Mathematical Modeling of the Blast Furnace Process
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By
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Dedicated to my teachers Professor Boris I. Kitaev
and Professor Sergey V. Shavrin
On the basis of modern concepts, the results of many years of research on the mathematical modeling of the blast furnace process are presented. A complex of mathematical models is described, including two-dimensional models of gas dynamics, heat transfer, reduction, cohesion zone, and balance equilibrium. On the basis of these models, a method for the analytical study of the blast furnace process, which essentially complements the experimental methods used in practice, has been developed. The calculation results are two-dimensional temperature fields of charge and gas, as well as the main parameters of the blast melting (coke consumption, productivity, composition of top gas), and two-dimensional fields of gas velocities and degrees of iron reduction.

Algorithms and software that implement the proposed mathematical models of the blast furnace process on personal computers have been developed. Algorithmic languages and corresponding compilers of the code in the object codes of the computer programs Microsoft Visual Basic (Microsoft Visual Studio) and Compaq Visual Fortran are used. Examples of the solution to practical problems of blast furnace smelting are given: improvement of the iron ore quality by improving the recoverability of the agglomerate; use of pulverised coal; and combined blast with high costs of natural gas and oxygen, high-potential heat. The features of temperature fields for various types of blast furnace melting are revealed. The mathematical models can be used (1) at research/design institutes and metallurgical enterprises to analyse the operation of blast furnaces and to develop recommendations for improving their performance, developing melting modes for various iron ore raw materials and coke substitutes, and developing blast furnace smelting control systems, and (2) at higher education institutions in the training of students in metallurgical specialties.

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INTRODUCTION

The blast furnace process is characterised by the variety and complexity of the phenomena of gas dynamics, charge transfer, heat transfer, recovery, softening, and other phenomena. Investigating these phenomena in a blast furnace in order to identify reserves of its efficiency—reducing coke consumption and increasing productivity—entails great financial, technological, and technical difficulties. Therefore, the use of mathematical models in the study of blast furnace smelting, when developing systems to control and manage the blast furnace process, is of great importance. The role of mathematical models increases with a lack of information about phenomena occurring in the furnace, such as the temperature of the charge and gas, the pressure and composition of the gas, and the degree of iron reduction in the furnace volume. This work considers the solution to mathematically describing the interrelated heat-exchange, gas-dynamic, and physical-chemical phenomena occurring during blast furnace melting, and its application to the study of processes determining the recovery of metals from multicomponent ores. The analytical method to investigate the blast furnace process is developed and applied practically. This method provides for two-dimensional control and forecast of the gas-dynamic and temperature fields in any vertical section of the furnace when changes occur in the blast parameters and loading systems, charge composition and quality of iron ore raw materials, and structural elements.

The method is based on a complex of two-dimensional mathematical models of the blast furnace process—gas dynamics, heat exchange, recovery, and balance that allows for determining the flow rate of coke and blast, the gas outlet, and the temperature of the fuming chamber are used as input parameters of the two-dimensional models. Work on the mathematical models of gas dynamics and heat transfer was started in 1977, a year after the author entered the full-time postgraduate course at the Department of Metallurgical Furnaces in the Urals Polytechnic Institute, S. M. Kirov (now the Ural Federal University named after the first President of Russia, B. N. Yeltsin, the department of “Thermophysics and Informatics in Metallurgy”). The scientific advisers are (1) professor and doctor of technical sciences Kitaev Boris Ivanovich, a scientist-heat engineer and the founder of the theory of heat exchange in a blast furnace,
and (2) associate professor, candidate of technical sciences (now professor and doctor of technical sciences) Sukhanov Evgeniy Leonidovich, a specialist in the field of the automation of the blast furnace process. Considerable assistance at this stage was also provided by associate professor, candidate of technical sciences Victor Borisovich Scherbatsky, who works in the field of mathematical modeling.

After finishing graduate school and defending the PhD thesis in 1979 on the topic “Investigation of the Thermal State of the Blast Furnace Shaft with Uneven Distribution of Charge and Gas Flows,” the author continued his work on the mathematical models at the Institute of Metallurgy of the Urals Branch of the Russian Academy of Sciences. Together with well-known a scientist and specialist in the field of physical chemistry and blast furnace process, Doctor of Technical Sciences Zakharov Ivan Nikitich, in the laboratory of “Complex Processing of Mineral Raw Materials”, the author developed a balance (equilibrium) mathematical model. In 1985, the work was carried out in the laboratory for the “Pyrometallurgy of Reduction Processes”, led by a well-known scientist and blast furnace smuggler, Professor Sergey Viktorinovich Shavrin. He provided broad support in the formulation and solution of practical problems of blast furnace smelting. In 1997, the author completed work on his Doctor of Technical Sciences thesis “Development and Implementation of the Analytical Method Research of a Blast Furnace Process on the Basis of a Complex of Two-Dimensional Mathematical Models”.

Subsequently, work was continued to improve the complex two-dimensional mathematical models and expand the range of tasks to be solved, including by incorporating a balanced logic-statistical model of the blast furnace process into the complex models.
PART I

GENERAL CHARACTERISTICS OF
MATHEMATICAL MODELS OF
THE BLAST FURNACE PROCESS
CHAPTER ONE

THE ROLE OF MATHEMATICAL MODELS
IN THE ANALYSIS, CONTROL, AND
MANAGEMENT OF BLAST FURNACE SMELTING

1.1. The use of mathematical modeling in the
development and improvement of pyrometallurgical
processes of processing complex raw materials

The mathematical description (computer simulation) of processes plays
the most important role in the study and optimization of the phenomena of
gas dynamics, motion of charge, heat exchange, mass transfer, reduction,
softening, filtration, and other phenomena occurring in existing
metallurgical aggregates, including the development of new schemes for
processing complex iron ore. This is because the experimental study of
operating units and the development of new ones are associated with great
financial, technological, and technical difficulties. At present, ferrous
metallurgy, especially the Urals, is experiencing an acute shortage of iron
ore raw materials for the production of pigmentary titanium dioxide,
titanium sponge, metal scrap, and so on.

At the same time, in the bowels of the Urals, there are huge reserves of
titan magnetite, iron-chromium-nickel, iron-alumina, siderite ores, and
leucoxene rutile-quartz sands. In these ores, besides iron, there are
vanadium, titanium, chromium, nickel, cobalt, copper, rare earth elements,
and alumina. Therefore, creating environmentally safe and resource-saving
schemes for processing the ores of these deposits is a pressing problem of
the Ural region. The economics of these schemes will be determined by
the depth and breadth of the extraction of iron and related elements, as
well as the use of waste enrichment and production. At the Institute of
Metallurgy of the Ural Branch of the Russian Academy of Sciences
(IMET UB RAS), since the 1950s, research has been carried out on the
metallurgical evaluation and development of technologies for processing
complex Ural ores, such as brown-chromium and oxidised nickel ore of
the Serovsky deposit; carbonate ores of Bakalsky deposits; titanomagnetite ores of the Kachkanarsky, Kopansky, and Medvedevsky fields; and leucoxene ores of the Komi Republic.

Much attention has been paid and continues to be given to the development and improvement of methods for studying metallurgical processes, which are integral parts of complex ore processing technologies. The research and the foundations of the theory of the stressed state of oxide systems applied to iron ore raw materials are formulated, the main regularities of high-temperature filtration slag melts through coke nozzles, a mathematical description of the recovery of ores by gases has been developed. On this basis, mathematical models of layered metallurgical processes have been created (shaft furnaces, conveyor machines, electric furnaces, rotary kilns). The use of computer methods and the availability of mathematical models for various stages of processing raw materials (redistribution) can optimise the developed schemes for energy and economic criteria. Mathematical models make it possible to apply new tools for the study of processes and phenomena—analytical research methods that allow for determining both one- and two-dimensional distributions in the volume of aggregates of velocities, temperatures, degrees of recovery, and other parameters.

In the proposed mathematical models of pyrometallurgical processes, along with conventional data, important indicators of the quality of iron ore raw materials and fuel—such as strength, reducibility, granulometric composition, softening and melting points, reactivity, and other indicators—are used as the initial information, determined experimentally under laboratory conditions. The latter attaches significant importance to not only mathematical but also experimental modeling. The most significant achievement is a complex of mathematical models of the blast furnace process, which includes two-dimensional models of gas dynamics, heat exchange, and recovery, and a balance model that allows one to determine coke and blast consumption, gas yield, and firing chamber temperature as input parameters of two-dimensional models.

1.2. Equilibrium states and the minimum coke consumption in blast furnace smelting

To determine all the output parameters of the blast furnace process through calculation, it is necessary to at least know the consumption quantities of coke, blast, and carbon for direct reduction (or gas composition). In order to find these three quantities, it is necessary to compile three equations—(1) the heat balance, including heat loss, combined with the
material balance (as calculations are conducted per unit of weight of cast iron); (2) the material balance of carbon; and (3) the degree of gas use (or the degree of direct recovery). The difficult point here, along with the definition of heat loss, is the choice between the degree of direct recovery and the degree of gas use. It should also be taken into account that the balance model should be able to both predict the actual smelting performance and determine the limit to the improvement of blast furnace technology.

Notably, between the degree of direct recovery and the output parameters of the blast furnace process, there are no logical dependencies, and even if one wants to determine the minimum coke consumption, it is not known what to set since the minimum value of the direct reduction (zero) is not the minimum coke consumption but, for example, low productivity with large heat losses. Regarding the degree of gas use, the problem is greatly simplified, at least when determining the minimum coke consumption. The limit of perfection of the blast furnace process is determined by the thermodynamic equilibrium in the individual zones of the furnace. The choice of these zones has long been of interest to scientists and blast furnace users [1]. In Table 1.1 and Figure 1.1, the degree of CO use proposed for calculating the minimum coke consumption is given. It can be seen that there are great differences in both the degree of carbon monoxide use and the form of an oxide-determining iron oxide. Of the authors listed in Table 1.1, only A. N. Ramm and subsequent researchers have addressed the matter with practical calculations.

**Table 1.1:** Equilibrium degrees of use of CO and basic types of iron oxides in determining the minimum coke consumption

<table>
<thead>
<tr>
<th>Authors</th>
<th>Literary source</th>
<th>$\eta_{CO}$</th>
<th>The determining oxide</th>
<th>The point number in Figure 1.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leibovich, M. M.</td>
<td>[1]</td>
<td>50</td>
<td>Fe$_3$O$_4$</td>
<td>1</td>
</tr>
<tr>
<td>Ramm, A. N.</td>
<td>[2]</td>
<td>50</td>
<td>Fe$_3$O$_4$</td>
<td>1</td>
</tr>
<tr>
<td>Austin, J.</td>
<td>[3]</td>
<td>32</td>
<td>FeO</td>
<td>2</td>
</tr>
<tr>
<td>Martin, P.</td>
<td>[4]</td>
<td>49</td>
<td>Fe$_2$O$_3$</td>
<td>3</td>
</tr>
<tr>
<td>Luban, A. P.</td>
<td>[5]</td>
<td>43</td>
<td>FeO</td>
<td>4</td>
</tr>
<tr>
<td>Kostylev, N. A.</td>
<td>[6]</td>
<td>63</td>
<td>Fe$_2$O$_3$</td>
<td>5</td>
</tr>
<tr>
<td>Ramm, A. N.</td>
<td>[7, 8]</td>
<td>55-63</td>
<td>Fe$_3$O$_4$</td>
<td>5–6</td>
</tr>
<tr>
<td>Rist, A. et al.</td>
<td>[9]</td>
<td>30</td>
<td>FeO</td>
<td>7</td>
</tr>
<tr>
<td>Rist, A. et al.</td>
<td>[10]</td>
<td>42</td>
<td>FeO</td>
<td>8</td>
</tr>
</tbody>
</table>
To the information presented in Table 1.1, we add the following. The assumption of equilibrium in work [2] A. N. Ramm developed in [7–8], where the coke consumption is determined from the condition of stepwise (sequential) reduction of Fe$_3$O$_4$ and FeO upon reaching the equilibrium gas composition at each stage, the degrees of use of gas at these stages, and the degree of direct reduction.

A. Rist and his co-workers developed a mathematical model of the blast furnace process, based on the thermal and material balance and the condition of thermodynamic equilibrium [9]. Initially, this model was not widely used. Subsequently, it was widely developed by J. Pisi and V. Davenport in Canada [14] and I. F. Kurunov in Russia [11]. In [9], the authors assumed equilibrium in the reduction stage of FeO ($t = 950^\circ$C, $\eta_{CO} = 30\%$). Subsequently, they came to the conclusion that, with a high reducibility of the iron ore charge, equilibrium is reached at a temperature of 670°C [10]. The additional savings of coke compared to a temperature of 950°C is 56 kg.

Figure 1.1: Equilibrium degrees of gas use, taken in calculating the minimum coke consumption (for the number of points, see Table 1.1)
The least logically contradictory proposal is to use the assumption of equilibrium at the point C in calculating the minimum coke consumption and taking Fe₃O₄ [12, 13] as the determining oxide.

Thus, the following facts are correlated: in the blast furnaces of Russia, operating at low coke consumption, the composition of the gas for the reduction of Fe₃O₄ is close to the equilibrium composition of the FeO reduction reaction; in connection with the insufficient level of preparation of charge (specifically, reducibility), there is a zonal regime of reduction, in which Fe₃O₄ is present in the main mass of the iron ore material, and FeO is on the surface. Further, the composition of the gas corresponds to equilibrium in this reaction; there is a reaction of CO₂ regeneration. In works [12, 13], similarly with equilibrium, when reducing CO, equilibrium is taken when hydrogen is reduced.

### 1.3. A brief history of the development of computational methods

The specific consumption of coke is of particular importance among the parameters of blast furnace smelting determined by calculation. The necessary equipment for its implementation is the thermal balance of the process [8].

The first attempt to compose the heat balance of blast furnace smelting was made in 1839 by R. Bunsen [18, 19]. It took into account the heating of the charge to 2000 °C; however, the heat introduced by the heated blast and the loss of heat to the external space were not taken into account, and the heat consumption greatly reduced recovery. The heat consumption for the melting of cast iron and slag was determined by the difference. This heat balance can be considered the first mathematical model of the blast furnace process.

After a quarter of a century, the calculation of the heat balance was carried out by De Water [20]. In spite of the errors contained in the calculation, as well as the lack and unreliability of the thermo chemical data, De Water’s calculation influenced further work in this field since his work first used the principle of the “initial and final state”, according to which the thermal effect of transformation depends only on the initial and final states of the system, and not on the path by which it is accomplished. Bell [21] replenished the heat balance with the articles of heat consumption that were absent from De Water’s work (the restoration of other elements except iron, the decomposition of carbonate salts, the loss of heat with cooling water, and the environment) and applied more reliable thermo chemical data. In his works, Bell widely used thermal balances to
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compare the performance of blast furnaces in different conditions and to analyse how numerous factors influence the consumption of coke and the possibilities of its saving.

The question of the role of direct restoration, a clear idea of which was absent in L. Bell’s work, was brought to the forefront by L. Grüner [22]. He developed a method for determining the carbon consumption for direct reduction in the composition of the top gas and a new method for calculating the amount of blast from the balance of carbon and oxygen (instead of calculating the carbon and nitrogen balance used previously). The method to reduce the heat balance proposed by L. Grüner was widespread and did not undergo significant changes until now. Almost simultaneously with L. Grüner, N. Ockerman [23] proposed a scheme for reducing the thermal balance, which is characterised by the fact that not all the heat developed by the oxidation of carbon in the furnace in CO₂ and CO is taken into account, but only the amount that it turns out to be the combustion of carbon from the tuyeres (minus the heat of decomposition of the moisture of the blast). In the same expenditure, it is not the heat of dissociation of the oxides, but the resultant thermal effect of the reactions of the reduction of iron and other elements by carbon and carbon monoxide. Since the thermal effect reduction of iron oxides with carbon oxide in Okerman’s work are equal to zero (which is close to the modern thermochemical data), then the heat consumption for the restoration of iron is taken into account only for the part reduced by carbon (i.e., direct way).

The scheme of L. Grüner is easily translated into N. Ockerman’s scheme by subtracting the amount of heat released in the oxidation of carbon through the direct reduction in CO, and the CO and H₂ through the indirect reduction in CO₂ and H₂O, both from the total heat input and from the heat of oxides dissociation in the consumption of heat.

M. A. Pavlov [24] then clarified the methods for calculating the thermal balance of blast furnace melting, particularly in his published summary of data on the heats of formation of chemical compounds and heat capacities, compiled on the basis of a critical analysis of a huge number of original physical and chemical studies.

In the literature of the late nineteenth and early twentieth centuries, notably, the works of O. Rokur [25] first compiled the equation of the thermal balance of the blast furnace process and used this equation to analyse the dependence between the coke consumption and the amount of carbon burned in tuyeres, in various melting conditions, and C. Brisker [8, p. 13] completed the first formula based on the heat balance for determining the carbon consumption for cast iron smelting. These works
are the first attempts to use the heat balance not only for analyzing the work of the operating blast furnaces but also for calculating the projected melting.

In 1936, the preliminary calculation method of coke consumption using the general thermal balance of the blast furnace process was proposed by A. N. Ramm [2] and subsequently improved by him. This method can be considered the first modern mathematical model of the blast furnace process and remains one of the best balance models.

To date, there is a significant number of both balance and kinetic-mathematical models of the blast furnace process. These are briefly considered below.

The balance logic-statistical model [26]—including the balance model, significant from the point of view of the final results of the regularity of heat and mass transfer, and statistical data—was developed at the Institute of Metallurgy of the Ural Branch of the Russian Academy of Sciences. As the raw data, only independent variables are used, including material characteristics such as cold and hot strength (according to GOST 15137-77, 19575-74, 5953-81), reducibility (according to GOST 17212-84), particle size distribution, and others; internal parameters, particularly the degree to which the reducing potential of the mining gas was utilised, and the temperature of the top gas are the results of the calculation. It provides the possibility of solving problems to optimise the properties of raw materials and smelting parameters, including the costs of oxygen and a hydrocarbon additive, taking into account their resources and mutual influence. The output parameter, along with others, is the productivity of the furnace (obtained by introducing a logic-statistical unit).

The main results of calculations using balance mathematical models are coke consumption; flow of blast; and yield, composition, and temperature of the top gas. In addition to balance sheets, there are mathematical models describing the individual phenomena of blast furnace melting: gas dynamics, charge flow, heat and mass transfer, and more.

Mathematical models of gas dynamics. One of the main problems in calculating the gas velocity field in a blast furnace is the task, or calculation, of the nature of the change in the height of the layer of lump materials of resistance to gas flow (pressure drop) [27]. The first systematic work on the resistance of a layer of lump material to a gas stream was carried out by S. Furnas [28]. However, the mathematical dependence between resistance to the gas flow and the properties of the gas and the charge layer was first established by P. Karman [29]. At present, the Ergan equation [30] is widely used to determine the loss of gas pressure in the moving bed of the charge:
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\[ \frac{dP_e}{dh} = 150 \frac{(1-\varepsilon)}{\varepsilon^3} \frac{\mu_G \cdot u_G}{(\Phi \cdot d_B)} + 1.75 \frac{(1-\varepsilon)}{\varepsilon^3} \frac{G_G \cdot u_G}{(\Phi \cdot d_B)} \]  

(1.1)

where \( d_B \) is the diameter of the burden piece; \( \varepsilon \) is the porosity (relative volume of voids in the layer); \( \mu_G \) is the dynamic viscosity of the gas; \( u_G \) is the conditional average gas velocity (in an empty mine); \( G_G \) is the specific gas consumption; and \( \Phi \) is the factor (factor) of the form (equal to 1.0 for spheres and to 0.6–1.0 for bodies of other forms).

It has been established theoretically and experimentally that this formula sufficiently and reliably reflects the nature of the change in resistance to the gas flow along the height of the burden layer.

One of the mathematical models of gas dynamics was proposed in UGTU–UPI. The model is based on the problem of filtering to a single draene (this concept exists in the theory of irrigation and drainage). In the model, the following assumptions are made: the motion of the gas corresponds to the laws of the potential isothermal flow; the porosity of the charge is constant throughout the volume; the distribution of the head losses linearly; and so on. This model allows for determining the local gas velocities in the layer of charge materials [24]. Unlike balance models, mathematical models of gas dynamics, in principle, make it possible to calculate the productivity of a blast furnace.

**Mathematical models of charge motion.** The main problems arising in the development of mathematical models of charge movement in a blast furnace are related to the discreteness of the structure of the materials composing the layer. On the one hand, the particle size of the charge is too small for the layer to be considered as an ensemble of point masses. On the other, it is large enough to inspire fears in the use of the theory of continuum mechanics. Most of the known studies related to the development of mathematical models of the charge movement, and they are rather small, use the so-called “bunker approach”, i.e., the assumption of the analogy of the movement of the charge, particularly of coke, to the furnaces of the blast furnace, and the flow of loose materials through the hopper openings. Here, the works of G. M. Malakhov, B. S. Fialkov, and E. V. Maksimov are notable [25–27].

V. I. Loginov [35] published a number of papers on the effect of the dynamic structure of the charge column, determined by the loading system and the blowing regime, on the course of blast furnace smelting. Existing mathematical models of charge flow do not allow for describing the movement of the material and the change of porosity in the furnace volume to a sufficient degree.
Mathematical models using the laws of heat and mass transfer. The most famous and widely used mathematical model of heat transfer in blast furnaces is the model of prof. B. I. Kitaev [36].

B. I. Kitaev also draws an analogy between the processes of heat transfer and reduction from the point of view of extinguishing the potential for furnace height: for heat exchange, the temperature difference between the flow of charge and gas; and, for reduction, the difference in the operating and equilibrium partial pressure of the reducing agent for isothermal conditions or its concentrations for constant pressure. This approach was used in joint development at the USTU–UPI and VNIIMT [37].

The Institute of Metallurgy of the Ural Branch of the Russian Academy of Sciences has developed a kinetic-mathematical model of the blast furnace process [38], which allows for analyzing the effect of the kinetic characteristics of blast furnace raw materials (iron ore and coke) on the smelting parameters. The calculation is carried out by the method of successive approximations. Preliminary calculations are carried out using the Ramm balance method with arbitrary values for the degree of direct reduction and temperature of the top gas given as initial data. Then the system of equations is used to calculate the heat mass exchange along the layers of ore and coke, starting from the top. The adequacy of the model was checked, and a number of practical problems were solved, such as the effect of the reactivity of coke on the parameters of blast furnace smelting and so on.

VNIIMT developed a kinetic model of the blast furnace process [39]. It includes the equations for the material and heat balance, heat exchange between gas flows and charge materials, and the kinetics of basic physics-chemical processes, boundary conditions, reflecting technological features of the blast furnace process. Subsequently, this model was supplemented by equations of dynamics and described as kinetic-dynamic. The adequacy of the model has been verified on the solution of a number of problems—the use of a combined blast of high parameters, metallised pellets, and hot reducing gases. A number of dynamic characteristics of blast furnaces have been determined through various control channels. Subsequently, the model was used to study the influence of the physicochemical properties of the charge on the performance of the blast furnace.

Mathematical models, including a number of submodels. Many known mathematical models consist of a number of mathematical models (or submodels). Some of them are considered here.

The Moscow Institute of Steel and Alloys (MISiS) proposed a set of mathematical models of the blast furnace process and a set of computer
programs for analyzing the operation of blast furnaces and developing new melting regimes [40]. The complex of models consists of models of recovery, gas dynamics, and heat transfer.

The reduction of iron oxides is described by a system of differential equations. The initial conditions were found by analyzing the composition of the top gas over the radius and the composition of the charge components, and by using the mathematical model of Rist, used to determine the ratio of direct and indirect recovery. The temperature of the charge was taken from the heat transfer equation. The output parameters of the recovery model are the lower boundary of the melting zone, the distribution fields of the degrees of reduction of the charge materials, and oxidation of the reducing gas. The basis of the mathematical model of gas dynamics is the boundary value problem with the use of partial differential equations. The pressure drop is described by the Ergan equation. The output parameters of the gas-dynamics model are the gas pressure distribution fields and their mass velocities. It is possible to determine a more frequent grid in the melting zone. The output parameters in the heat exchange model were the temperature fields of the material and the gas in the space of the blast furnace. The tasks of gas dynamics and heat exchange were solved as axisymmetric by the finite element method. With the help of a set of models, the operating mode was investigated using fine-coke coke substitutes (coke, coke briquettes, coal) in an amount of 10% in a mixture with an agglomerate.

In Australia, the central research laboratory of the firm “Broken Hill Proprietary” (BHP) developed a mathematical model [41, 42] that has the following features. The flow of solid material is calculated using the theory of potential flow; coke and ore flows are solved separately. The gas flow is compressed; the density varies depending on the temperature, pressure, and composition of the gas. The pressure loss and the corresponding gas velocity vectors are determined using the well-known Ergan equation. Heat transfer is calculated using the Rantz equation with empirical modifications. The gas velocities are calculated according to local rates of chemical reactions taken from the literature. It is accepted that the iron ore charge is softened at 1200 °C and melts at 1400 °C. The data on the distribution of materials was obtained by applying the RABIT model, modernised by BHP, and data on thermal and material balances of the BHP model. Using this model, a long period of operation of blast furnace No. 5 in Port Kembla was investigated. In particular, the causes of the formation of crusts have been studied, and the temperature distributions of materials for ten distribution options have been calculated.
One of the most complete mathematical models was developed at the USTU–UPI [43]. It is assumed that the shaft furnace of an arbitrary profile is filled with lumpy material with porosity and clearness, which are functions of the shape of the loaded pieces and coordinates. From below, a gas is injected through the tuyeres, which gives the heat to the charge and enters into a chemical interaction with it. As a result of solving the system of equations of continuity, the following is determined: movement of gas; energy; state; heating pieces of material; the law of change in viscosity and thermal conductivity of gas, mass transfer, and recovery; the distribution of velocities, pressures, and temperatures; and gas concentrations along the height and cross-sections of the furnace.

The most complete model that has been used practically is the two-dimensional mathematical model of the firm “NipponSteel”, named BRIGHT [44]. It consists of six submodels: distribution of charge, gas flow, material flow, chemical reactions, melt flow, and heat exchange.

In the gas flow model, the Ergan equation, with respect to the pressure drop in the dense layer, is extended to a two-dimensional shape as equation motion and is combined with the equation of continuity, which satisfies the material balance of gas. The flow of gas in the blast furnace is determined by the solution of this combination of equations; the distribution of pressure in the furnace is determined by the propagation of the two-dimensional Ergan equation to the Laplace equation.

Notably, the motion of the charge in the blast furnace has not yet been theoretically clarified. Based on the analysis of the results of disassembly of cooled blast furnaces and experiments on physical models, it is assumed that the behavior of descending particles is similar to a potential flow. The equation of continuity introduces the value, which takes into account the decrease in the volume of the charge due to burning and gasification of coke, shrinkage and melting of the ore.

Calculations for these submodels are carried out in the order listed above in the cycle. Information from sensors as input is not used, but the model has access to them for adapting the model or analyzing the phenomena taking place in the blast furnace. The further the submodel is in the indicated list, the more information it uses from the previous submodels. The cohesion zone is judged by the shrinkage of layers of ore. Shrinkage is determined by the temperature of the charge, which is calculated according to the degree of recovery and the data from the heat exchange sub-model. The volume of the furnace is considered a zone of cohesion, if the shrinkage of the charge layers is in the range of 0.05–0.75. The settlement cycle is repeated until the discrepancy between the coordinates of the boundaries of the zone of cohesion in two cycles does
not exceed 0.5 m. As a result of calculations using the BRIGHT model, we can determine (in a graphic form) the following: charge distribution, current flow lines and charge, temperature distribution of charge and gas. The calculation time on the FACOM M340 computer is 65 minutes. In addition to determining the zone of cohesion, the BRIGHT model was used to analyse the effect of furnace capacity and the reducibility of the sinter on blast furnace smelting processes. In addition to developing the blast furnace operating modes, the model can be used to analyse new processes and the influence of changes in parameters that cannot be studied on an operating blast furnace.

At present, a mathematical model of the blast furnace process is being developed in Japan [45], based on the finite element method (FEM), and combined with the continuous medium model (MCC). A hybrid model where gas particles, liquids, and fines are analysed by MCC, and the flow of solid particles and melt by the MDE method, provides analysis of many phenomena in a blast furnace. Here, the directions for further developing this model are formulated.

In addition to the classical methods of mathematical modeling, methods based on the use of artificial intelligence systems are used to analyse and manage the blast furnace process [46–48]. It is of interest to consider the possibility of using neural networks to calculate the effect of the parameters of combined blast (flow natural gas and oxygen content in the blast) on the smelting parameters—furnace capacity and coke consumption.

The main drawback of most mathematical models, with the exception of the USTU–UPI model, is the lack of consideration for the features of the tuyere blower. For more detailed descriptions of mathematical models, see the literature. In 1977, A. B. Shur and Yu. A. Shur believed that kinetic-mathematical models could only be used by their developers. These words could also apply to the beginning of the twenty-first century.

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2.1. Statement of the problem

The purpose of the development of a complex of mathematical models is the creation of a method for the analytical study of a blast furnace process that makes it possible to carry out a computer experiment to study the phenomena of blast furnace melting, including for new technical solutions and for deviations from the normal operating conditions. This allows for analysing the influence of various parameters, including the metallurgical properties of the complex iron ore raw materials, and the melting parameters and temperature fields of the charge and gas, the location and shape of the cohesion zone while accounting for the uneven flow of charge and gas flows. In accordance with this goal, the physical formulation of the problem is as follows (Figure 2.1).

In a shaft furnace with a radius $R_0$ and a height $H_F$, continuously along the lines, the gas and charge with moving temperatures $t_G'$ and $t_M'$ move towards each other with initial temperatures, respectively. It is assumed that the tuyere focus, located at distance $L_F$ from the furnace wall, serves as a point source of gas and a waste of material. The heat capacity of the gas flow $W_G$ and the total heat transfer coefficient $\alpha \Sigma$ are functions of the length of the streamlines $h$, i.e., the velocity of the gas at a given point; the temperatures of the onset of softening and melting, apart from the chemical and mineralogical composition, are a function of the degree of reduction. Such a formulation of the problem allows for staying confined to the following mathematical models: the balance (equilibrium) and two-dimensional models, consisting of models of gas dynamics, heat exchange, reduction, and cohesion zone.
Figure 2.1: To the formulation of the problem of constructing a mathematical model of the blast furnace process: \(H_f\) and \(R_0\) are the height of the charge layer and the radius of the furnace, respectively; \(L_T\) is the distance from the furnace wall to the center of the tuyere chamber; \(\mathbf{v}_M\) and \(\mathbf{v}_G\) are the velocity vectors of material (burden) and gas, respectively; \(t'_M\) and \(t'_G\) are the temperatures of charge and gas, respectively at the entrance to the bed; and \(h_i\) is the distance from the level of the charging to the calculated point along the line of current \(i\).
As a result of solving the equations and systems of heat balance equations, gas flow, heat, and mass transfer, the main parameters of blast furnace smelting (coke consumption, direct reduction, blast flow, quantity, composition, temperature of top gas, zonal thermal balances) should be determined: two-dimensional distributions of gas velocity, charge and gas temperatures, the degree of iron reduction, and the position and shape of the cohesion zone.

### 2.2. Balance (equilibrium) mathematical model

The following prerequisites are the basis of the mathematical model. The heat transfer in the blast furnace is complete, i.e., at a certain part of the altitude; there is a small temperature difference between the gas and the charge (Figure 2.2).

![Diagram](image)

**Figure 2.2:** Scheme of division of a blast furnace into zones (a) and temperature distribution of charge and gas in height (b).

In a certain zone of the furnace during the reduction of magnetite, the recovery reaction of wustite tends to reach the thermodynamic equilibrium (points C and C′) (Figure 2.3). The equilibrium state is characterised by practically constant equilibrium parameters: temperature $t_E = 700 \, ^\circ\text{C}$, and the degree of use of carbon monoxide and hydrogen $\eta_{\text{CO}} = 0.405$ and $\eta_{\text{H}_2} = 0.305$, respectively.
Figure 2.3: The equilibrium diagram of Fe-O-C and Fe-O-H.

Small changes are possible due to the weak dependence of the gasification reaction constant of carbon on pressure:

\[ T_E = 680 + 2.5 \cdot P_H + 9 \cdot P_T \text{°C}; \]  
\[ \eta_{H_2} = 0.256 + 0.000392 \cdot (t_E - 575); \text{ and} \]  
\[ \eta_{CO} = 0.981 - 0.002408 \cdot (t_E - 575). \]

This assumption was made on the basis of an analysis of the operation of a large number of blast furnaces for a long time [1], according to which the gas composition at the magnetite reduction stage is close to, or in some cases equal to, equilibrium in the wustite reduction reaction. The actual values of the gas composition at this stage are determined by the composition of the top gas by excluding oxygen, which is released during the transition of hematite to magnetite.

The solution of the system of equations for the lower zone (\( t > 700 \text{°C} \))—consisting of the condition of thermodynamic equilibrium, carbon material balance and heat balance supplemented by the thermal balance of