The Fluid Catastrophe

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By

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Cambridge
Scholars
Publishing

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This book first published 2019

Cambridge Scholars Publishing

Lady Stephenson Library, Newcastle upon Tyne, NE6 2PA, UK

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

ISBN (10): 1-5275-3206-2
ISBN (13): 978-1-5275-3206-9

Contents

Foreword

We arrived at Buckles Bay, Macquarie Island on MV Nella Dan in December 1963. We clambered down the ship's side on a rope ladder into an amphibious vehicle and were driven ashore. As the vehicle ground its way up the steep pebble beach, I recall being not overly impressed by my new surroundings. Four ton elephant seals clashed their chests in mock battle and needed to be persuaded to get out of the way of our vehicle. Despite the brisk, cold wind whipping through the tussock grass, the place stank to high heaven. The smell came from the piles of rotting bull kelp strewn along the beaches. This was going to be my home for the next 12 months. It neither looked nor smelled hospitable.

The southern aurora (Aurora Australis) is most active at the latitude of Macquarie Island, which lies 1200 km SE of Tasmania, Australia. There is a band of activity known as the Auroral Oval which encircles the South Magnetic Pole located at the edge of Antarctica south of Tasmania. I had taken a job with the Australian Antarctic Division as Auroral Physicist. This entailed operating various semi-automatic instruments which measured various aspects of the aurora. Long nights of visual observation were no longer part of the job description. There was an "all-sky-camera" which took a ten second exposure of the whole sky once per minute during the hours of darkness, a spectrophotometer which measured auroral colours and an instrument called a "riometer" which measured radio wave absorption, which occurred when the aurora was active.

All of this high-tech equipment was new. I was one of the first of a new breed of auroral physicists being sent south. First we had to build the auroral lab, then install the new gear, and then keep it operational. The prefabricated lab was erected behind a small hill to shield the instruments from stray light from the base.

I spent the whole year in a frantic attempt to keep it all working. The station experienced frequent power failures due to contaminated diesel fuel, and these led, in turn, to failures in the lab DC power supply which supplied all the instruments. Solid-state electronics was very new back then. I had barely even seen a transistor before I arrived. Eventually I redesigned the lab power supply to make it sufficiently robust to handle the power failures and brown outs. I also developed about 5 km of all-sky-camera film. It was a hectic year but I coped and in the end stayed on an extra three months to show the new guy the ropes. I had no time to do any research. I was a maintenance technician.

But this wild and windy place had got into my blood. In some way, I grew up there.

It teemed with wildlife. Several species of penguin, fur seals and elephant seals, skuas, giant petrels and other sea birds were all to be found near the main base at Buckles Bay at the northern end of the island. The wandering albatross nested on the coastal plain and could be approached without their attempting flight. They are the world's largest flying bird – so large that they only become airborne with difficulty. The beautiful, light-mantled sooty albatross nested among the giant grass tussocks which covered the steep slopes. Occasionally, on the plateau among cushion plants and rocks, a feral cat would be glimpsed from afar. They seemed larger than domestic cats and were generally ginger in colour.

On the coastal plains of grass and feather-bed bog were rabbits and wekas, introduced as a food source by sealers in the 19th century. The Stewart Island weka is a flightless rail similar to the kiwi. They are amusing little creatures, like fluffy, chestnut bantams. They occupied small territories with fiercely contested, but invisible, boundary lines. On the beaches and around the base, numerous fat elephant seals were moulting, mating or fighting, depending on the time of the year. It was like living in a big zoo.

The riometer is really a radio telescope with a very wide beam antenna pointing upwards and operating at the lowest frequency at which the ionosphere is nearly transparent. It receives radio noise from the Milky Way as it passes overhead, giving a smooth curve on the paper charts. During times of auroral activity, radio waves are absorbed and big dips appear in the daily curve. It was a reliable and sensitive instrument and, unlike optical methods, operated well in daylight and on cloudy nights. I became interested in the fine structure in the riometer absorption records. It was very tantalizing; certain rough or "noisy" patches seemed to correspond to the X-ray "microbursts" which had recently been observed by means of high altitude balloons launched from the island. The trouble was, the riometer did not have enough resolution to isolate the individual pulses; what was needed was a faster riometer.

In late 1966 I returned to Macquarie Island with the world's first operational "fast-response riometer" designed using a state-of-the-art, low-noise radio receiver developed at the University of Tasmania's Physics Department. It had 50 times the resolution of a conventional riometer. This was my PhD project. Over the next three months I recorded the first observations of cosmic noise absorption pulsations. They had an unexpected sawtooth shape. Their slower decay was a measure of electron-ion recombination in the D-region. I returned home in March 1967 and published my discovery in a Letter to Nature. Later I was able to show that other auroral-zone phenomena, Pi1 micropulsations, were also asymmetrical and caused by the same mechanism.

This was all, of course, entirely useless; the purest of pure research. There were no practical reasons for studying the ionosphere following the advent of communication satellites. No-one today has the slightest interest in cosmic noise absorption pulsations nor in Pi1 micropulsations apart from a few researchers in closely related fields. This may change if there is a nuclear war and the issue of EMP (electromagnetic pulse) becomes important.

I learned a lot. I learned how important it is to have instruments that work properly and are properly calibrated. I learned that auroral physics is not about plugging a computer into the sky and pressing the ON button. Science is about interacting with the real world; you have to manipulate things in order to understand them, and you have to understand them in order to manipulate them. That is how understanding, the end-product of science, is built up. The idea that you can sit indoors at a computer terminal and determine how the real world *must* work is nonsense.

I remember what my supervisor, Prof G.R.A. (Bill) Ellis, told me before I went south with my gear. He had said:

When you first use an instrument with much higher resolution than before, not only do you see the same things bigger, you see new things.

This is what happened to Galileo. When he first looked at the planet Jupiter in 1610, he not only saw a sphere on a larger scale, he saw four of Jupiter's moons as well. This was the first observation of the satellites of another planet. It strongly supported the new heliocentric theory and ultimately led to the development of modern astronomy and physics.

When I operated my fast riometer for the first time, I too saw new things; I saw sawtooth pulsations with a decay time related to the electron density 80 km above the earth.

That is what scientific research is about; it is about exploring the Universe, about broadening understanding, about seeing new things. It formalizes the natural human tendency to explore. It involves a sense of wonder. But we also have another, conflicting tendency which inhibits our understanding; we have a tendency to kowtow to authority and to suppress ideas which threaten that authority. We live in social hierarchies based on shared dogma. Science can be subverted by such dogma.

The famous story of King Canute commanding the waves is told by chronicler Henry of Huntingdon, who lived within 60 years of the death of Canute (1035 AD). When at the summit of his power, Canute ordered a seat to be placed for him on the sea-shore when the tide was coming in. Then, before a large group of his flattering courtiers, he spoke to the rising sea, saying,"Thou, too, art subject to my command, for the land on which I am seated is mine, and no one has ever resisted my commands with impunity. I command you, then, o waters, not to flow over my land, nor presume to wet the feet and the robe of your lord." The tide, however, continued to rise as usual, dashing over his feet and legs without respect to his royal person. Then the King leaped backwards, saying: "Let all men know how empty and worthless is the power of kings, for there is none worthy of the name, but He whom heaven, earth, and sea obey by eternal laws."

Canute was the most powerful monarch ever to rule England. He was, at once, King of England, Denmark, Norway and Scotland. He was a Viking, a Christian and an educated man. He performed an experiment by which he demonstrated to his superstitious courtiers the objective reality of natural laws and the limitations of human agency. To him the idea that a human being could override God's "eternal laws" was blasphemy. The belief in the existence of laws of nature is a consequence of monotheism. It led ultimately to the scientific revolution which accompanied the Renaissance.

For the first time in a millennium, scholars now question the existence of natural laws. Evidently Canute was wrong and science is merely a social construct. From now on, global temperature will be controlled politically, by decree. The Scientific Method is to be abandoned, it seems.

This book is intended as a robust refutation of this fashionable tendency. It is written in praise of the Scientific Method as a means of comprehending and manipulating the natural world.

In Part I, Popper's description of the Scientific Method is used to address and rectify a persistent myth: the myth of the continuum, the rationalist belief in the universal applicability of differential calculus to fluid processes.

In so doing, emphasis is placed on time series rather than continuous functions, and a methodology for dealing with time series is developed, *Discrete Time Spectral Analysis*; this is discussed in Part II. It allows rigorous methods of statistical inference to be applied to time series and to the time series of global average temperature in particular. Most of this mathematical development could be heavy going for the non-specialist and can be skipped; it is the implications which are important (page 85).

In Part III, we abandon mathematical rationalism and adopt an empirical approach, in order to speculate about a variety of natural processes such as the recurrence of Ice Age Terminations, the growth of wind-seas, the effect of subaqueous volcanism on ocean circulation and the dynamics of the Earth's interior.

If a movie were to be made of the workings of the Universe and then shown backwards, the only Law of Physics that would not be obeyed is the Second Law of Thermodynamics. It is the only Law in which the direction of time is important. In the reversed movie, the motions of the planets in their orbits would look much the same. Planetary motions are determined solely by Newton's differential equations and do not involve the Second Law. On the other hand, a breaking wave would look very wrong indeed when viewed backwards in time. Waves never "unbreak". Breaking waves involve the Second Law and cannot be adequately described by differential equations. That is what this book is about.

Part I

The Myth of the Continuum

Chapter 1

The Scientific Method

The Scientific Method

The fundamental things we know about the physical world are either hard-wired into our brains or we found them out by experience. Watch toddlers playing with kitchen utensils. They manipulate objects, they experiment. They find out by experience that big things don't fit inside small things. As we grow up, the results of those early experiments are automatically understood but the actual experiments have been forgotten. The same applies to all learned behaviour about cause and effect, about the nature of flowing water and the danger of fire. As we go through life, we continue to find out about the world around us by experiencing it, by living in it, by manipulating it, by carrying out millions of informal "experiments", few of which we remember except for one or two epiphanies.

However, as humans we also have the gift of language so that our knowledge of the world is far broader than our personal experience. The vast majority of things people know about the world are learned from other people. Humans have a strong desire to learn through communication: firstly from our parents, then from our teachers, then from our playground friends and peers. In addition, we learn from various media: books, television, social media, and so on. Nevertheless, "facts" are often communicated and remembered as summary principles, tendencies, or trends rather than as observed details, which would be far too numerous to remember.

But there is a problem. It is a very big problem.

Beliefs about the world, the most general and important ones, are also political banners which both unite and divide people. It is human nature to bond with those who share a belief and to reject those who do not as alien, as foreign, as other. At best we regard people whose beliefs conflict with our own as unsound, eccentric or downright crazy. If we are told, by those in authority, that such-and-such is the truth, then to publicly air doubts can be seen as a mad or traitorous act. No doubt this socially unifying tendency of belief once served an evolutionary purpose in uniting believers against a common enemy – "survival of the loyal". At this level, a belief becomes a religion or an ideology.

Such unifying beliefs are usually concerned with politics and value judgements: "Henry Tudor is the rightful King of England", "Communism is evil", "There should be an equal number of women and men on company boards" and so on. However that is not always the case. For example, the belief that the Earth is the centre of the Universe is value free, but in 1633 Galileo was forced to recant under threat of torture when he proposed otherwise. At that time the Christian Church in Europe was the absolute authority on the legitimacy of a belief; validity or otherwise was determined solely by the authority of the Church which, in turn, regarded the word of the Bible as the ultimate arbiter, subject to its own interpretation of course.

Francis Bacon, a contemporary of Shakespeare, was the foremost exponent of the Scientific Method of the early modern era. A great legacy of Bacon was the description, in his *Novum Organum* (1620), of "Idols of the Mind": beliefs which commonly obstruct the path to correct scientific reasoning. These are:

1. Idols of the Tribe: *The human understanding is of its own nature prone to suppose the existence of more order and regularity in the world than it finds ... The human understanding when it has once adopted an opinion draws all things else to support and agree with it.*

2. Idols of the Cave: *The Idols of the Cave are the idols of the individual man. ... men look for sciences in their own lesser worlds, and not in the greater or common world.*

3. Idols of the Marketplace: *names of things which do not exist and names of things which exist, but yet confused and ill-defined, and hastily and irregularly derived from realities.*

4. Idols of the Theatre: *… in the plays of this philosophical theatre you may observe the same thing which is found in the theatre of the poets, that stories invented for the stage are more compact and elegant, and more as one would wish them to be, than true stories out of history. Idols which have immigrated into men's minds from the various dogmas of philosophies.*

In modern language these are equivalent to:

1. *To a man with a hammer, every problem is a nail.*
2. *I've made up my mind. Don't confuse me with facts.*
3. *How many devils can sit on the head of a pin?*
4. *Never spoil a good story for the sake of the truth.*

In the ensuing centuries, natural philosophers – scientists – have striven to purge science of these conceptual errors. Some are still there, embedded in the fabric of sciences such as Fluid Dynamics and the environmental sciences.

In fluid dynamics, an Idol of the Tribe is the widespread belief in the universal applicability of differential calculus in describing the behaviour of fluids. According to fluid dynamicists, every fluid is a *continuum* and so is continuous and differentiable almost everywhere. This despite the overwhelming evidence in support of the atomic theory from nuclear physics, and in support of the granularity of action space from Quantum Mechanics.

The mystique of Chaos Theory now dominates some fields despite this granularity. Mathematical Chaos is a an Idol of the Marketplace whereby a thing may exist (chaos) but is "yet confused and ill-defined and hastily and irregularly derived from realities". Mathematical Chaos bears no resemblance to reality nor to the $\chi\alpha o\zeta$ of the Greeks. It is a fashionable buzzword intended to mislead us into believing that the pathological behaviour of differential equations somehow provides a profound insight into the nature of the physical world. By its very existence, Chaos Theory misdirects attention from stochastic methods which provide a more fruitful approach.

By the 16th century, the invention of the printing press meant that ideas could be disseminated widely and rapidly. This was the Church's problem with Martin Luther. A century later, other inventions such as the telescope enabled people to discover the nature of reality for themselves, and what they saw often contradicted Church authority. This was the Church's problem with Galileo. Furthermore, Europe was in a state of religious ferment as Christianity fragmented into various splinter groups.

A handful of thinkers in England found a way out of this mess. It was to draw a strong distinction between science and religion and to separate religion from the secular aspects of human inquiry. In 1663, the "The Royal Society of London for Improving Natural Knowledge" was founded. Its motto, "Nullius in verba", broadly translates as "take nobody's word for it". Thinkers like Newton and Priestley were often intensely religious people, but they were careful to keep their religion separate from their scientific work.[1]

This separation of modes of thought is an aspect of civilization. The great sagas of the past, such as the Iliad, pre-dated writing and were part of an oral tradition. They were written as poetry to be more easily remembered. As such they were a mixture of poetry, religion, history and technology. Parts of the Finnish saga, the Kalevala, appear to have been a handbook for finding and refining bog iron. Another confluence of myth, poetry and technology is perhaps the legend of King Arthur and the sword Excalibur. It may well have been an Iron Age myth about making weapons from iron meteorites. "He drew the sword out of the stone"; red hot iron is "drawn" by beating it with a hammer because the melting point of iron is too high for primitive furnaces.

How can we know about the world?

We can continue to experiment and to see for ourselves as we did as children, but it is a very impractical solution and beyond the reach of even the best-resourced individual. The recent observation of gravitational waves by LIGO cost one third of a billion dollars and involved hundreds of scientists and technicians.

We are inevitably forced to accept the beliefs of other individuals or groups of people who have done the experiments themselves. We have to take their word for it. But how are we to know that these beliefs are based, ultimately, on experiment and observation and are not the expression of some tribally unifying ideology to which humans are so prone?

[1] Priestley was first to isolate an interesting gas which he called "dephlogisticated air". It was later renamed "Oxygen" by Lavoisier. Priestley emigrated to America, was a personal friend of Jefferson and, like him, passionately believed in the separation of church and state.

We do not know. We have to trust them. There is no other way.

The people we must trust are the scientists. It is their task to draw conclusions from observations uninfluenced by their personal ideologies in the tradition of Newton and Priestley and the 17th century Royal Society. The stock-in-trade of scientists is *understanding*. Understanding is their product. As with music, once a new idea has been seen or heard or understood, it is impossible to go back and un-hear it or un-understand it. This makes it difficult to adequately reward creative scientists and for them to protect their work. Musicians have copyright; scientists have peer-reviewed papers. As with music, much that is published may turn out to be dross, but that which is not can last indefinitely. As a civilized society we pride ourselves on our symphony orchestras. Our scientific institutions should have a similar role.

Our trust need not be absolute. We can, to some extent, assess for ourselves whether a particular piece of research is cutting-edge or fairly ordinary or even suspect, but in order to do so, we too must put ideology aside. We too must be conscientious and do our homework. We can judge to some extent by the way the expert talks about what they have done. It is similar to talking to one's doctor or motor mechanic: we may not understand the detail, but we can judge the validity of their insights by their general demeanour, by whether the things they say measure up to what we do already know, and, ultimately, by whether they actually work. If we find out that our G.P. is a closet naturopath, we may start to have doubts about what he/she is telling us. Likewise, if we discover that a scientist is a passionate Green, we may have doubts about his/her prognoses on species numbers or climate change unless he/she has made some obvious effort to guard against his/her ideological predilections.

There are two major streams in European thought: Rationalism and Empiricism. In the case of the former, the power of human reason is assumed to be so great that, starting with a few general principles that are obvious to everyone, it is possible to sit at a desk in a closed room and by reason alone deduce the nature of the Universe and all its workings. The Ancient Greeks were Rationalists who believed that their geometry was an exploration of the properties of space. The great modern Rationalist, Immanuel Kant, talked about the *synthetic a priori* postulate, a statement about the real world that we somehow just *know* to be true. Examples are cause and effect (i.e. *every effect must have a cause which precedes it*) and the axioms of geometry.

Empiricists on the other hand believe that all knowledge is based on experience derived from the senses. In order to understand the world, we must observe it, and observation is paramount. It is a fundamental part of the scientific method that all hypotheses and theories must, ultimately, be tested against observation.

Once again, the big breakthrough came in the 17th and 18th centuries with the Empiricist philosophers, Locke, Berkeley and Hume. With it came the Enlightenment and the rise of the Scientific Method put into practice by early scientists such as Newton, Halley, Galileo, Hooke and Boyle. These people did not completely abandon Rationalism but tempered it with empirical observation. Many people think that physicists such as Newton and Einstein produced their great unifying theories out of thin air without recourse to observation, but that is not the case. Newton experimented extensively with pendulums and Einstein recognized the need for observation in verifying his theories of relativity. For Einstein, time *is* that thing which is measured with clocks, and space *is* that thing which is measured with measuring rods.

When I was a student, a post-apocalyptic science fiction novel was very popular. It was *Earth Abides* by George R. Stewart. As I recall, the protagonist helps his people as they descend into savagery by teaching them how to make and use the bow and arrow, so saving them thousands of years of technical evolution. The idea has stayed with me. In a similar situation I would attempt to teach them the Scientific Method for similar reasons.

This interaction between ideas and observations is complex. Young research scientists spend many years learning the skills of their trade. There is no simple formula, no button to press: you have to learn how to do it under the tutelage of skilled practitioners, much like a musician.

The Scientific Method set out by Bacon in the early 17th century was further refined by Newton and others and set out by Popper (1962) in the form of his Seven Principles:

1. *It is easy to obtain confirmations, or verifications, for nearly every theory if we look for confirmations.*

2. *Confirmations should count only if they are the result of risky predictions; that is to say, if, unenlightened by the theory in question, we should have expected an event which was incom-*

patible with the theory, an event which would have refuted the theory.

3. *Every "good" scientific theory is a prohibition: it forbids certain things to happen. The more a theory forbids, the better it is.*

4. *A theory which is not refutable by any conceivable event is non-scientific. Irrefutability is not a virtue of a theory (as people often think) but a vice.*

5. *Every genuine test of a theory is an attempt to falsify it, or to refute it. Testability is falsifiability; but there are degrees of testability: some theories are more testable, more exposed to refutation, than others; they take, as it were, greater risks.*

6. *Confirming evidence should not count except when it is the result of a genuine test of the theory; and this means that it can be presented as a serious but unsuccessful attempt to falsify the theory.*

7. *Some genuinely testable theories, when found to be false, are still upheld by their admirers for example by introducing ad hoc some auxiliary assumption, or by reinterpreting the theory ad hoc in such a way that it escapes refutation. Such a procedure is always possible, but it rescues the theory from refutation only at the price of destroying, or at least lowering, its scientific status.*

These principles are descriptive not prescriptive and pre-date Popper by three centuries. Popper was not stating *how* Science *ought* to be done but how it actually *is* done. Most working scientists would recognize these ideas and would support their application in their own research field. Popper describes how Science frees itself from Bacon's Idols.

The remarkable advances in science and technology witnessed in the modern era are largely the result of the meticulous application of the Scientific Method. When a theory is tested against observation and fails the test, new insights into the underlying reality are gained, whereas clinging tenaciously to a "correct" theory can only lead to a sterile absolutism. This is the fundamental difference between Physics and Applied Mathematics. It is also the fundamental difference between science and superstition.

In recent times some aspects of Physics itself appear to have abandoned the Scientific Method as described by Popper. An example is superstring theory of Theoretical Physics:

> The possible existence of, say, 10^{500} consistent different vacuum states for superstring theory probably destroys the hope of using the theory to predict anything. If one picks among this large set just those states whose properties agree with present experimental observations, it is likely there still will be such a large number of these that one can get just about whatever value one wants for the results of any new observation.[2]

According to Popper then, superstring theory, the most advanced (and glamorous) form of contemporary theoretical physics, is not even science. It is incapable of making risky predictions about the real world and so is evidently a form of pure mathematics, and not science at all.

There is another so-called “science” which has abandoned the empirical and lost touch with reality. That is Fluid Dynamics or at least those aspects of Fluid Dynamics which have ignored observation in favour of a mathematical ideal.

[2] Woit (2006) p122.

Chapter 2

Numbers and Entropy

Numbers

> God made the natural numbers. Everything else is Man's handiwork.[1]

Leopold Kronecker

> All the mathematical sciences are founded on relations between physical laws and laws of numbers, so that the aim of exact science is to reduce the problems of nature to the determination of quantities by operations with numbers.

James C. Maxwell

> I believe that mathematical reality lies outside us, that our function is to discover and observe it, and that theorems which we prove, and which we describe grandiloquently as our "creations" are simply the notes of our observations.

G.H.Hardy

There are different sorts of numbers. Sometimes numbers are just codes, like telephone numbers, but mostly numbers represent quantities, i.e. things in the real world such as the number of potatoes in a shopping bag or the radius of the earth. Some numbers, like π and e,

[1] Die ganzen Zahlen hat der liebe Gott gemacht; alles andere ist Menschenwerk.

come out of mathematics itself as if the mathematical world has its own objective reality as Hardy's quote above suggests.

In fact numbers have evolved. It started with the natural numbers. The natural number "5" is the property of any set of objects that can be put into one to one correspondence with the fingers of my right hand. Add a new member to this set and it then has the number property "6" which is called the "successor" of "5". Every natural number has a successor, but there is a natural number that is not a successor to anything. It is called "1".

Later on the number zero was invented. Zero started out as a place-holder when the Arabic system replaced the Roman numerals. Arithmetic operations such as multiplication became a lot easier. Then came negative numbers, which were a boon to accountants. Having negative money meant that you owed money. The negative numbers, zero and the natural numbers are called "integers".

Then came fractions. You have 2 acres of land to split equally between 3 sons so they each get 2/3 of an acre. These are called rational numbers. A rational number is an "ordered pair" of integers, (p, q), which is usually written as p/q or $p \div q$ or $N\frac{p}{q}$ where N is also an integer. There are rules for manipulating rational numbers which we called "doing fractions" in primary school.

Then around 300 BCE came the first scientific catastrophe which we will call "The Rational Number Catastrophe", commonly known as "Euclid's proof that the square root of two is not a rational number".

The proof is easy to follow, even for a non-mathematician. It works by showing that the assumption that there is a rational number (i.e. a fraction) whose square is 2 leads to a contradiction. It goes as follows: Suppose

$$\frac{p}{q} = \sqrt{2} \tag{2.1}$$

where p and q are integers with no common factor, i.e. p/q is in its simplest form. Then

$$\frac{p^2}{q^2} = 2 \tag{2.2}$$

Therefore

$$p^2 = 2q^2 \tag{2.3}$$

so that p^2 is an even number. If p^2 is an even number, then p must also be an even number, i.e.

$$p = 2a \tag{2.4}$$

where a is an integer. Then

$$4a^2 = p^2 = 2q^2 \tag{2.5}$$

and so

$$2a^2 = q^2 \tag{2.6}$$

Therefore q^2 is even which implies that q itself must also be even. Therefore both p and q are even numbers which contradicts our original assumption.

More than two millennia later, the square-root-of-two-problem is still with us. $\sqrt{2}$ was termed "irrational". Other numbers such as π and e were even worse; they are not only irrational, they are "transcendental".

From the point of view of mathematics, the problem of irrational numbers was solved in the late 19th century by Dedekind, Wieirstrasse and others who devised the "real numbers". This was done using the concept of limits. Think of the set of all rational numbers which are less than $\sqrt{2}$. The upper limit of this set is called the "supremum". Now think of the set of all rational numbers which are greater than $\sqrt{2}$. The lower limit of this set is called the "infimum". It can be shown that the supremum and infimum are the same number. That number is the definition of the real number, $\sqrt{2}$. Real numbers make mathematics self-consistent.

From a physics point of view, however, this is fairyland. The mathematicians have defined a set of numbers which scientists are unable to use to perform calculations. Real numbers are not computable. Real numbers are useless for dealing with the real world. All the calculations done by scientists involve rational numbers not real numbers. Computers use rational numbers like p/q, where $q = 2^{64}$.

Real numbers are not real. What happened was that mathematics became more and more refined and elegant until it ceased to be useful. As Einstein once said, "Elegance is for tailors". Belief in the relevance of *real* numbers to the natural world is an Idol of the Tribe.

Entropy

Contrary to popular belief, James Watt did not invent the steam engine. The Newcommen steam engine had been around for seventy years. It was used for pumping water out of mines to prevent them flooding. The Newcommen engine used steam to force a piston along a cylinder. The hard part was getting the piston back to its original position. This had involved cooling the piston itself with water before reheating it again for the "power stroke". As a result more heat was used to reheat the cylinder than was converted into mechanical energy. Watt's invention allowed the steam to be cooled outside the cylinder in a separate apparatus called a *condenser.* The condenser sucked the steam out of the cylinder leaving the cylinder hot and ready for the next stroke. There was a huge increase in efficiency of steam engines; much less fuel was required to pump the same amount of water. Imagine a Cornish tin miner having to import from Wales, all the coal to drive his pumps. To him, a more efficient engine meant lower costs.

Following this breakthrough there were numerous incremental improvements, such as converting reciprocal motion to rotary motion to drive lathes, presses and the like. Steam engines enabled mass-production and so revolutionized manufacturing. Stevenson's locomotive in 1829 revolutionized transport as well.

Fuel costs money, and enormous effort went into attempts to improve efficiency. Clearly heat was a form of energy (the First Law of Thermodynamics). People asked why couldn't *all* the available heat be converted to mechanical energy?

In 1850, the German physicist Rudolf Clausius published a paper in which he proposed the Second Law of Thermodynamics:

It is impossible to construct a device which operates in a cycle and produces no other effect than the transfer of heat from a cooler body to a hotter body.

which is equivalent to:

It is impossible to construct a device which operates in a cycle and produces no other effect than the transfer of heat from a single body in order to produce work.

At this time an important thought experiment was developed to help gain an understanding of the implications of the Second Law. It is called the *Carnot Cycle* and comprises a piston in a cylinder operating between two heat reservoirs, rather like a steam engine, except that the cylinder contains an ideal gas rather than steam. The heat reservoirs have absolute temperatures T_H and T_C and heat is passed between the cylinder and the reservoirs in a four phase cycle. It is done very slowly or "reversibly" so that the gas is always in equilibrium and friction can be ignored. It turns out that the efficiency, η, is given by

$$\eta = 1 - \frac{T_C}{T_H} \tag{2.7}$$

i.e. efficiency depends solely on the ratio of the absolute temperatures of the reservoirs and it is always less than one. Furthermore it can be shown that this is the best case. There is no other heat engine that can convert heat to work more efficiently than the Carnot Cycle. It is the perfect heat engine.

The work, W, done by the Carnot Cycle is succinctly described by the equation:

$$W = (T_H - T_C)(S_B - S_A) \tag{2.8}$$

where S_B and S_A are the initial and final *entropy* of the two reservoirs. The change in entropy of a reservoir, ΔS, is defined by

$$\Delta S = \int_A^B \frac{dQ}{T} \tag{2.9}$$

where a quantity of heat, dQ, is transferred to the reservoir at absolute temperature, T.

This equation (2.9) is the thermodynamic definition of entropy or, at least, of entropy change. It is a macroscopic, observable quantity which is measurable using thermometers, calorimeters and the like.

But what does it mean at a molecular level? Heat is the total kinetic energy of molecules in a gas, temperature is related to the average kinetic energy of the gas molecules while pressure is the sum of forces per unit area when molecules collide with the boundaries of the container.

But what is entropy?

There have been a number of definitions, starting with Boltzmann's famous:

$$S_B = k_B \ln(W) \tag{2.10}$$

where S_B is the entropy and k_B is Boltzmann's constant and ln() is the natural logarithm.

Boltzmann's paradigm was an ideal gas of N identical particles, of which N_1, N_2, etc. are the numbers of particles in various microscopic conditions of position and momentum. Using the formula for permutations

$$W = \frac{N!}{N_1! \times N_2! \times \cdots} \tag{2.11}$$

where ! denotes factorial,[2] W is the number of microstates associated with a particular observed macrostate. Although more difficult to understand than Einstein's famous $E = mc^2$, Boltzmann's entropy formula is equally profound. It is engraved on his tombstone in Vienna. It is the first formula in physics to relate a measurable, supposedly continuous, physical quantity to probability.

An easy way to understand macrostates and microstates is to consider the "entropy" of a deck of cards.

State 1

Suppose the top 26 cards are all red and the bottom 26 cards are all black. These represent two microstates, M1 and M2, say. The number of possible permutations of both M1 and M2 are $26 \times 25 \times ... \times 2 \times 1$, i.e. factorial 26 (written 26!). Likewise the number of permutations of the whole deck is 52! so that (2.11) becomes

$$W = \frac{52!}{26! \times 26!} \tag{2.12}$$

for this particular case. Substituting in (2.10) and assuming $k_B = 1$ for cards gives $S_B = 0.042$.

State 2

Now suppose the top 13 cards are hearts and the bottom 39 are the other three suits so that W becomes

$$W = \frac{52!}{13! \times 39!} \tag{2.13}$$

Now $S_B = 0.065$. The entropy is greater because this is a less orderly arrangement of the cards.

A transition from State 1 to State 2 results in an entropy increase of $\Delta S_B = .065 - .042 = .023$.

Suppose we use two decks of cards so that W for State 1 is given by

$$W = \frac{104!}{52! \times 52!} \tag{2.14}$$

[2]e.g. 6!=6 X 5 X 4 X 3 X 2 X 1 = 720

and for State 2 by

$$W = \frac{104!}{26! \times 78!} \tag{2.15}$$

This time the entropy change is $\Delta S_B = .024 - .016 = .008$. i.e. less than the value of .023 in the single deck case.

This example indicates that entropy change depends on the number of entities being shuffled – it depends on the *granularity* of the system under investigation. It depends on the total number of grains in the system. It does not work for a continuum.

There are other statistical definitions of entropy, such as those of Gibbs and (in a different context) Shannon:[3]

$$S_B = -k_B \sum_i p_i \ln(p_i) \tag{2.16}$$

If we made a movie of the Universe and showed it backwards, all the laws of physics would still be true except for the Second Law of Thermodynamics. This is the only Law in which time has a direction. It says something about the nature of time itself; time cannot go backwards, entropy can only increase with time.

The three entropy equations, (2.8), (2.9) and (2.10), indicate that thermodynamic systems are coarse-grained. They are coarse grained because of quantum physics. Not only is matter itself coarse-grained according to the atomic theory, but dynamical systems for which a Hamiltonian exists must also be coarse-grained or quantized in "action space"[4].

The Second Law of Thermodynamics is perhaps the most profound of all the Laws of Physics; it arose from a desire to make mining more profitable. The Second Law leads to the concept of entropy. Entropy is a measure of how energy is ordered in a stochastic, granular system. It has no meaning in a deterministic continuous system. The idea that deterministic equations relating continuous physical quantities can provide a comprehensive picture of physical reality is an Idol of the Tribe. It is false because such a model cannot account for the Second Law of Thermodynamics.

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[3]The two definitions, (2.10) and (2.16), are not as different as they may first appear because of an approximation for the logarithm of a factorial known as Stirling's Theorem.

[4]*Action* is a physical quantity which has the units of energy×time or momentum×length.