the determination of the chosen main quantity [5].

So, realized in the form of a computer program, the mathematical model is a kind of computational experimental unit [27] that has several advantages over the conventional technology experimental construction:

- universality, because for the study of a new version of the computing installation it is only necessary to introduce new background information, whereas the technologically realized experiment will need a lot of raw materials and sometimes reinstalling, reconstruction and even full-scale installation of the new design;
- the possibility to obtain complete information about the effect of process parameters on the temperature field of the interacting bodies.

However, the array of information provided by the computing unit has a very large volume, making it difficult to process it.

At the same time, implementation of the full-scale experiment at the test conditions of the process equipment would be fraught with even greater difficulties. In order to be able to compare the numerical calculations and the experimental data, it is necessary to hold at least the same number of experiments, with the options as calculated by the computer. To make the experimental data statistically significant, it is needed to organize three to five replications in each experiment. This will lead to a further increase in labor costs and an increase in the duration of the experiments, which, in turn, affects the accuracy of the experimental data.

On the other hand, it is obvious that with random, haphazard use of any sorting options, usage of the fastest computer does not provide optimal solutions. It needs a deliberate and planned recognition of these options. However, not all parameters equally affect the researched process. So, the reduction of the number of quantities to a minimum on the basis of their relative influence and the selection of essential process quantities is the most important goal in the correct formulation of the problem. For this reason, the active principles of the theory of experimental design [28] are most valuable.

There are various methods for global sensitivity analysis of an output data model. Numerous statistical and probabilistic tools (regression, smoothing, tests, statistical training, Monte Carlo, random balance, etc.) are aimed at determining the input quantities which most affect the selected target quantity of the model. This value may be, for example, the variance of the output quantity. Three types of methods are distinguished: screening (coarse sorting being the most influential among a large number of inputs), the measure of importance (quantitative sensitivity indices) and in-depth study of the behavior of the model (measuring the effects of inputs on their variation range) [29]. As an example of the organization and usage of phases of the sensitivity analysis, we will discuss the method of random balance here.

In the method of random balance, linear effects and pairwise interactions are eliminated. But, at the same time, there is an additional constraint: it is assumed that the number of significant effects is significantly less than the total number of effects taken into consideration.

The application of random balance in the study of any process has, in principle, two features. The solution to any practical problem will be of great value when the independent quantities are used as generalized criteria, rather than individual factors of the physical dimension. The rationale for this approach is justified in [30]. In this case, the monitoring process is less