

A Critical Perspective  
of Entropy Generation  
Minimization in  
Thermal Analyses  
and Optimizations



# A Critical Perspective of Entropy Generation Minimization in Thermal Analyses and Optimizations

By

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# PREFACE

Thermal phenomena, including heat transfer and heat-work conversion, are very common and basic in human life. From day to night, from the sun to the carbon nanowires, from cooking in a kitchen to working in spacecraft, one can easily see the transport or conversion of heat almost everywhere in nature. However, the understanding and application of thermal phenomena are by no means easy. As is known, it took so long a time for people to find that there is no perpetual motion. The controversy between the caloric theory of heat and the thermal-motion theory also lasted many years. With the untiring efforts from generation to generation, the understanding of the thermal phenomena has deepened, many technologies have been developed, and human life has been improved accordingly.

Nowadays, we are also facing many thermal problems. For instance, thermal design is often required to ensure that the equipment are working under proper temperatures. In practical applications, thermal optimization is also very necessary to obtain the best thermal performance so that the energy utilization efficiency can be increased and the economic cost can be decreased. When trying to solve the thermal problems, researchers have developed many different theories and methods, the vast majority of which can only be used in specific fields to solve specific problems.

However, in academic circles, researchers have a preference for finding a unified theory. Even in a very narrow field, they always have a beautiful expectation that they could use only one theory or method to

solve all problems. Generally, this goal is not easy to achieve no matter how narrow the field is. This is also true in thermal analyses and optimizations. Up to now, we still do not have any unified theory in this area. During the past decades, the entropy generation minimization has been widely used and found to be effective in many cases. Sometimes, it was even used without checking the applicability. It seems that this theory is the unified one that we want. But, is it true? In this book, this question can be discussed in detail, and the applicability of entropy generation minimization is analyzed with theoretical derivations and different numerical examples.

The optimization directions and the application preconditions of entropy generation minimization are summarized and discussed. In thermal engineering, when the design objectives are inconsistent with the optimization directions of entropy generation minimization or the application preconditions are not satisfied, it is shown that the entropy generation minimization may not lead to the objectives. In heat transfer and heat-work conversion, this point is demonstrated clearly with different examples, in which the entropy generation may not be the minimum, but the maximum or an intermediate value when the objectives are achieved. As there are many different design objectives in thermal analyses and optimizations, only some typical ones are discussed in this book, including the maximum heat transfer rate, the minimum average temperature of the heated domain, the maximum heat exchanger effectiveness, the minimum thermal resistance, the system mass minimization, the minimum economic cost, the maximum output work, the maximum thermal efficiency, the best thermo-economic performance, etc.

Especially, the “entropy generation paradox” is analyzed in detail. It is shown clearly that the entropy generation rate in heat exchangers does not

always decrease with increasing heat exchanger effectiveness or heat transfer rate, and the “entropy generation paradox” may not disappear even if one considers the effect of heat exchangers on the whole energy system in which the heat exchangers are organs. In heat exchanger networks, it is also shown that the entropy generation minimization does not always give the best system performance. Therefore, entropy generation is not a proper parameter to evaluate the heat transfer performance of heat exchangers and heat exchanger networks.

In addition, the applicability of the dimensionless parameters of entropy generation, the entropy resistance minimization and the exergy destruction minimization is discussed and found to be conditional. In other words, these parameters and methods are not omnipotent or perfect, either. The limitations of entropy generation minimization are also pointed out directly and discussed in detail. It is clearly revealed that the entropy generation minimization has a definite application scope, beyond which the theory may not be effective.

All in all, although the entropy generation minimization has been widely used, it is not the unified theory in thermal analyses and optimizations, and cannot be used to solve all thermal problems. Therefore, before its application, one should pay enough attention to the applicability to avoid the abuse of the theory.



# CHAPTER ONE

## INTRODUCTION

Thermal analyses and optimizations are very ubiquitous in academic research and engineering applications because they are of significant importance to the improvement of system performance, the increase of energy utilization efficiency and the decrease of economic cost<sup>1</sup>. For instance, the application of heat pipes can enhance heat transfer<sup>2</sup>. However, the number of heat pipes may be limited with the consideration of economic cost. Therefore, the distribution of the limited heat pipes should be analyzed and optimized to make the heat transfer rate as high as possible. In the “Volume-to-Point” heat conduction problem, the distribution of the limited high thermal conductivity material should be analyzed and optimized to obtain the minimum average temperature of the heated domain<sup>1d</sup>. In thermal power plants, the operation and structure

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<sup>1</sup> a) Schneider P J. Conduction Heat Transfer. Addison-Wesley, New Jersey, USA, 1955.

b) Bergles A E. Heat transfer enhancement-The maturing of second-generation heat transfer technology. Heat Transfer Eng, 1997, 18: 47-55.

c) Webb R L. Principles of enhanced heat transfer. John Wiley & Sons, New York, USA, 1994.

d) Guo Z Y, Zhu H Y, Liang X G. Entransy-A physical quantity describing heat transfer ability. Int J Heat Mass Transfer, 2007, 50: 2545-2556.

e) Cheng X T, Liang X G, Guo Z Y. Entransy decrease principle of heat transfer in an isolated system. Chin Sci Bull, 2011, 56: 847-854.

f) Zhang X R, Ren J X, Liang X G, et al. Optimization of the thermal network in parallel connection of manned spacecraft (in Chinese). J Tsinghua Univ (Sci & Tech), 2002, 42: 462-465.

<sup>2</sup> Cheng X T, Liang X G. Entransy flux of thermal radiation and its application to enclosures with opaque surfaces. Int J Heat Mass Transfer, 2011, 54: 269-278.

parameters should also be analyzed and optimized to obtain the maximum output power<sup>3</sup>. In different thermal systems, we may have different design objectives, such as the minimum heat transfer temperature difference<sup>1d</sup>, the maximum heat exchanger effectiveness<sup>4</sup>, the system mass minimization<sup>5</sup>, the homogenization of temperature field<sup>6</sup>, the maximum coefficient of performance (COP) in heat pump systems<sup>7</sup>, etc. When trying to achieve these objectives, researchers and engineers have developed many design methods and theories, such as the constructal theory<sup>8</sup>, the biological evolution method<sup>9</sup>, the entransy theory<sup>1d</sup>, etc. In this book, we mainly focus on the entropy generation minimization<sup>10</sup>, and discuss its applications in thermal analyses and optimizations.

From the viewpoint of thermodynamics, thermal processes in nature are irreversible, and the irreversibility can be measured by the concept of entropy generation. Prigogine<sup>11</sup> showed that the entropy generation of a linear unbalanced system always decreases with time until the entropy

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<sup>3</sup> Sun C, Cheng X T, Liang X G. Output power analyses for the thermodynamic cycles of thermal power plants. *Chin Phys B*, 2014, 23: 050513.

<sup>4</sup> Guo Z Y, Zhou S Q, Li Z X, et al. Theoretical analysis and experimental confirmation of the uniformity principle of temperature difference field in heat exchanger. *Int J Heat Mass Transfer*, 2002, 45: 2119-2127.

<sup>5</sup> Zhou B, Cheng X T, Liang X G. Conditional extremum optimization analyses and calculation of the active thermal control system mass of manned spacecraft. *Appl Thermal Eng*, 2013, 59: 639-647.

<sup>6</sup> Cheng X T, Xu X H, Liang X G. Homogenization of temperature field and temperature gradient field. *Sci China Ser E: Tech Sci*, 2009, 52: 2937-2942.

<sup>7</sup> Cheng X T, Liang X G. Entransy and entropy analyses of heat pump systems. *Chin Sci Bull*, 2013, 58: 4696-4702.

<sup>8</sup> Bejan A. Constructal theory network of conducting paths for cooling a heat generating volume. *Int J Heat Mass Transfer*, 1997, 40: 779-816.

<sup>9</sup> Xia Z Z, Li Z X, Guo Z Y. Heat conduction: High-conductivity construction based on biological evolution. *Proceedings of the 12th International Heat Transfer Conference, AIHTC, Grenoble, 2002, 2: 27-32.*

<sup>10</sup> a) Bejan A. Entropy generation minimization: The new thermodynamics of finite-size devices and finite-time processes. *J Appl Phys*, 1996, 79: 1191-1218.

b) Bejan A. Entropy generation minimization. CRC Press, Florida, USA, 1995.

<sup>11</sup> Prigogine I. *Introduction to thermodynamics of irreversible processes (3rd ed.)*. John Wiley & Sons, New York, USA, 1967.



generation gets a minimum value when the system achieves a non-equilibrium steady state. This is the minimum entropy production principle, with which the minimum value of entropy production has been related to the steady state and we can judge whether a linear unbalanced thermal system reaches its steady state or not. However, the concept of entropy generation has not been related to thermal optimization yet.

Furthermore, Bejan<sup>10b, 12</sup> developed the expressions of entropy generation for heat and fluid flows, and analyzed the minimum combined entropy generation induced by heat transfer and fluid viscosity as the objective function to optimize the geometry of heat transfer tubes and to find the optimal parameters for heat exchangers and thermal systems. By this time, the concept of entropy generation has been related to thermal optimization, and the entropy generation minimization has been proposed for thermal analyses and optimizations.

From above, it can be seen that the difference between the minimum entropy production principle and the entropy generation minimization is obvious. For the thermal processes in a thermal system, they may not be the optimal one that we want. However, the minimum entropy production principle is always tenable, and indicates that the entropy generation must be a minimum value when the system achieves its steady state. On the other hand, we may change the structure and operation parameters of the system and obtain different steady states and different entropy generations. The entropy generation minimization claims that the entropy generation of the optimal steady state should be the minimum.

From the viewpoint of entropy generation minimization, the decrease of entropy generation means the decrease of irreversibility and could improve the thermodynamic performance of thermal systems. In heat-work conversion, the minimization of entropy generation could lead

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<sup>12</sup> Bejan A. Entropy generation through heat and fluid flow. John Wiley & Sons, New York, USA, 1982.

to the maximum output mechanical work for heat engine systems and the minimum input mechanical work for refrigeration systems, respectively. In heat transfer enhancement design, the entropy generation minimization indicates that the thermal conductance should be improved to decrease the heat transfer temperature difference and entropy generation for fixed heat transfer rate, while it shows that the thermal conductance should be decreased to decrease the heat transfer rate and entropy generation for fixed heat transfer temperature difference in thermal insulation design. In one word, the minimum entropy generation becomes the design objective for thermal systems. With this theory, researchers have done much work and many interesting results have been obtained<sup>10,12,13</sup>. For instance, Myat et al.<sup>13a</sup> analyzed an absorption chiller and found that the minimization of entropy generation leads to the maximization of the COP. Ereğ and Dincer<sup>13b</sup> analyzed a latent heat storage module and the results showed that entropy generation is crucial in such systems and should be minimized in order to increase the exergy efficiency. Pussoli et al.<sup>13c</sup> also optimized peripheral finned-tube evaporators with the entropy generation minimization.

However, it was noticed that the entropy generation minimization does not always lead to the design objectives in many cases. In heat transfer, Bejan<sup>14</sup> himself found that there is an entropy generation paradox when

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<sup>13</sup> a) Myat A, Thu K, Kim Y D, et al. A second law analysis and entropy generation minimization of an absorption chiller. *Appl Thermal Eng*, 2011, 31: 2405-2413.

b) Ereğ A, Dincer I. An approach to entropy analysis of a latent heat storage module. *Int J Thermal Sci*, 2008, 47: 1077-1085.

c) Pussoli B F, Barbosa J R, Silva L W, et al. Optimization of peripheral finned-tube evaporators using entropy generation minimization. *Int J Heat Mass Transfer*, 2012, 55: 7838-7846.

d) Saghi H, Lakzian E. Optimization of the rectangular storage tanks for the sloshing phenomena based on the entropy generation minimization. *Energy*, 2017, 128: 564-574.

<sup>14</sup> Bejan A. *Advanced engineering thermodynamics* (2nd ed.). John Wiley & Sons, New York, USA, 1997.

the entropy generation minimization was used to analyze the balanced counterflow heat exchangers. The heat exchanger effectiveness does not always increase with decreasing entropy generation number, but decreases under some conditions. Shah and Skiepko<sup>15</sup> discussed the relationship between heat exchanger effectiveness and entropy generation for eighteen kinds of heat exchangers and found that the heat exchanger effectiveness can be maximum, intermediate or minimum at the maximum entropy generation. Sahiti et al.<sup>16</sup> found that not all definition forms for the entropy generation number lead to the right conclusions when they optimized a double-pipe pin fin heat exchanger. In conductive heat transfer, convective heat transfer and radiative heat transfer, it was also found that sometimes the design objectives cannot be achieved with the entropy generation minimization<sup>2,17,18</sup>. In heat-work conversion, the entropy generation minimization may not lead to the optimal system performance, either. For instance, Klein and Reindl<sup>19</sup> found that minimizing the entropy generation rate does not always result in the same design as maximizing the system performance unless the refrigeration capacity is fixed. Haseli<sup>20</sup> showed that the entropy generation

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<sup>15</sup> Shah R K, Skiepko T. Entropy generation extrema and their relationship with heat exchanger effectiveness-Number of transfer unit behavior for complex flow arrangements. *ASME J Heat Transfer*, 2004, 126: 994-1002.

<sup>16</sup> Sahiti N, Krasniqi F, Fejzullahu Xh, et al. Entropy generation minimization of a double-pipe pin fin heat exchanger. *Appl Thermal Eng*, 2008, 28: 2337-2344.

<sup>17</sup> Wei S H, Chen L G, Sun F R. Constructural entransy dissipation minimization for "Volume-Point" heat conduction based on triangular element. *Thermal Sci*, 2010, 14: 1075-1088.

<sup>18</sup> Chen Q, Liang X G, Guo Z Y. Entransy theory for the optimization of heat transfer-A review and update. *Int J Heat Mass Transfer*, 2013, 63: 65-81.

<sup>19</sup> Klein S A, Reindl D T. The relationship of optimum heat exchanger allocation and minimum entropy generation rate for refrigeration cycles. *J Energy Res*, 1998, 120: 172-178.

<sup>20</sup> a) Haseli Y. Performance of irreversible heat engines at minimum entropy generation. *Appl Math Model*, 2013, 37: 9810-9817.

b) Haseli Y. Efficiency of irreversible Brayton cycles at minimum entropy generation. *Appl Math Model*, 2016, 40: 8366-8376.

minimization neither correlates with the maximum thermal efficiency design nor with the maximum work output criterion when he analyzed the performance of irreversible heat engines. You et al.<sup>21</sup> also got similar conclusions when they analyzed the Dual-Miller cycle. It was shown that the entropy generation minimization is neither equivalent to the maximum thermal efficiency nor equivalent to the maximum power production in general.

From the discussion above, we can see that the entropy generation minimization has limitations in thermal analyses and optimizations although it is effective in many cases. In other words, the entropy generation minimization is not always applicable. This point should be carefully noticed when this theory is used. Our comments here do not reduce the importance of entropy generation minimization because it is very common to have limitations for scientific theories. In one system, one theory may be only effective for definite design objectives, which means that the applicability of the theory may change when the objective changes. For instance, the constructal theory is effective to decrease the highest temperature of the heated domain in the “Volume-to-Point” heat conduction problem<sup>8</sup>. However, if our design objective is changed to be the minimum average temperature of the heated domain, it was found that the entransy theory may lead to lower average temperature<sup>17,18</sup>. Furthermore, one theory may be effective in this system, but not effective in another one. In other words, the applicability of the theory may change when the system changes. We can take the Fourier heat conduction law as an example. As is known, the law is widely used and works well in many thermal systems. However, nowadays, it has met great challenges in heat conduction at ultra-small scale, both temporal and spatial scales, where energy transport plays a very important role for material designs and new

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<sup>21</sup> You J, Chen L G, Wu Z X, et al. Thermodynamic performance of Dual-Miller cycle (DMC) with polytropic processes based on power output, thermal efficiency and ecological function. *Sci China Tech Sci*, 2018, 61: 453-463.

power resources<sup>22</sup>. Therefore, when we use a theory to analyze one system, the limitations of the theory should be carefully noticed to make sure whether the theory is suitable for the system and the design objective or not.

In this book, the applicability of entropy generation minimization to thermal analyses and optimizations is discussed. The optimization directions and the application preconditions are summarized and analyzed. Some typical examples in heat transfer and heat-work conversion are presented to verify our analyses, and some related literatures on the applicability analyses of entropy generation minimization are reviewed and discussed. To avoid the misuse of entropy generation minimization, the limitations of the theory are also pointed out and analyzed in detail.

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<sup>22</sup> Wang M, Yang N, Guo Z Y. Non-Fourier heat conductions in nanomaterials. *J Appl Phys*, 2011, 110: 064310.



## CHAPTER TWO

# OPTIMIZATION DIRECTIONS AND APPLICATION PRECONDITIONS OF ENTROPY GENERATION MINIMIZATION

In this chapter, the applicability of entropy generation minimization to steady thermal systems is analyzed and discussed in theory. Based on the entropy balance equation, some basic concepts and formulas are presented with theoretical derivations. With these concepts and formulas, the optimization directions and the corresponding application preconditions of entropy generation minimization in heat transfer and heat-work conversion are summarized and discussed. An additional discussion is also made on irreversibility and entropy generation.

### **Part I Optimization direction of entropy generation minimization**

In thermal analyses, we have two kinds of systems, transient and steady thermal systems. Sometimes, we focus on the transient characteristics, while sometimes we concern more on the steady characteristics. For instance, when using an air conditioner, we sometimes pay attention to the change rate of the room temperature, and sometimes concern the power consumption for a steady room temperature. For the operation of the thermal control systems in spacecrafts, we should focus on both the steady and transient characteristics to ensure that the spacecrafts can work safely and stably. When we analyze transient thermal

systems, we should consider the variations of the physical parameters with time. Therefore, the analyses would be more complex. With this consideration, we only analyze steady thermal systems to simplify our discussions and make our conclusions clearer below.

For any thermodynamic system, the entropy balance equation is<sup>23</sup>

$$dS = \delta S_f + \delta S_g, \quad (2-1)$$

where  $dS$  is the entropy change with time,  $\delta S_f$  is the entropy flow, and  $\delta S_g$  is the entropy generation. For steady systems,  $dS = 0$ , so the entropy generation is

$$\delta S_g = -\delta S_f. \quad (2-2)$$

Considering the definition of entropy flow, we have<sup>24</sup>

$$S_g = \int \delta S_g = -\int dS_f = S_{f-out} - S_{f-in} = \int_{A_{out}} \frac{q_{out}}{T} dA - \left( \int_{A_{in}} \frac{q_{in}}{T} dA + \int_V \frac{Q_V}{T} dV \right), \quad (2-3)$$

where  $S_{f-out}$  is the entropy flow out of the system, while  $S_{f-in}$  is that into the system;  $T$  is the temperature;  $A$  is the area;  $q_{out}$  is the heat flow density out of the system through the corresponding boundary area  $A_{out}$ , while  $q_{in}$  is that into the system through the corresponding boundary area  $A_{in}$ ;  $V$  is the volume, and  $Q_V$  is the inner heat source that is assumed to be positive. In Eq. (2-3), we can obtain that<sup>24</sup>

$$\int_{A_{out}} \frac{q_{out}}{T} dA = \frac{\int_{A_{out}} q_{out} dA - \int_{A_{out}} q_{out} \left(1 - \frac{T_0}{T}\right) dA}{T_0} = \frac{Q_{out} - E_{out}}{T_0}, \quad (2-4)$$

<sup>23</sup> a) Moran M J. Availability analysis: A guide to efficient energy use. Prentice-Hall Inc., Englewood Cliffs, USA, 1982.

b) Zeng D L, Ao Y, Zhang X M, et al. Engineering thermodynamics (in Chinese). High Education Press, Beijing, China, 2002.

<sup>24</sup> Cheng X T, Liang X G. Discussion on the applicability of entropy generation minimization to analyses and optimizations of thermodynamic processes. Energy Convers Manage, 2013, 73: 121-127.



$$\begin{aligned}
 \int_{A_{in}} \frac{q_{in}}{T} dA + \int_V \frac{Q_V}{T} dV &= \frac{\int_{A_{in}} q_{in} dA - \int_{A_{in}} q_{in} \left(1 - \frac{T_0}{T}\right) dA + \int_V Q_V dV - \int_V Q_V \left(1 - \frac{T_0}{T}\right) dV}{T_0} \\
 &= \frac{\left(\int_{A_{in}} q_{in} dA + \int_V Q_V dV\right) - \left[\int_{A_{in}} q_{in} \left(1 - \frac{T_0}{T}\right) dA + \int_V Q_V \left(1 - \frac{T_0}{T}\right) dV\right]}{T_0} \quad (2-5) \\
 &= \frac{Q_{out} - E_{in}}{T_0}
 \end{aligned}$$

where  $T_0$  is the environment temperature;  $Q_{out}$  is the total heat flow out of the system, while  $Q_{in}$  is that into the system;  $E_{out}$  is the exergy flow out of the system, while  $E_{in}$  is that into the system; and

$$Q_{out} = \int_{A_{out}} q_{out} dA, \quad (2-6)$$

$$Q_{in} = \int_{A_{in}} q_{in} dA + \int_V Q_V dV, \quad (2-7)$$

$$E_{out} = \int_{A_{out}} q_{out} \left(1 - \frac{T_0}{T}\right) dA, \quad (2-8)$$

$$E_{in} = \int_{A_{in}} q_{in} \left(1 - \frac{T_0}{T}\right) dA + \int_V Q_V \left(1 - \frac{T_0}{T}\right) dV. \quad (2-9)$$

Substituting Eqs. (2-4) and (2-5) into Eq. (2-3) and considering the energy conservation lead to

$$S_g = \frac{Q_{out} - E_{out}}{T_0} - \frac{Q_{in} - E_{in}}{T_0} = \frac{E_{in} - E_{out}}{T_0} - \frac{Q_{in} - Q_{out}}{T_0} = \frac{E_{net}}{T_0} - \frac{W}{T_0}, \quad (2-10)$$

where  $E_{net}$  is the net exergy flow into the system, and  $W$  is the output work of the system. Then, we have<sup>24,25</sup>

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<sup>25</sup> Salamon P, Hoffmann K H, Schubert S, et al. What conditions make minimum entropy production equivalent to maximum power production? J Non-Equilib Thermodyn, 2001, 26: 73-83.

$$E_{\text{net}} = T_0 S_g + W \quad (2-11)$$

From the viewpoint of exergy, it can be seen that some of the net exergy flow into the system becomes the output work, and the rest is destroyed because of irreversibility. Especially, if the system is a heat transfer system, there is no output work, and all the net exergy flow into the system is destroyed. There is

$$E_d = E_{\text{net}} - W = T_0 S_g, \quad (2-12)$$

where  $E_d$  is the exergy destruction, which also means the loss of ability to do work. This is the Gouy-Stodola theorem, which indicates that the entropy generation directly reflects the exergy destruction or the loss of ability to do work because the environment temperature is fixed in general. Therefore, if the minimum entropy generation of thermal systems becomes the design objective, it is very clear that the optimization direction is to find the design result that gives the minimum exergy destruction and the minimum loss of ability to do work.

## **Part II Application preconditions of entropy generation minimization**

As above, it is shown that the optimization direction of entropy generation minimization is definite. When we design or optimize a thermal system, we also should have a definite direction to achieve our objective. If the two directions are consistent with each other, it is very sure that the entropy generation minimization can lead to the design objective, while the entropy generation minimization may not give the objective if the two directions are not coincident. Whether the theory can be used to design or optimize a system depends on its consistency with the design objective.

As shown in Chapter One, we may have many different design objectives in different cases. If all these objectives could be achieved with

the entropy generation minimization, the optimization direction of the entropy generation minimization should be coincident with all the different objectives. Hence, the design directions or optimization directions of different objectives should be the same. This is impossible. Otherwise, the objectives are probably not different ones, but the same. Therefore, it is not strange that the entropy generation minimization is not always applicable.

Sometimes, the relationships between entropy generation and our design objectives can be set up. In these cases, the application preconditions of entropy generation minimization can be obtained. For instance, the relationship between entropy generation and output work can be got with Eq. (2-11). As the environment temperature is fixed in general, the entropy generation minimization leads to the maximum output work for fixed net exergy flow into the system. Therefore, the precondition for the application of entropy generation minimization to optimizing output work is that the net exergy flow into the system should be fixed, and the applicability of entropy generation minimization is conditional. If the precondition is not satisfied, the minimum entropy generation may not result in the maximum output work<sup>24</sup>.

The relationship between entropy generation and output work can also be set up in another way, which can be shown below. For the heat flow inlet and outlet through which heat flows into and out of the system, the equivalent thermodynamic forces weighted by the corresponding heat flows can be defined as<sup>26</sup>

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<sup>26</sup> a) Onsager L. Reciprocal relations in irreversible process I. Phys Rev, 1931, 37: 405-426.

b) Onsager L. Reciprocal relations in irreversible process II. Phys Rev 1931, 38: 2265-2279.

c) Sauar E, Kjelstrup R S, Lien K M. Equipartition of forces: A new principle for process design and optimization. Ind Eng Chem Res, 1996, 35: 4147-4153.

d) Nummedal L, Kjelstrup S. Equipartition of forces as a lower bound on the entropy production in heat exchange. Int J Heat Mass Transfer, 2001, 44: 2827-2833.

$$F_{\text{out}} = \frac{S_{F_{\text{out}}}}{Q_{\text{out}}} = \frac{\int_{A_{\text{out}}} \frac{q_{\text{out}}}{T} dA}{Q_{\text{out}}}, \quad (2-13)$$

$$F_{\text{in}} = \frac{S_{F_{\text{in}}}}{Q_{\text{in}}} = \frac{\int_{A_{\text{in}}} \frac{q_{\text{in}}}{T} dA + \int_V \frac{Q_V}{T} dV}{Q_{\text{in}}}, \quad (2-14)$$

where  $F_{\text{out}}$  and  $F_{\text{in}}$  are the equivalent thermodynamic forces of the heat flow outlet and inlet, respectively. Then, based on Eq. (2-3), we have<sup>26e</sup>

$$S_g = Q_{\text{out}} F_{\text{out}} - Q_{\text{in}} F_{\text{in}} = (Q_{\text{in}} - W) F_{\text{out}} - Q_{\text{in}} F_{\text{in}} = Q_{\text{in}} (F_{\text{out}} - F_{\text{in}}) - W F_{\text{out}}. \quad (2-15)$$

It can be seen that the entropy generation minimization leads to the maximum output work when the heat flow into the system and the equivalent thermodynamic forces are prescribed. Here, the optimization direction of entropy generation minimization is to maximize the output work, and the corresponding application preconditions are also definite. If the heat flow into the system is not given or the equivalent thermodynamic forces are not prescribed, the entropy generation minimization may not lead to the maximum output work<sup>26e</sup>.

In addition, although Eq. (2-11) is from the viewpoint of exergy and Eq. (2-15) is from the viewpoint of thermodynamic force, it can be seen from above that Eqs. (2-11) and (2-15) are both derived from the entropy balance equation, and they are equivalent to each other. In the two equations, the net exergy flow into the system is fixed when the heat flow into the system and the equivalent thermodynamic forces are prescribed. Therefore, the preconditions for the application of entropy generation minimization shown in both equations are also equivalent to each other. In other words, they are the two sides of one coin.

When analyzing heat pump systems, we can see that Eq. (2-15) is still

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e) Cheng X T, Liang X G. Discussion on the applicability of entropy generation minimization and entransy theory to the evaluation of thermodynamic performance for heat pump systems. *Energy Convers Manage*, 2014, 80: 238-242.

tenable. However, the value of the output work is not positive, but negative because the work is input the system. So, we can obtain that<sup>26c</sup>

$$S_g = Q_{in} (F_{out} - F_{in}) + W_H F_{out} = W_H F_{in} - Q_{out} (F_{in} - F_{out}), \quad (2-16)$$

where  $W_H$  is the work input the heat pump system, and equals  $-W$ . In heat pump systems, heat is delivered from low temperature heat source to high temperature heat source, so  $F_{in} > F_{out}$ . The physical meaning of  $Q_{in}$  is the heat flow pumped from the low temperature heat source, while that of  $Q_{out}$  is the heat flow into the high temperature heat source. Considering the definition of COP, we have

$$S_g = Q_{out} \left[ \left( \frac{1}{COP} - 1 \right) F_{in} + F_{out} \right], \quad (2-17)$$

$$S_g = W_H \left[ F_{in} - COP (F_{in} - F_{out}) \right]. \quad (2-18)$$

Therefore, when the thermodynamic forces are given, the entropy generation minimization leads to the maximum COP for fixed input work, heat flow pumped from the low temperature heat source or heat flow into the high temperature heat source. When the entropy generation minimization is applied to the analyses and optimizations of heat pump systems, we also have definite optimization direction and application preconditions.

In Eq. (2-15), if it is a heat transfer system, the output work should be zero, and

$$Q = Q_{in} = Q_{out}, \quad (2-19)$$

where  $Q$  is the heat flow transported in the system. So, we have<sup>27</sup>

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<sup>27</sup> a) Cheng X T, Liang X G. Analyses of entropy generation and heat entransy loss in heat transfer and heat-work conversion. *Int J Heat Mass Transfer*, 2013, 64: 903-909.

b) Cheng X T, Liang X G. Analyses of coupled steady heat transfer processes

$$S_g = Q(F_{\text{out}} - F_{\text{in}}) \quad (2-20)$$

It can be seen that the entropy generation minimization leads to the minimum transferred heat flow for fixed equivalent thermodynamic force difference and the minimum equivalent thermodynamic force difference for fixed heat flow, respectively. We also have definite optimization directions and the corresponding application preconditions when the entropy generation minimization is used in heat transfer processes. If the preconditions are not satisfied, the entropy generation minimization may not result in the corresponding objectives, either.

From the analyses above, it can be seen that the application of entropy generation minimization is never unconditional. If we cannot set up the relationships between the entropy generation and the objectives, it is very clear that the entropy generation minimization may not be applicable. If we can set up the relationships, we should further pay attention to the optimization directions and the corresponding application preconditions. If the optimization directions of entropy generation minimization are not consistent with our objectives or the preconditions are not satisfied, the entropy generation minimization may not be applicable, either.

### **Part III An additional discussion on irreversibility and entropy generation**

As is known to all, the concept of entropy generation can measure the irreversibility of physical processes, and the entropy generation minimization can minimize the irreversibility. This is the physical basis for the application of entropy generation minimization. As below, an additional discussion on the physical basis is presented.

In nature, irreversibility is an important characteristic of physical