

Muon Physics

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By

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The author dedicates this book to his late wife, Antonina Popova.

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ABSTRACT

The muon plays an important role in elementary particle and nuclear physics. The muon was discovered in cosmic radiation by Carl D. Anderson and Seth Neddermeyer in 1936. The muon was a puzzle for physicists for a long time as the only difference between a muon and an electron seemed to be the difference in their masses: the mass of a muon is about 207 times greater than that of the electron. “Who ordered that?” asked Isidor Rabi.

At present the muon is very important in the framework of the Standard Model. With the discovery of the charm quantum number muon and the accompanying muon, the neutrino plays an important role in the quark-lepton model of elementary particles. Namely, the muon and the muon neutrino, together with their correspondent quarks, have been combined in the second generation of the Standard Model.

Muonic processes provide important information on the low energy limit of a weak interaction. The equality of the muon decay coupling constant and the vector one of beta-decay proved the Conserved Vector Current (CVC) hypothesis. The most accurate information on induced (via a strong interaction) pseudoscalar coupling (g_ρ) was obtained from the muon capture process. The most important problem is the exchange-current effects in the nuclear muon capture, as well as a possible T-violation beyond the kaon system.

A negative muon entering a gaseous mixture of light elements initiates a complicated chain of atomic and molecular processes, which forms an unavoidable background to the experimental investigation of a weak muon capture by hydrogen and helium nuclei. The experimental investigation of atomic and molecular processes, ending with nuclear fusion of light elements induced by muons (called Muon Catalyzed Fusion: μCF), is an interesting test for the verification of different approaches to theoretical calculations.

The investigation of muonic atoms gives essential information about the structure of light and heavy atomic nuclei, including nuclear formfactors. A very precise value of the root-mean-square charge radius of the proton was obtained from the measurement of the Lamb-shift in a $p\mu$ atom. The investigation of muonic atoms enables quantum electrodynamics to be tested using the high order of perturbation theory.

The behaviour of the muon and muonium in condensed matter is currently very topical, due to the fact that it is important for solid state physics.

Finally, muonic research is widely used in chemistry, biology, and medicine.

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1. INTRODUCTION

The research into muon participation plays an important role in elementary particle and nuclear physics; as well as in applied physics. There are many excellent reviews concerning muon physics (see references, 1–4) that have been published some years ago. I will now provide a contemporary description of this topic.

The muon was discovered in cosmic rays in 1936 [5] and it was first named “muon” in 1947 [6]. However, this particle puzzled physicists for a long time as the only distinction between a muon and an electron appeared to be their mass: a muon is ~ 207 times heavier than an electron. Despite the numerous investigations into the muon, the nature of this particle remained unclear for a long time, leading Isidor Rabi to declare: “Who ordered this?” The numerous attempts to find any additional muon interactions in comparison with electrons, which could prove responsible for its mass, were unsuccessful. However, when the Standard Model was developed it proved that the muon plays an important role in the quark-lepton scheme of the elementary particles. The muon and the muon neutrino were combined with the correspondent quarks to the second generation of the Standard Model [7]; this evolved naturally after the discovery of charm particles.

Muonic processes provided important information on the low energy limits of this weak interaction [8]. The equality of the muon decay

coupling constant to vector one of beta-decay proved the Conserved Vector Current (CVC) hypothesis [9].

The investigation of muon capture by proton provided the best information on the induced pseudoscalar interaction [10]. The kinetics of muonic atoms in the hydrogen isotopes mixture is very important for weak muon capture. The investigation of muon catalyzed fusion (μCF) was predicted ~ 70 years ago [11] and it was effectively investigated over a thirty-year period [3, 12–14]. However, the hope that it would be possible to use μCF as an alternative source of energy has met with difficulties and is yet to be realized. At the same time, the numerous theoretical and experimental results that were obtained within the μCF investigations have made a very important contribution to mesic atomic physics.

In addition, the investigation of muon capture in light nuclei (e.g., by helium) is an important process in the research on weak interaction; this also includes the particle correlations for the determination of weak interaction constants. The possible exchange currents could prove to be very important when explaining the difference between the pseudoscalar formfactor in nuclei and that of the free proton. The information about nuclear formfactors and the verification of quantum electrodynamics in high order perturbation theory was obtained through the investigation of the muonic atoms; this also included mesic hydrogen [15].

Heavy muonic atoms are used to investigate problems in nuclear fission [16]. The prompt (by cascade transitions) and delayed (by weak muon capture) fissions will be discussed by comparing the experimental results with the theoretical explanation.

It is also very important to use the muon as a probe for the verification of the structure of condensed matter (e.g., the behaviour of muonium in the solid-state target) [3, 17]. However, these problems could form the subject of another book, so I have only provided a brief discussion on this topic.

Finally, muon research is used in biology and medicine [18–20]. With regard to medical goals, the characteristic gamma-ray emission of muonic atoms is used for diagnosis; this also included cosmic medicine.

The aim of this monograph is to demonstrate the wide role of muon physics when discussing problems in elementary particles and nuclear physics. This also includes applied research.

2. MUON IN THE VACUUM

2.1 Mass and spin

Different methods to determine particle masses exist [21]. The numerous amounts of data on muon mass have been obtained by measuring muon tracks using photo-emulsion and gamma-spectra from πp reactions. Additionally, X-rays from C , P , and Si muonic atoms were published in 1954. The determination of the muon mass (m_μ/m_e), as well as the muon magnetic moment (μ_μ/μ_p), has been obtained using the precision measurement of the ground state hyperfine structure interval of muonium (μ^+e^-), which is a hydrogen-like atom consisting of a positive muon and an electron [22, 23]. The muonium Zeeman hyperfine transitions, $(1/2, 1/2) \leftrightarrow (1/2, -1/2)$ and $(-1/2, -1/2) \leftrightarrow (-1/2, 1/2)$ under a high magnetic field, have been made at LAMPF using the microwave magnetic resonance spectroscopy method [2]. The Hamiltonian of the interaction between the magnetic moments of two particles is:

$$H = -\frac{8}{3}\pi\vec{\mu}_\mu \cdot \vec{\mu}_e\delta(\vec{r}_\mu - \vec{r}_e) \quad [24], \quad \text{where} \quad \vec{\mu}_i = \frac{e\hbar}{m_i c}\vec{S}_i$$
. This is the magnetic moment operator of corresponding particle ($i = \mu, e$) with the spin operator, \vec{S}_i , and known constants e, \hbar, c ; m_i is the mass of the particle. The magnetic moment depends on the muon mass, which can be obtained from this experiment: $(m_\mu/m_e) = 206.768277(24)$ with $(\mu_\mu/\mu_p) = 3.18334513(39)$. The recent experimental data of the muon mass, $m_\mu =$

105.6583715(35)MeV [4] and $(m_\mu/m_e) = 206.768276(24) \leq 1.2 \times 10^{-7}$ [23], coincided with earlier experimental data [22].

There has been research into muon spin since 1941 [25]. The comparison of these calculations with the experimental data from cosmic rays indicated a muon spin 1/2 [26]. However, the direct proof of the muon spin 1/2 follows on from the gyromagnetic factor, $(g_\mu/2) =$

1.0011659203(15) [3,27], where g_μ is determined from
$$\vec{\mu}_\mu = g_\mu \frac{e\hbar}{2m_\mu} \vec{S}_\mu$$
.

The experimental data for $a_\mu = (g_\mu - 2)/2$ was presented in [23] using

$a_\mu^{\text{exp}} = 11659209.1(5.4)(3.3) \times 10^{-10}$, where the first errors are statistical and the second errors are systematic. The recent data for $g_\mu - 2$ has also been provided [see reference 28].

2.2 The lifetime and decay spectrum

The lifetime of the muon, τ_μ , can be obtained by using μ^+ decay in the reaction, $\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$. The analogous reaction of μ^- decay, $\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$, can compete with the muon capture reaction in a proton or nuclei. Here, ν_e and ν_μ are the electron and the muon neutrinos, respectively, while $\bar{\nu}_e$ and $\bar{\nu}_\mu$ are the antineutrinos.

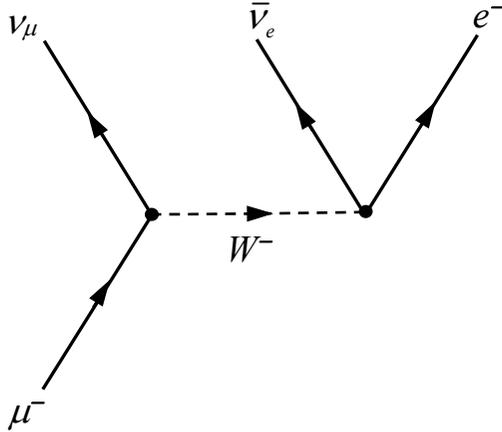


Fig.1. A muon transmutes into a muon neutrino by emitting a W^- boson. The W^- boson subsequently decays into an electron and an electron antineutrino.

The initial investigations of τ_μ began in 1941 using cosmic rays [29, 30].

The data for τ_μ were first obtained in the 1970s and 1980s [27, 31] and later, as evidenced in reference 32.

The recent determination of τ_μ was performed in the Paul Scherrer Institute (PSI) [33]. The experiment was conducted using a time-structured and nearly 100% polarized surface muon beam, along with a segmented, fast-timing plastic scintillator array.

Approximately $1,6 \times 10^{12}$ positrons were accumulated and the following was obtained, $\tau_\mu = 2.1969803(22) \times 10^{-6} s$; this is 30 times more precise than previous generations of lifetime experiments. The weighted average of all results gives a lifetime of $\tau_\mu = 2.1969811(22) \times 10^{-6} s$. The detailed description of this experiment [33] has also been given in the reference 4.

The measurement of the lifetime of μ^- in flight resulted in $2.1948(10) \times 10^{-6} s$ [34, 3]. As indicated above with regard to the stopped negative muon and muon decay, the muon capture process is possible. Therefore, the measurement of the negative muon's lifetime is determined by the total disappearance rate; this includes the decay and capture rates.

The contribution of the capture rate to the disappearance rate varies from $\sim 0.1\%$ in hydrogen to $>90\%$ in heavy elements [4]. Despite the complicated extraction of the lifetime of μ^- , due to the atomic and molecular processes that occur in the hydrogen isotope mixtures, the lifetime of μ^- from the singlet state of μp atom coincides with the lifetime of μ^+ [10]. The muon decay spectrum was considered in, for example, reference 35. All the measurements of the μ^\pm decay have been successfully described by the V-A lepton number conserving four-fermion interaction. The members of each generation's weak isospin doublet are

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix}, \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}, \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}.$$

These assigned lepton numbers are conserved under the Standard Model.

Electrons and electron neutrinos have an electronic number of $L_e = 1$, while the number for muons and muon neutrinos is $L_\mu = 1$. For τ the number is $L_\tau = 1$. Antileptons have a lepton number of -1. The conservation of e^-, μ^- and τ^- - type lepton numbers indicates that $\Delta L_e = \Delta L_\mu = \Delta L_\tau = 0$.

The initial consideration of the electron (positron) spectrum (beyond the Standard Model) was calculated using the Michel parameter ρ ; however, the QED corrections ($\hbar = c = 1$) have been ignored:

$$\Gamma dx = 12\Gamma_0 \left[1 - x - \frac{2}{9}\rho(3-4x) \right] x^2 dx ,$$

where $x = E/E_{\max}$ with $E_{\max} = m_\mu/2$ and $\Gamma_0 = \tau_\mu^{-1} = \frac{G_F^2 m_\mu^5}{192\pi^3}$ is the total decay rate with $G_F = 1.1663787(6) \times 10^{-5} (GeV)^{-2}$ [33] as the weak interaction constant. The spectrum dependence over the parameter, ρ , is given in Fig. 2, which has been taken from reference 1b. However, in the framework of the Standard Model using a lepton number conservation and

a weak V-A interaction of $\rho = \frac{3}{4} = 0.75$, the electron spectrum is given

$$\text{by } \frac{d\Gamma}{dx} = 2\Gamma_0(3-2x)x^2 .$$

The following is the differential decay probability for $\mu^\pm \rightarrow e^\pm$ decay with respect to muon polarization:

$$\frac{d^2\Gamma}{dx d\cos\mathcal{G}} = \Gamma_0 [3 - 2x \pm P_\mu(2x-1)\cos\mathcal{G}] x^2 .$$

Where the upper sign corresponds to μ^+ decay and the lower sign to μ^- decay, P_μ is residual muon polarization and \mathcal{G} is the angle of electron (positron) momentum with respect to the muon polarization vector \vec{P}_μ .

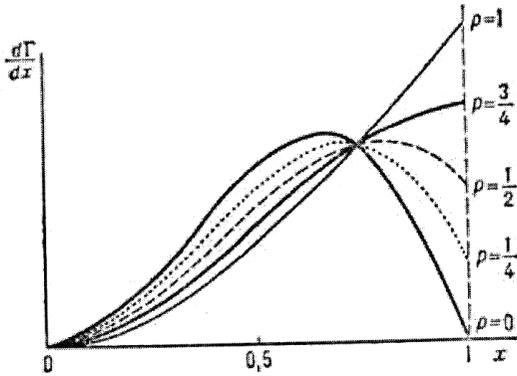


Fig. 2. The energy spectrum of electrons (positrons) from muon decay using the different Michel parameters taken from 1b.

The experiments agree with the standard Michel parameter, $\rho = 3/4 = 0.75$, for the emission of both the neutrino and antineutrino. In the case of $\rho = 0$, the electron (positron) emission could be accompanied by two neutrinos or two antineutrinos, which contradicts the conservation of the lepton numbers. The energy spectrum of positrons is described in Fig. 3.

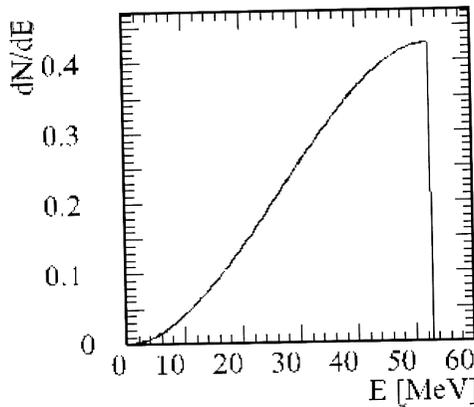


Fig.3. The Michel spectrum of positrons from muon decay at rest ($\rho = 3/4$).

The positron asymmetry, $N(\vartheta, E) \propto 1 + A(E) \cos \vartheta$, where $A(E) = (2x - 1)(3 - 2x)^{-1}$, which for high energy positrons ($x \rightarrow 1$) approaches $A = +1$, so positrons are preferentially emitted in the muon spin direction (i.e. opposite to the muon momentum). For low-energy positrons ($x \rightarrow 0$) the asymmetry approaches $A = -1/3$ and positrons are preferentially emitted opposite the muon spin direction. When they are integrated over the energy distribution of emitted positrons, the asymmetry is $A = +1/3$ (i.e., positrons are preferentially emitted alongside the muon spin). The muon decay is realized with P - and C -violation for parity. The backward asymmetry is for emitted electrons.

2.3 Rare decays and other processes

The fundamental law of flavour conservation has been confirmed through observations that set an upper limit on flavour conservation-violating processes, such as $\mu^\pm \rightarrow e^\pm + \gamma$, $\mu^\pm \rightarrow 3e^\pm$ or $\mu^\pm N \rightarrow e^\pm N$ and so on. The conservation of lepton numbers means that when the particles interact the number of the same type leptons remains the same. The process, $\mu^\pm \rightarrow e^\pm + \gamma$, represents the ultra-rare decay mode. The decay mode, $\mu^+ \rightarrow e^+ + \gamma$, was researched at PSI [36]. The decay mode features the back-to-back emission from both a positron and a gamma ray, each with energies that equal $m_\mu/2 \approx 53 \text{ MeV}$. A tracking drift chamber system and high-resolution liquid xenon scintillation detector were also [4]. Their results to data ratio set the limit, $BR_{er} < 5.7 \times 10^{-13}$ (90% CL.) [36].

Searches for the $\mu \rightarrow 3e$ also resulted in the upper limit of flavour violation [37]. The total energy of three electrons should be close to the muon's mass. Therefore, possible background processes, such as internal-conversion decay, generally yield lower event energy sums and distort the momentum balance; this means they should not be considered. So, the results to data ratio of the $\mu \rightarrow 3e$ set the limit $BR_{3e} < 1.0 \times 10^{-12}$ [37, 4].

The reactions, $\mu^- N \rightarrow e^- N$, were researched using different nuclei, including *Ti* and *Pb* [8]. This recent experiment was performed on *Al* [38, 4]. The signature for the (μ^-, e^-) reaction is particularly simple and unique as the electron will become mono-energetic with an energy of $\sim 106 MeV$, while, in the free muon decay, an electron only reaches $\sim 53 MeV$. This process involves the formation of a muonic atom with a nucleus (i.e. μAl – atom) followed by the coherent conversion of the muon to an electron, which is then ejected with an energy close to the muon rest mass. The coherent conversion measurements will correspond to

$$BR_{\mu e} = \frac{\Gamma[\mu^- + (A, Z) \rightarrow e^- + (A, Z)]}{\Gamma[\mu^- + (A, Z) \rightarrow \nu_\mu + (A, Z - 1)]}, \quad \text{where the denominator}$$

corresponds to the muon capture rate. Searches for the (μ^-, e^-) reaction have set an impressive limit on lepton number conservation: $BR_{\mu e} < 7 \times 10^{-13}$ [38, 4]. Therefore, the searches for all of these reactions have indicated the placement of strong limits on lepton number conservation.

2.4 The muon and the Standard Model

The Standard Model of elementary particles is illustrated in Fig. 4. The second degeneration of the model is represented by muon μ and the muon neutrino ν_μ , as well as the strange s and charm c quarks. At the same time, the first degeneration corresponds to the electron e and its neutrino ν_e ; it also relates to both the down d and up u quarks. The third degeneration is determined by tau meson τ , its neutrino ν_τ , and the bottom b and top t quarks. The interaction between fermions is realized by the photon γ , gluon g , and gauge bosons, W and Z . The Higgs boson (H) with a mass of about 126GeV using the Large Hadron Collider (LHC) at CERN was confirmed to exist on 14th March 2013. The Higgs boson was predicted by Standard Model's explanation of spontaneous breaking of gauge symmetry and appearance of masses of elementary particles.

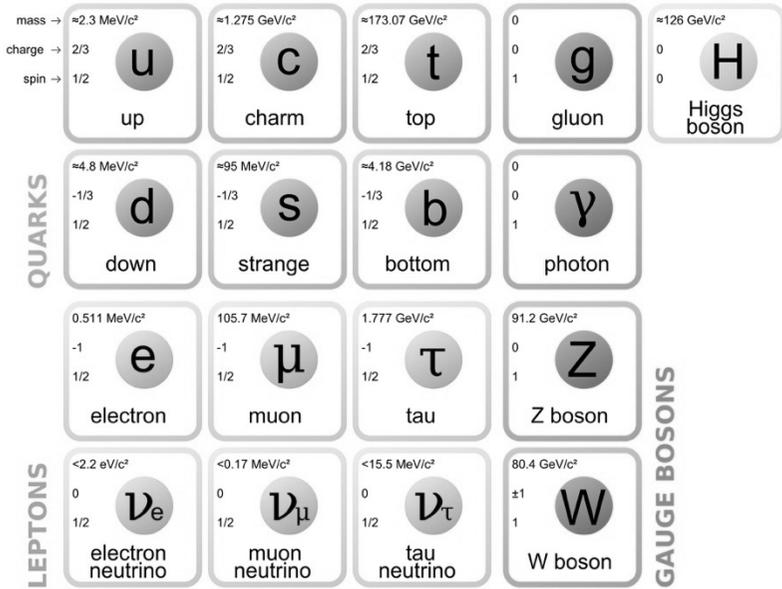


Fig. 4. The Standard Model of elementary particles

Four interactions exist in nature: electromagnetic, weak, strong, and gravitational.

Table 1. Fundamental interactions

| Force | Range (cm) | Particle | Strength | Boson |
|------------------|---------------------|--------------------|-------------------|--------------------|
| Electromagnetism | ∞ | charged particles | 10 ⁻² | photon |
| Weak | ≈ 10 ⁻¹⁶ | leptons and quarks | 10 ⁻⁶ | W [±] , Z |
| Strong | ≈ 10 ⁻¹³ | quarks | 1 | gluons |
| Gravity | ∞ | all particles | 10 ⁻³⁸ | graviton |

The initial unification of these interactions corresponds to the electromagnetic interaction within Maxwell's unification of electrostatic and magnetic interactions. The local gauge invariance corresponds to the electrodynamics used to conserve electric charge. The range of electromagnetic interactions is infinite and, therefore, is in the agreement with the quantum mechanics principle of uncertainty $R \sim \hbar/mc$ we should have $m = 0$ for the photon's mass realized the interaction between fermions in the agreement with the gauge invariance. At the same time the ranges of weak and strong interactions are limited by the distances indicated in Table 1. Therefore, the requirements of the gauge invariance are violated. However, the initial ideas formulated in reference 39–41 allowed us to keep the gauge invariance when formulating the Standard Model of elementary particles. The initial gauge invariance with massless bosons can be breached because of a spontaneous broken symmetry due to the interaction of fermions with the scalar field, as demonstrated by the Higgs boson [40–42]. The result is that the bosons get their masses. The discovery of Higgs boson in 2013 was a very important result that confirmed the legitimacy of the Standard Model. The Nobel Prize in Physics 2013 was awarded to P. Englert and P. W. Higgs for their prediction of the Higgs boson in 1964. The Higgs boson H was obtained by research into the following decay: $H \rightarrow \gamma\gamma$ and $H \rightarrow 4l$, where $l = e^\pm$ or μ^\pm . The experimental lifetime of H is $\tau_H \geq 10^{-24} s$ and this agrees with the theoretical prediction $\tau_H = 1,6 \cdot 10^{-22} s$.

Further investigations into the LHC could be connected to the following areas of research: $H \rightarrow \tau^+\tau^-$ ($\sim 6\%$), $\rightarrow \mu^+\mu^-$ ($\sim 0,02\%$), $\rightarrow Z\gamma$ (\sim

0,15%), and so on. However, the Standard Model does not explain gravity whilst model of super-symmetry is far from complete. It is unable to explain dark matter and dark energy. Cosmological observations tell us that the Standard Model explains only $\sim 5\%$ of the energy present in the Universe. The Standard Model cannot explain matter-antimatter asymmetry in the initial stages of the Universe.

The proton radius puzzle is another problem for the Standard Model. The Standard Model makes precise theoretical predictions regarding the radius of proton of ordinary hydrogen and that of muonic hydrogen. However, the extremely precise extraction of the proton radius from the measured energy difference between the 2P and 2S states of muonic hydrogen (Lamb shift) disagrees significantly with that extracted from electronic hydrogen or elastic electron-proton scattering. The definition of the proton's charge radius from e-p scattering is: $r_p = 0,879(8) \text{ fm}$ [43]. Collectively the charge radius from hydrogen spectroscopy is: $r_p = 0,8758(77) \text{ fm}$ [23]. At the same time, the muonic Lamb shift of μP atom induced by laser frequency, corresponding to ΔE_{2s-2p} results in $r_p = 0,84087(39) \text{ fm}$ [44,45a]. The comparison of different results for the proton charge radii was presented in Fig.1 of reference 45b. The μd value is also smaller than the r_d value from the electronic deuterium spectroscopy. The so-called proton radius puzzle remains unsolved. Finally, according to the Standard Model, neutrinos are massless particles with left-handed helicity for a neutrino and right-handed helicity for an antineutrino. However, neutrino oscillation experiments have shown that neutrinos do have mass.

The mass of the muon neutrino

W. Pauli proposed the third particle (i.e., a neutrino) in the final state of the reaction to explain the solid spectra of beta-electrons [46]. A neutrino is a massless Dirac particle within the framework of the Standard Model. F. Reines and C. Cowan first detected the antineutrinos that are emitted by a nuclear reactor [47]. In 1957, B. Pontecorvo suggested that multiple types, or flavours, of neutrinos exist and that they can change, or oscillate, from one to another [48]. The existence of both ν_e and ν_μ was observed at Brookhaven National Laboratory in 1962 by L. Lederman and M. Schwartz. A third type of neutrino, ν_τ , was predicted in 1975 and discovered in 2000. R. Davis found that the experiment detected only 30% of solar-neutrinos according to ν_e [49] (i.e., the deficit with respect to the prediction of the Standard Solar Model as developed by Bahcall). This discrepancy can only be explained by the oscillation of neutrinos during their travel from the Sun to the Earth [50]. Finally, in 1998, Kajita used the data from the Super-Kamiokande Observatory experiment to show that the ratio of ν_e to ν_μ was different on the opposite sides of the Earth. This means that these neutrinos oscillated as they passed through the Earth. At the same time, McDonald and colleagues in the Sudbury Neutrino Observatory (SNO) experiment reported in 2001–2002 how many of the ν_e that are produced in the Sun change into ν_μ or ν_τ as they travel to the Earth. This was possible due to the fact that the SNO experiment could measure the number of neutrinos of all flavours that arrived from the Sun, as well as the number of ν_e that also travelled to Earth from the Sun. These measurements allowed McDonald and his colleagues to both

confirm theoretical prediction of the solar ν_e flux and also show that about two-thirds of the solar ν_e change flavour by the time they reach the Earth; this result agrees with R. Davis' experiment. This means that the neutrino oscillations were proved and it, therefore, implies that the neutrino has a nonzero mass, due to its state as a Majorana fermion [51] beyond the framework of the Standard Model. This neutrino violates the conservation of its lepton number also being its own antiparticle. There are many very good reviews connected with neutrino oscillations [52]. Information about neutrino mass can be obtained by measuring the β -decay spectrum near the end point of the tritium, as well as from astrophysical data. E. Fermi proposed the first theory of β -decay [53] which was based on the neutrino hypothesis of Pauli [46]. The form of solid β -spectrum is given by

$$\frac{dN}{dE} \propto p\tilde{E}(Q-E)\sqrt{(Q-E)^2 - m_{\nu_e}^2}$$
, where p, \tilde{E}, E is the momentum, energy, and kinetic energy of the electron; Q is the maximum kinetic energy of the electron in the case of zero neutrino mass. We can ignore the $F(\tilde{E})$ and the nucleus recoil, where $F(\tilde{E})$ is the Fermi function. This takes into account the Coulomb interaction between the electron and the daughter nucleus. The Kurie plot is, therefore, a straight line for $m_{\nu_e} = 0$. In the case of $m_{\nu_e} \neq 0$, the Kurie plot is:

$$K(E) = \{(Q-E)\sqrt{(Q-E)^2 - m_{\nu_e}^2}\}^{1/2}$$

The Kurie plot $K(E_e)$ is a convenient linearization of the beta spectrum

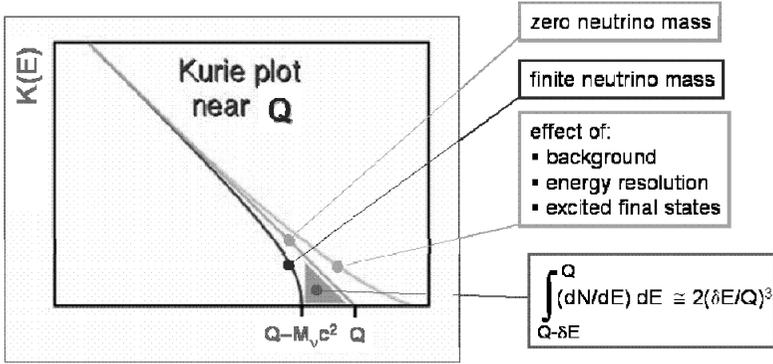
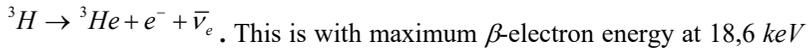


Fig. 5: Kurie plot

Information about the neutrino mass can be obtained by measuring the spectrum of electrons near the end point in the tritium decay:



and a lifetime of 13.3 years. The most stringent upper boundary in the $\bar{\nu}_e$

mass was obtained in reference 54's experiment: $m_{\bar{\nu}_e} < 2.05\text{eV}$ at 90%

CL. A similar result was obtained in the experiment from reference 55:

$m_{\bar{\nu}_e} < 2.3\text{eV}$ at 95% CL. In March 2013 the Planck Collaboration

published their first constraints on $\sum_j m_j$ using cosmological and

astrophysical data. Assuming the existence of three light massive neutrinos, they reported an upper limit for the sum of the neutrino masses:

$\sum_j m_j < 0.57\text{eV}$ at 95% CL [56] and later $\sum_j m_j < 0.23\text{eV}$ at 95% CL. It

follows from this data that neutrino masses are much smaller than the

masses of charged leptons and quarks: $m_j/m_{l,q} \leq 10^{-6}$, $l = e, \mu, \tau$, $q =$