# Metalinguistic Discourses

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Edited by

Viviane Arigne
and Christiane Rocq-Migette

Cambridge
Scholars
Publishing

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This book first published 2015

Cambridge Scholars Publishing

Lady Stephenson Library, Newcastle upon Tyne, NE6 2PA, UK

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library



ISBN (10): 1-4438-7258-X
ISBN (13): 978-1-4438-7258-4

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# ACKNOWLEDGEMENTS

We wish to thank Nicolas Ballier, Pierre Cotte, Geneviève Girard-Gillet, Jacqueline Guéron and Philip Miller, members of the scientific committee for the conference held at Paris 13 University in October 2012. We are also grateful to Eugene Brennan, Isabelle Gaudy-Campbell, Albert Hamm, Paul Larreya, Susan Moore, Eithne O'Neill, Jean Pamiès, Rose-May Pham Dinh, Susan Pickford, Steven Schaefer, Jennifer Vince and Stephan Wilhelm, who all provided kind assistance during the various stages involved in the preparation of this publication. This book could not have come about without the help and support of François-Xavier Mas, Houda Schiratti and Bérénice Waty of the "cellule recherche" of the UFR LSHS, to whom we are also greatly indebted. Last but not least, we thank Claire Parfait, head of the CRIDAF research unit (EA 453), and her successor Françoise Palleau-Papin, currently deputy-head of the Pléiade research unit (EA 7338), for making it possible to hold this conference on *Metalinguistic Discourses* in 2012 and now rendering possible the publication of this book.

# INTRODUCTION

VIVIANE ARIGNE
AND CHRISTIANE ROCQ-MIGETTE

Contemporary linguistic research is diverse and uses a great variety of concepts and terms, which pertain to its fields of study (phonology, morphology, syntax, semantics), to its various theoretical trends (Guillaume's psycho-mechanics, generativist, enunciativist, cognitivist trends, etc.) or sub-trends (trace theory, binding theory, construction grammars, cognitive grammars, etc.). Other labels are used to refer to methods or working practices, such as field linguistics, corpus linguistics, automatic treatment of natural languages. All this leads to a proliferation of theoretical terms as well as theoretical discourses and to the consequent fragmentation of knowledge. Starting from this observation, we aimed at conducting an inquiry into the heterogeneity and diversity of linguistic theories, in order to assess their descriptive and explanatory power. A conference on the subject was held at Paris 13 University in October 2012, for which we suggested a number of directions and angles from which to analyse and possibly confront metalinguistic theoretical discourses. The eight contributions presented here constitute the result of this attempt.

The first part is dedicated to formal and theoretical issues. The first and third chapters both deal with the notion of theoretical representations, assessing their validity and the role they may play at the different levels of their hierarchy, while the other two sections analyse the historical contexts of the emergence of the linguistic trends of cognitive linguistics and systemic functional grammar, giving an account of how their respective theoretical stands originated in former linguistic positions or traditions. In "Metalinguistic Discourse and Formal Representation," Jean Pamiès questions the very notion of metalinguistic discourse and observes that a formalized theory can only be confronted to empirical data through some naive theory interpretation, a naive theory providing a conceptual grid indispensable for turning raw observation into data, while a formalized theory captures in artificial language the structure of naive theory. He questions what is understood by formal representation and notational variant and also analyses the nature of formal symbols, seen as pure ipseity. The author also argues that algebraic and geometricized representations are as complementary as the blind man and the cripple, the former not knowing where he is going and the latter being unable to follow any of the routes he can see. This contribution is a preliminary study meant to prepare further research upon representation in its cognitive, cerebral, semantic and intentional aspects. The second chapter is entitled "Generative Grammar and Cognitive Linguistics: on the History of a Theoretical Split in American linguistics." In this contribution, Jean-Michel Fortis examines the current conflict between generative grammar and cognitive linguistics, the origin of which he sees in the disagreement that opposed generative semantics to the mainstream syntax-centred Chomskyan brand of generative grammar. He analyses the motivations of the semanticist linguistic movement, showing how it paved the way for cognitive linguistics. Three case-studies focusing on linguists Lakoff, Langacker and Talmy explain how these linguists, bereft of any theoretical affiliation following the failure of generative semantics, adopted theoretical stands which constituted the pivotal ideas of cognitive linguistics. In the next chapter, "Metalinguistic enunciative systems. An example: temporality in natural languages," Jean-Pierre Desclés and Zlatka Guentchéva explore the internal structure of the theoretical model of cognitive linguistics. More specifically, they examine the way different levels of metalinguistic representations connect and interact by means of an explicit compiling process. Within the "generalised compilation" hypothesis, metalinguistic representations are seen as intermediate representations between cognitive representations and linguistic expressions. Accordingly, the fundamental commitments of linguistic semantics are to specify the syntax of each metalinguistic level as accurately as possible, to minutely describe the architecture of relations between related levels, as well as to give an account of the modes of translation of the expressions from one level of representation to another. The approach is illustrated by the analysis of the representation of tense and aspect within an enunciative framework, which shows that propositions underlying utterances are not directly expressed by linguistic expressions, as different enunciative operations are taken into account. In chapter four, "From functions to metafunctions: the sources of British functional linguistic*s*," Charles-Henry Morling endeavours to study the genesis of the notion of metafunction put forward by Halliday. Analysing the development of Halliday's work, he places the functionalist turning point of his systemic functional grammar in the two years between 1966 and 1968. He shows that those functions of language Halliday calls metafunctions are indeed defined on empirical grounds and originate in the more traditional syntactic functions, and cannot therefore be said to be a priori categories or to mark any significant theoretical advance. Not acknowledging any real qualitative difference between these syntactic functions and his own metafunctions, Halliday's argumentation firmly established the latter in the empirical tradition of the London school.

The contributions gathered in the second part evaluate the results of a certain number of linguistic studies, putting various theories to the test in the fields of syntax, semantics and discourse analysis. Olivier Simonin analyses some of the conceptual problems raised by linguistic categorization. In a chapter entitled "Gradients, scales and clines," he addresses the questions of tokens which apparently belong to two categories. After summarising the two major models of categorization (the Aristotelian one versus categorization seen as a continuum) he introduces and discusses Aarts' notions of intersective gradience and subsective gradience. Basing his claim on three case-points, he argues that it is possible for two tokens to pertain to two categories. This is providing there is a structural ambiguity between two syntactic interpretations amounting to the same general meaning, or where one interpretation appears to be superimposed onto another. Kate Judge's contribution studies the semantic domain of modality and explores the explanatory power of two commonly opposed theoretical stands on the subject, namely the formal and functional approaches. Under the title "Are possible worlds necessary? Evaluating theories of modality," she pleads for a moderate attitude towards both and warns against focusing too exclusively on either of the two theoretical positions. Her inquiry leads her to remark about the importance of acknowledging the respective dimensions of cognition, sociocultural parameters and pragmatics for a proper understanding of the semantics of modality. Also, possible worlds may not be necessary but should not be too hastily discarded altogether. Being aware of various angles is probably one of the conditions for a better account and a more thorough analysis of the semantic distinctions between root and epistemic meanings or even between evidential and epistemic interpretations. The following chapter is entitled "Irony in two theoretical frameworks: relevance theory and argumentative polyphony theory." Focusing on discourse analysis, Tomonori Okubo draws a comparison between the theoretical frameworks of relevance theory and argumentative polyphony theory, supported by their respective analyses of irony. In this light, it appears that much of what is described in terms of cognitive pragmatics might be better accounted for in linguistic terms, and this probably applies to a greater number of linguistic phenomena than is usually thought.

Polyphony is constitutive of language and it is language itself which commands the tones and textual functions of an utterance at a particular time of utterance. In the last chapter, "Pragmatic vs. enunciative views of spoken English discourse; interpreting the prosody of certain parenthetical 'comment clauses'," Steven Schaefer focuses on the contrast between functional or pragmatic and constructivist or "enunciativist" approaches to the prosodic realization of two specific types of comment clause (*I think* and *you know*). Basing his arguments on a prosodic analysis of a selection of spoken corpora, he considers that the pragmatic view often taken in the literature misses the pertinence of the semantic origin of these discourse markers and does not take into account all the aspects due to discourse positioning; he also shows that intonation units do not primarily echo the syntactic structure of language as has been traditionally thought. Taking an utterer-centred approach, he argues that exchanges can be formally construed from the standpoint of abstract enunciative positions taken by the enunciator and co-enunciator as linguistic constructs, subject to modulation in context by prosodic means. This theoretical approach makes it possible to reconsider the concept of epistemic stance. A corpus-based spectrographic analysis of the prosodic realization of several examples shows the pertinence of suprasegmental phenomena which accompany these markers.

This book addresses the complex epistemological issues of metalinguistic discourses, questioning the very theoretical constructs or some fundamental concepts linguistic analyses are built on, at times confronting different theories, at others, analysing the history of their birth and development, or again assessing their explanatory power in the light of empirical linguistic data. This enterprise may result in discarding certain theoretical stances or ways of reasoning; it can also be conducive to a clearer view and a better analysis of what real and indeed deep contradictions may be found between theoretical positions. It may also very well bridge the gap between certain theories so far almost completely impervious to each other, by highlighting similarities of commitments or analysis. This sort of consideration has already been put forward in the literature, notably—were we to mention only two of the theoretical currents which are being scrutinised or put to the test in the present book—regarding cognitive linguistics and utterer-centred, i.e. enunciative, theories.[1] We hope that *Metalinguistic Discourses* can be a step towards this acknowledgement of common aims and a finer understanding of the vocabulary and concepts of theoretical discourses.

# Chapter One

# Metalinguistic Discourse and Formal Representation

Jean Pamiès

## Introduction

As understood here, the term 'metalinguistic discourse' will be taken to cover all the means the theoretician uses to state what he has to say:

(a) about the object he has chosen to study (including specific descriptions, generalisations or explanations offered as analysis);
(b) about the epistemological framework he has elected to guide his investigation (including chosen specific goals, accepted idealisation, or ontological claims for posited entities and constructs); and
(c) from a reflexive perspective, about the concepts and constructs he uses to achieve his goals.

To find or devise the expressive resources needed, such metalinguistic discourse will typically draw both:

(i) on natural language vocabulary and syntax (with, if needs be, a profusion of technical jargon vocabulary and phraseology extensions); and
(ii) on an open class of artifactual diagrams, schemas or formulas, including those of artificial languages or formal systems. Whenever a reflexive consideration of type (c) concerns a type (ii) element and is couched in a formulation using a type (ii) item, we shall say that, on the one hand, the concerned element of type (ii) is treated as pertaining to the (artificial) object-language and, on the other hand, that the type (ii) item used in the reflexive consideration belongs to the (by definition, artificial) meta-language.[1]

In the framework of this quite general definition, for reasons of available space, the work presented here is only the first half of a more specific study on the Protean use of the concept of representation in formalised, or at least partially formalised (mainly Chomskyan), linguistic theories. The aim of this over-all study is to shed some light on what holds together (or fails to hold together) the motley conglomeration of intended acceptations for the term in such collocations as 'formal representation,' 'cognitive representation,' 'cerebral representation,' 'graphic representation,' 'scriptural[2] representation,' 'symbolic representation,' 'semantic representation' or 'intentional[3] representation.'

Focussed on the key concept of 'formal representation,' the first half we present here is chosen because it can stand on its own feet while laying indispensable foundations for the second. Our guide-line throughout being that in the quest for adequate tools of types (i) and ii), considerations pertaining to (a) and (b) must be viewed as two sides of the same metalinguistic coin and in section 1, we shall dwell on the epistemological framework without which it would make no sense at all to talk about formal representations. In section 2, we shall introduce two basic kinds of formal representations, underscore their dual nature, counter formalist attempts at nipping in the bud any idea of formal 'representation', explain what it is that formal representations (each taken as a whole) are representations of, and define 'notational equivalence' amongst formal representations. In section 3, we shall concentrate on the representing side of a key sub-part of formal representation, the formal symbol. By first establishing what this representing side cannot be, we will then determine what it actually is, and a few observations will be adduced in support of the claim. In section 4, we shall concentrate on the other side of the (by definition, uninterpreted) formal symbol: that which, as such, it might be a representation of. Thanks to specific type (b) assumptions, we will then show what formal object a symbol *per se* is a representation of, as a result of which process of radical abstraction, and what light this sheds on the dialectics of formalisation and interpretation. In section 5, essential features of the way quantifiers are used are shown not to be a problem after all for the ontological claim made for the formal symbol in the preceding section, a claim for which an elucidation of the unlikely oxymoron 'variable constant' is shown to bring support. Finally, reasons are given to conclude that currently entertained acceptations for 'individual' and 'domain of discourse' are too narrow and too broad, respectively.

So far, consideration of a handful of simple, made-up examples will already have served to illustrate the distinction between naïve and formal theory and explore the nature and expressive potential of such type (ii) inscriptions as ‘algebraic’ [4] and ‘geometricised’ [5] representations. In addition, the delineation of classes of notational variants among such inscriptions will have begun to raise issues of identity, equivalence and distinction and to re-emphasise the importance of the role of abstract mathematical objects in formalised empirical investigation. However, in section 6, we shall delve more specifically into the arcana of fig. 1-1–20, both to illustrate and comfort already formulated contentions and to work on the compared merits and/or demerits of algebraic and geometricised formal representations.

As a conclusion, after a recapitulation of our essential claims, in a prospective note we shall outline the main features and guiding principles of the sequel to come.

Throughout, we shall try to reduce to reasonable size such technical material as the full definitions and formal underpinnings for fig. 1-1–20, but, in the notes, the interested reader will find more detail, and precise references for the entire paraphernalia.

Finally, we shall use the following internal cross reference conventions:

(A) from beginning to end of this work, the successive stages of the main overall argumentation (premises, definitions, conclusions or results) will be numbered using bracketed Roman numerals [from (I) to (XL)], with italics for contentions tentatively introduced only to be discarded further as untenable [*(VI)*, *(VII)*, *(VIII)*, *(X)*], and bold type for claims of greater importance [**(I)**, **(II)**, **(V)**, **(XI)**, **(XII)**, **(XIII)**, **(XV)**, **(XIX)**, **(XXI)**, **(XXII)**, **(XXVI)**, **(XXXI)**, **(XXXIII)**, **(XXXIV)**, **(XXV)**, **(XXXVI)**];

(B) within the narrower confines of a particular section or subsection, the successive steps of a sub-argumentation locally buttressing one or more mainstream claims will be numbered using Arabic numerals of various types [for instance, within section 6, steps 1.–20., with intermediate results $\boxed{1}$-$\boxed{5}$ will be adduced in support of conclusions (XXXV)-(XL)];

(C) whenever [within a note (e.g. note 8) or from note to associated main text (e.g. note 24)] an otherwise unreferenced English passage in double quotes is coupled with a fully referenced quotation in French, it is to be understood that the former is our translation for the latter.

## 1. Formalisation: naïve theory, formalised theory, abstraction and empirical content

Our starting point here will be the distinction between naïve and formalised theory.

In the framework of our type (i) and (b) assumptions, a 'naïve'[6] (or 'intuitive')[7] theory Tn is by definition an unformalised theory, that is to say a theory resorting almost exclusively to natural language (occasionally with loosely illustrative drawings or schemas[8]) for its expressive tools. In this context, the use of 'naïve' echoes a sense of dissatisfaction with the error-inducing imprecision and ambiguity of the former and the uncontrolled intuitive and "[over-]powerful analogical import"[9] of the latter.

To improve on a theory Tn thus perceived to be "lumbered with too much resort to intuition"[10] and replace it by a 'formalised theory' Tf, the formalising strategy is to use an "artificial language"[11] AL generated by a "formal system"[12] FS (or "calculus"[13]), "of algebraic type," the "expressions" of AL comprising the well-formed "formulas" (wwfs) generated by the "production rules" of FS and, among those wwfs, the subset of "theorems" obtainable *via* the "derivation rules"[14] of FS.

While, in Tn, "the meaning of the terms plays an important part in the way they are used and combined"[15] to form larger constructs, one major 'improvement' in Tf is meant to be that, in the way the rules of FS apply, ideally[16], any such consideration of how the symbolic material constituting the formulas of LA might be interpreted is "completely eliminated"[17].

"The calculus [operating] thus 'by transforming formulas according to certain prescribed laws',"[18] crucially, "in order to transform a [formula] into another, the pre-established rules" of FS apply "regardless of the properties of any entity those symbols might be held to represent,"[19] and it is precisely in this sense that FS can be said to be a "a calculus […], that is to say a system consisting of nothing but its syntax."[20]

On the one hand, then, Tf would be at best a formal, not a formal*ised* theory[21] if, once achieved the initial formalising move away from Tn, no attempt was made to return from Tf to Tn *via* a process of 'interpretation' assigning to each formula or relation of Tf some naïve content expressible in the terms of Tn.[22]

And on the other hand, given that, in order to "curb any form of reliance on what is merely felt to be obvious," the strategy is "to establish a mapping from the content-laden objects of Tn and their relations onto symbolic formulas *per se* devoid of such content,"[23] the move from Tn to Tf is one of abstraction—where 'abstraction' is "an operation" which consists in either “isolat[ing] a property from a certain type of object (concrete or ideal) to consider that property in itself, or in cut[ting] off a property or a relation from one or several objects to retain for consideration what is left after the severing.”[24]

Hopefully culminating in the full extirpation of “the structural properties” of Tn as “symbolically expressed”[25] in Tf, the formalising process may be intended to be partial or complete,[26] and the abstracting away from intuitive content a matter of degree[27] between the absolutely naïve and the totally abstract.[28]

Ideally, if it is possible to “demonstrate 1) that each theoretical, intuitive proposition [of Tn] is derivable in [FS] (i.e. has a corresponding theorem[29] in FS [of which this proposition is an interpretation, and] 2) that each theorem of FS can be interpreted by a theoretical, intuitive proposition [of Tn],” then the relation between Tf and Tn may be said to be one of “adequation,”[30] and “the structural properties” of FS may be held[31] “indeed to be those of Tn.”[32] But with this *proviso* that, in actuality, adequation between a given Tn and some proposed Tf is itself a matter of degree and, for principled reasons can never be proven to be fully established. Essentially because, due to the obvious impossibility for the formalising process to magically transmute the naïve into the formal prior to formalisation, it is in the nature of the case that ampliative (non-demonstrative) inferences only are accessible by way of justification—which of necessity falls short of the called-for demonstration.[33]

But the “intuitive naïvety”[34] of Tn should not be confused with unsophistication, and the somewhat condescending connotations of the terminology should not obscure the fundamental importance of naïve theory.

In the particular case of empirical science,[35] unformalised as it may be, Tn is an indispensable prerequisite to investigation.

Without the mode of questioning and the conceptual grid provided by such a theory, it would simply be impossible to “go beyond appearances.” A naïve theory alone can provide the means to apprehend as data the otherwise overwhelming and intractable flux of raw empirical phenomena and is elaborate enough to conceivably undergo formalisation.[36]

In other words, empirical data are indispensable intuitive-laden constructs of Tn obtained *via* its selective, oriented conceptualisation and organisation of empirical phenomena. And it is from those constructs, regardless of their intuitive content, that Tf extirpates a structure[37] by abstracting away from the structured.

So that, in a nutshell, contrary to their interpretations, as such,

**(I)**A formalised theory Tf and its constructs have no empirical content,[38] and naïve theorising is inescapably the first stage in the formalising process,[39] a precondition for further formalisation and interpretation.

Furthermore, the relation between Tn and Tf is one of constant, mutual interaction. On the one hand, FS is commonly devised while 'thinking ahead' of some targeted Tn interpretation. So that the specific organisation of the constructs of Tn dictating, as it were, the 'terms and conditions' of the tacit formalising contract, this anticipatory concern for successful adequation acts as stimulating goad for on-demand innovative formal imagination—so that Tn has an impact on Tf. And on the other hand, the formal results and demands of Tf in terms of explicitness and precision may suggest or impose for Tn small adjustments or serious re-thinking, if not radical re-consideration[40]—so that Tf has an impact on Tn.

Finally, this constant "oscillation between the naïve and the formal"[41] and mutual feed-back between Tf and Tn is so vital for the dynamics of theoretical change and progress in empirical investigation that, when it is claimed that Tn is the first stage in the formalising process, this does not mean that the naïve theorisation process becomes 'old hat' as soon as Tn has served its ancillary purpose in the ascent to the formal spheres of Tf. On the contrary, it needs to be permanently sustained as an ever-renewed and ever-renewing constitutive part of the over-all formalising dialectical process.

## 2. Formal representations (algebraic and/or geometricised): dual nature, formalist strictures, abstract formal objects and notational variants

In this section, we now turn to what we may learn from a consideration of the made-up examples of fig. 1-1–20[42] about the kind of notational apparatus that may be used as Tf representations, or 'formal representations' FR.

A cursory glance at such FR representations as fig. 1-1, 1-13, 1-14 or 1-10 is enough to show that they are *per se* unintelligible for the lay reader, that there is no guessing for sure what they stand for—in our type (i) and (ii) terms that, unless appropriate 'deciphering instructions' $\mathrm{DI}_{\mathrm{FR}}$ are provided, there is no grasping[43] what it is that they are supposed to be the '*locus tenens*' $\mathrm{LT}_{\mathrm{FR}}$ for. Without such specific instructions, a would-be formal representation FR reduced to just an orphan *locus tenens* (or 'place-holder') is no representation at all, nothing but useless graphic junk.

In other words, in a formalised theory, the notations[44] *per se* are illegible, so that, properly speaking, a formal(ised) representation FR consists of an ordered pair (notations, deciphering instructions for those notations). In our (i) and (ii) terms,

**(II)** A formal representation FR is an ordered pair ($LT_{FR}$, $DI_{FR}$).[45]

Focussing now on such FR representations as fig. 1-1, 1-14, 1-13, 1-10, 1-11 and 1-17, what they have in common is that they are 'algebraic,' that is to say couched in the standard scriptural notations of ordinary writing and printing, but[46] drawing on the full distinctive potential of the whole gamut of typographical resources.[47]

It is in the nature of scriptural techniques of notation that a character like, for instance, the letter 'A' is to be sharply distinguished from the class of its concrete realisations, the former being one and the latter innumerably many—and (owing to manufacturing imperfection) inevitably materially non-strictly identical. The distinction between abstract character and concrete inscriptions being analogical to the distinction between abstract phoneme and concrete phones, in our type (i) terms we shall draw a distinction between (abstract) 'grapheme' and (concrete) 'inscription' on some material support.[48]

From which it follows that the *locus tenens* of an algebraic formal representation cannot be an inscription. Otherwise, from one copy of the present article to another, the slightest difference in shade of black in the material realisation on paper[49] of, say, fig. 1-13 would have to count as resulting in the transmutation of one formal representation into another, reputedly distinct, formal representation. With the absurd consequence that, since such nuances in shade are technically inevitable, no two copies of the present article could ever be held to have been written about the same things.

In other words (without even taking (I) into account), in our type (i) terms,

**(III)** An algebraic formal representation FR cannot possibly consist of a concrete inscription.

If we now turn to such FR representations as fig. 1-2, 1-3, 1-8, 1-15, 1-16, 1-19 and 1-20, they have in common being 'geometricised,'[50] that is to say, by our definition, constructed by resort (commonly, together with 'algebraic material'[51]) to such geometry-like items as points and segments of curves or straight lines), then exactly the same point as (III) can be made, for essentially the same reason. The distinction between (abstract) 'enriched oriented graph' and '(concrete) inscription' being, again, analogous to the distinction between 'abstract phoneme' and 'concrete phone,'[52] it follows, this time, that the *locus tenens* of a geometricised formal representation cannot be an inscription. Otherwise, from one version of the present article to another, the slightest, half a degree difference in the orientation of one segment featuring in, say, fig. 1-2 would again have to count as resulting in the transmutation of one formal representation into another, reputedly distinct, formal representation, just as absurdly with regard to standard notational practice.

In other words (again without even taking (I) into account), in our type (i) terms,

> **(IV)** A geometricised formal representation FR cannot possibly consist of a concrete inscription,

and, subsuming (III) and (IV) (still without even taking (I) into account):

> **(V)** A formal representation FR, algebraic and/or geometricised, cannot possibly consist of a concrete inscription.

More importantly, by very much the same token, we may hope to shed some light on the "strong[53] formalist" conception of mathematical objects, which claims that

> **(VI)** "Mathematics consist of an array of concrete symbols."[54]

The matter is of some importance here, given that, if it could be maintained, *(VI)* would entail that the whole course we have followed so far is misguided and that it does not make sense to talk of such things as $DI_{FR}$ ($LT_{FR}$) because

> **(VII)** Formal notations are not representations (scriptural, formal, whatever), since they do not represent mathematical objects at all: they *are* those mathematical objects.

To make sense of the strong formalist claim *(VII)*, one might take it more precisely to mean that

> **(VIII)** Mathematical/formal objects are nothing but inscriptions.[55]

But such considerations as led to (V) immediately dismiss the suggestion: Since no two material inscriptions can ever be materially strictly identical, it would otherwise follow from *(VIII)* that in a formal text, nothing could be held as an occurrence of anything because it would be all tokens and no types,[56] and that by way of keeping track, it would have to be all *non sequiturs* throughout, since in scatter-brain fashion no two passages could ever be held to be about the same thing. Such implications are so absurd that even the strictest of nominalists[57] soon had to concede[58] that

**(IX)** Mathematical/formal objects cannot possibly consist of inscriptions.

Failing *(VIII)*, to make safer sense of formalist contentions that mathematics are nothing but rule-governed manipulations of notations, a last-ditch option was to try reducing mathematical objects, not to concrete inscriptions, but, in our type (i) and (ii) terms[59] to abstract orphan *locus tenens* (graphemes and/or or graphs), that is to retreat from *(VIII)* to

**(X)** Mathematical objects are nothing but abstract notational types of which concrete inscriptions are tokens.[60]

That way, by reducing the unmanageable myriads of disconnected tokens to a controllable multiplicity of subsuming types, one could hope to dodge the unsolvable difficulties that plagued *(VIII)*.

However, it is our contention that a consideration of such pairs as fig. 1-1, fig. 1-2 is enough to dismiss even this weakened[61] version of *(VIII)*.

To show this, the problem we shall now address is to decide whether such two figures may come to be associated with the same (in whichever way conceived) mathematical object and if so to understand how.

Under formalism, strong or weakened, the answer has to be that there cannot be any such association.

In the case of the strong formalist claim *(VIII)*, because if mathematical objects *are* inscriptions, then, given that no two inscriptions are identical, it follows that, apprehended as concrete inscriptions, fig. 1-1 and fig. 1-2 cannot possibly *be* the same mathematical object (which, under the strong formalist version, would have been the only way for them to be associated with the same mathematical object).

And similarly in the case of the revised formalist claim *(X)*. This time, because if mathematical objects *are* abstract types of which concrete inscriptions are tokens, then, given that there is no conceivable way the concept of ‘type’ could be so stretched as to have such typographically unmatchable notations as fig. 1-1 and 1-2 count as realisations of the same underlying graphemes and/or graphs, it follows that, when apprehended as the abstract types of which they are tokens, fig. 1-1 and fig. 1-2 cannot possibly *be* the same mathematical object (which, under the revised formalist version, would again have been the only way for them to be associated with the same mathematical object)—so that the final formalist answer to our problem is to decide that there is no way fig. 1-1 and 1-2 could conceivably be associated with the same mathematical object.

But, crucially, it turns out that what is deemed to be impossible in the formalist framework proves to be easily manageable in the representational framework. As we shall see in the next section, it is quite possible to find ways of associating such notations as fig. 1-1 and fig. 1-2 with exactly the same mathematical object (a 'canonical tree') which they have in common to represent.[62] And, furthermore, we shall argue that this kind of optional notational variation may be counted as a blessing with a rich potential for heuristic insights, formal rigour, inventiveness and imagination.[63]

From which we will conclude[64] that the formalist conception of mathematics, if taken seriously, would impose such strictures on the use of mathematical notations as to freeze notational flexibility, stifle formal imagination, and render literally incomprehensible current fruitful representational practices. To sum up:

**(XI)** The formalist approach to mathematics (strong or weakened) is untenable.

Our representational approach being thus cleared of formalist invalidation, we may now return freely to our notations of note 45 and continue to assume that such entities as '$DI_{FR}$ ($LT_{FR}$)' designates do exist, and concentrate on what these entities might be.

We have just seen that a given $DI_{FR}$ ($LT_{FR}$) can neither be the abstract graphemes and/or graphs of the orphan *locus tenens* $LT_{FR}$, nor any of the infinitely many distinct concrete inscriptions that may potentially be concrete realisations of $LT_{FR}$.

If we now return to (I), given a formalised theory Tf resorting to a formal representation FR with $LT_{FR}$ as its *locus tenens*, and given the open class[65] $T_N = \{Tn_1, Tn_2, \ldots\}$ of the possible naïve interpretations for Tf,[66] it follows immediately from (I) that the entity designated by '$DI_{FR}$ ($LT_{FR}$)' cannot be any of the naïve constructs of any of the naïve theories of $T_N$.[67]

But then, the reason why the distinct naïve theories of $T_N$ (each with its specific constructs) are federated as members of one equivalence class is that, by hypothesis, all these naïve theories have in common organising their particular constructs in essentially the same way. So that, at the end of the day, it is this common mode of organisation, or structure[68] that '$DI_{FR}$ ($LT_{FR}$)' designates.

The term 'structure' being so far left undefined, given that Tf is supposed to abstract away from the structured, we shall take it that what Tf extirpates is precisely a 'structure' in the mathematical sense:[69]

**(XII)** As such, a formal representation represents a structure composed of one or more sets together with one or more relations defined on those sets.

Accordingly, we can give a general definition of 'notational variant' which we shall use in the next section to sort out fig. 1-1–20 into appropriate equivalence classes: with the notations of note 45,

**(XIII)** Two formal representations $FR_1$ and $FR_2$ are notational variants of each other *iff* $LT_1 \neq LT_2$[70] and $DI_1 (LT_1) = DI_2 (LT_2)$.[71]

## 3. The formal symbol as *locus tenens*: abstraction, inscriptions, graphemes and pure distinctiveness

In this section, we shall concentrate on the first of two[72] issues concerning sub-parts of formal representations.

This issue has to do with the nature as orphan *locus tenens* of a fundamental component of algebraic representations: the 'letter symbol' of some (non-committal, as we shall see) 'naïve' dictionary definitions.[73]

So far, we have seen that, as *locus tenens*, a type (ii) formal symbol could not be a material inscription but had to be more abstractly apprehended, as a grapheme. On closer inspection, however, a number of converging observations tend to show that this move from inscription to grapheme may still not be abstract enough.

For instance, when one reads in a classical text-book introduction to Predicate Calculus[74] that "the choice of the symbol $x$ in $(x)$ $M(x)$ is arbitrary, and the same proposition could equally well be denoted[75] by $(y)$M$(y)$, $(z)M(z)$, etc.," the suspicion dawns that the choice of terms is rather misleading, and that it would be more accurate to consider that in this type of instance the very same symbol can be equally represented either as the grapheme $x$, or the grapheme $y$, or the grapheme $z$, etc. Which means that in the naïve 'letter symbol' duet, the letter should play an ancillary part, and the symbol be restored to its far more abstract identity.

This suspicion is borne out by the fact that any uniform distinction-preserving graphemic substitution[76] for the technical type (ii) symbols of a mathematical demonstration[77] will not affect its validity, which tends to show that some essential invariant is maintained in spite of the graphemic merry-go-round.

To capitalise on the insight that the grapheme is only half-way in abstraction from concrete inscriptions to properly conceived formal symbols, our suggestion[78] is to work with a three-fold distinction between 'symbol' (of a 'vocabulary'), 'grapheme' (of an 'alphabet') and 'inscription' (on a material support).

The symbol being given a new definition:

> **(XIV)** The formal essence of any one of the *n* 'formal symbols' of a finite 'vocabulary' V consists in nothing but its radical distinction from any one of the *n*-1 remaining members of V,[79]

or, in less technical terms,

> **(XV)** In its formal essence of abstract *locus tenens*, a formal symbol is pure distinctiveness from each of the other members of a vocabulary,

in this definition, the link between symbols, graphemes and inscriptions is that a formal symbol may be graphically represented (*via* 'scriptural assignment') by a grapheme, which may in turn be 'realised' by (a potentially infinite number of) concrete, material 'inscriptions.'

From then on, a few pieces of the puzzle may fall into place.

First, if it is a requirement of formalisation to manipulate such tenuous, quintessential place-holders as defined in (XV), then formalisation would just be too mind-boggling a task for the human intellect to accomplish, unless some technique be provided to set anchor into something less evanescent, something materially stable to get a grasp of and come back to at will. Hence to allow selective focussing on and avoid short-term working memory saturation. So much for the need.

By now, it should be clear why graphemic techniques are such precious means of fulfilling that need: by transmuting the unmanageable continuum of inscriptional variation into a small number of discrete character-types, the grapheme is the ideal intercessor to bridge the gap between extreme formal *locus tenens* abstraction and more down-to-earth support for scrutiny and re-examination.

At the same time, it is just as clear why it is of no importance *which* graphemes are picked up for scriptural assignment: all that matters is that one or another set of (bundles of) distinctive graphemic features,[80] no matter which, be made available to filter out non-distinctive graphic residues and in one way or another legibly discreticise the inscriptional material. And this explains why one-to-one distinction-preserving substitution of one alphabet of graphemes for another is always an open option.[81]

Furthermore, all of this is comforted by the way it dovetails with antecedent work by Jack Goody and Sylvain Auroux, which equally support the contention that, without the intellectual technological innovation of (in its very essence, graphemic) alphabetical writing[82], sustained formal abstract reasoning and computation on entities deprived of intuitive content would have remained beyond the reach of human capacities.[83]

## 4. The symbol as a formal object: individuals, radical abstraction, ipseity and Plenitudinous Platonism

The preceding section having dealt with the first, we shall now concentrate on the second of the two issues we wished to address concerning sub-parts of formal representations. This issue has to do with how we can make ontological sense of the formal reduction to pure distinctiveness of the symbol as *locus tenens*.

For this, we shall work on the relations between formal symbols, individuals, formalisation and abstraction, under certain type (b) assumptions.

As a starting point, ignoring counter-claims,[84] we shall tentatively[85] subscribe to the ontology of a version of Platonism[86] (Mark Balaguer's "plenitudinous Platonism," *alias* 'FBP'[87]) which ascribes enduring reality, not only to structures,[88] but to all conceivable mathematical objects.

The reason for the move is that, under FBP, even though the symbol is not a structure, it may be granted the same ontological status as any other formal object in the plenteous cornucopia of mathematical entities.[89]

To proceed one step further, as the term is currently used in work on formalisation, the 'individuals' relevant to "the context of a particular discussion" are broadly defined as "compris[ing] a set" "[-] called the domain (or universe) of discourse for that discussion [- which] […] contains everything that we might want to talk about, and [from which] we can arbitrarily exclude […] things that are irrelevant for the purpose at hand. In a mathematical discussion, for example, the domain of discourse might contain [such individuals as] positive integers, sets of positive integers, collections of such sets," "collections of these collections, etc." "A linguistic discussion might presuppose a domain of discourse containing [such Tn constructs as] words, sentences, phrase markers, grammars, etc., but not, say, motorboats or guitars."[90] Thus defined, because of its fundamental link with instantiation and quantification in the

Predicate Calculus,[91] the individual plays an important role in logic and mathematics [92] and is not unrelated to a rich philosophical tradition traceable back to Aristotle, Avicenna and the scholastics.[93]

Yet one more step, and we may ask, since formalisation is consubstantially linked with abstraction,[94] what it means to abstract away from an individual and how far one can abstract away from the individual without altogether dissolving it.

Our suggestion then is that

> **(XVI)** Abstracting away from an individual is to apprehend it as deprived of one or more of its attributes,[95]

and that the utmost possible degree of abstraction in the formal apprehension of an individual is attained when it has been deprived of all but one of its attributes, retaining that which makes it be irreducibly itself and no other, i.e. its 'individual difference,'[96] or to use what we shall take to be a synonym:

> **(XVII)** When maximally subjected to formal abstraction, an individual is reduced to its sheer ipseity.[97]

Then, to finally to tie the knot between formal symbols, individuals, formalisation and abstraction, we shall suggest that

> **(XVIII)** What, as *locus tenens*, a formal symbol is a place-holder for is pure ipseity,[98]

and, to combine (XV) and (XVII),

> **(XIX)** In its essence as abstract *locus tenens*, the formal symbol is pure distinctiveness; in its essence as abstract formal object, the formal symbol is pure ipseity.

Granted those assumptions, we can make ontological sense of the way formal symbols can be put to remarkably flexible use, and a few more pieces of the puzzle may fall into place.

More precisely, if we accept, under FBP, that, apprehended as a mathematical object, a formal symbol has the status of an individual stripped of all its attributes, reduced to its sheer ipseity (and irretrievably lost-sight-of in any given detail of its original fully-fledged integrity), then, crucially, it makes perfect sense that (preserving its ipseitic core) it could be used as a kind of universal hanger on which to put, or off which to take, at will, any number of attributes.

As a matter of fact, in the first place, this is exactly what goes on in typical meta-linguistic stipulations of the 'let *X* be …' type.[99] For instance, halting at each intermediate stage to comment on a made-up, archetypal case:

> 'let μ' (so far, nothing but pure ipseity) 'be a number' (one attribute on). 'If μ is an even number…' (one more attribute on). 'Now, if μ is an odd number…' (one attribute off, one attribute on). 'But if μ is a prime number…' (one more attribute on), etc.

And secondly, as can easily be checked by thumbing through volumes picked at random on the shelves of the mathematics section of a library, since an abstract symbol is ontologically a universal attribute-hanger, μ may just as well (in another article, another book, or in the next paragraph) be stipulatively associated with a set, a function, a mathematical structure, or so on or so forth, *ad libitum*.

Thirdly, in like fashion, since, on the one hand, in the chain from abstract symbol to graphemic type, to (ultimately) inscriptional tokens, the link (under scriptural assignment) between abstract symbol and grapheme is arbitrary, it follows that in any given text in which μ is chosen to graphically represent a formal symbol, uniform replacement of all inscriptional tokens (or realisations) of μ by inscriptional tokens (or realisations) of another grapheme *Y* will yield a free notational variant of that text, as long as two conditions are respected: 1) that either *Y* had previously no occurrence in the text, or, if it had, that the uniform replacement of all previous realisations of μ by new realisations of *Y* be accompanied by vice versa uniform replacement of all previous inscriptional tokens of *Y* by new inscriptional tokens of μ; and 2), that the replacement be carried out, not only in the expressions of the object-language, but, crucially, in the accompanying expressions of the meta-language as well.

And fourthly, by the same token, since once whittled down to its last ontological tether, the 'original' individual cannot be retrieved from its ipseitic core, it makes perfect sense that the mapping from (the formal representations of) formal theory Tf to (the naïve-laden, unformalised constructs of) naïve theory Tn should be potentially one-many and not necessarily one-to-one.[100]

## 5. In defence of ipseity: quantifiers, variables, nonspecific constants, 'individual' and 'domain of discourse' revisited

Returning now to the Predicate Calculus,[101] in this section we shall concentrate on some specificities of the notations for quantifiers and variables and one or two terminological pitfalls which could easily obscure the relevance of our contention (XIX), and are thus in need of clarification.

### 5.1. Quantifiers and variables

The first specificity has to do with the seemingly abbreviatory nature of the notations used for quantifiers. For instance the universal quantifier '$\forall$' in, say, $(\forall x)$ P(x) and the existential quantifier '$\exists$' in, say $(\exists x)$ P(x) may be held, respectively, to subsume "a kind of generalized conjunction" and "a generalized form of disjunction" "extending over the entire domain of discourse," that is to say over the entire set comprising all and only the "constants" which, by definition, may serve as possible "instantiations of P($x$)."[102]

Now, such symbolic condensation may be deemed necessary because, given that the set of possible instantiating constants is an open class, such symbolic notations for the underlying generalised conjunctions and disjunctions as (respectively) '$P(a) \wedge \mathrm{P}(b) \wedge \mathrm{P}(c) \wedge \ldots$' and '$P(a) \vee P(b) \vee P(c) \vee \ldots$' can only be indicative and are not satisfactory.

But, useful as it may be, this symbolic condensation is somewhat confusing because it results in a kind of indistinct coalescence of two heterogeneous sets of objects: a tight class of connectives and an open class of constants—and this first thrust at the concepts does not take us very far.

Furthermore, if only one quantifier could be used per well-formed formula, and if all predicates were one-place predicates, no variable at all would be needed, since, after all, if would be enough to just write '$\forall P$' or '$\exists P$.'

But such is not the case, since multiple quantification and $n$-uples of arguments are available options in the formation rules of Predicate Calculus. And it is precisely in such cases that resorting to variables provides useful notational resources.

For instance,[103] in such simple well-formed formulas (or 'propositions') as i) '$(x)(\exists y)xLy$' or ii) '$(\exists y)(x)(yLx \rightarrow xLy)$,' as a first approximation, trying to dispense with variables by somehow mixing or reshuffling constants and generalised conjunctions and disjunctions to find equivalent algebraic notations proves hopelessly unpromising, if not totally infeasible.

The second specificity has to do with the triple function played by the notations used for variables[104] in such cases as i) and ii).

Since there exists a proper subset of formulas of the Predicate Calculus (the 'propositional functions') in which certain variables play a less complex role, it is by this relatively simpler case that we shall begin.

In the case of such formulas (or 'propositional functions') as iii) '*xLy*' or iv) '*yLx→xLy*,'[105] then, free variables fulfil only two functions:

a) each occurrence of a free variable marks a position (a rank) in the formula where an instantiation may take place (by replacement of that occurrence by an occurrence of some suitable constant symbol); and
b) the occurrences of free variable symbols present in a propositional function *pf* collectively impose combinatorial constraints amongst instantiations taking place at different positions in *pf*.

More precisely, two positions in *pf* may be instantiated by two (tokens of) distinct graphemes representing each a distinct constant symbol only if those two positions are held by two (tokens of) distinct graphemes representing each a distinct variable. Or, in more easy-going, rule-of-thumb style: before you introduce two distinct constants by instantiation, make sure you are replacing two distinct variables; if you introduce twice the same constant, as long as it is free variables you are replacing, feel free to do so.

As a result [cf. the 'may … only if'—and not 'must … whenever' in the formulation of b)], given a) and b), and in potentially disturbing relation with (XIX),

> **(XX)** the graphemic distinction of two occurrences of free variables does not guarantee graphemic distinction within the pairs of occurrences of constant symbols that may legitimately instantiate them.[106]

For instance, under a) and b), a legitimate instantiation for '*yLx→xLy*' [our iv) *supra*] is v): '*aLa→ aLa*'

Returning now to the more complex cases, before analysing the third function played by certain variables in such wffs as i) or ii), we must first say another word about quantifiers (this time without trying to proceed *via* generalised conjunction or disjunction).

Each occurrence *Q* of a quantifier in a given well-formed formula *wff* of the Predicate Calculus PredC signals that, in certain positions in *wff*, either any instantiation is valid [when *Q* is an occurrence of the universal quantifier] or that at least one instantiation is valid [when *Q* is an occurrence of the existential quantifier]. But Q by itself does not specify in which positions those instantiations are supposed to take place—a vital specification if instantiation is to be defined as a formal operation.

If the formation rules of PredC could guarantee that *Q* is the only occurrence of a quantifier in *wff*, then the problem could be solved by the default stipulation that [q.v., *mutatis mutandis*, function a) *supra*] the set of relevant positions in *wff* is coextensive with the set of positions occupied by a (bound) variable in *wff*. But such is not the case, and something else must be done to cope with wffs comprising more than one occurrences Q—as for instance in i) and ii).

One further step towards a solution is then to resort to scope-indicating brackets (and scarcity of) to circumscribe the substring of *wff* in which the positions governed by each particular Q must be located. If the formation rules of PrdC could guarantee that, for any two occurrences Q, their scopes can never overlap, then that further move would suffice. But such is not the case—as exemplified, again, by i) and ii).

So that, for instantiation to be definable as a formal operation, something more must be done to somehow link each occurrence of a quantifier with the positions it governs and exclusively those. That is to say, in Richard C. Jeffrey's terms,[107] the "linkage" problem remains to be solved.

From this perspective, resorting to quantifiers and bound variables may then be seen to be one way of completely solving the problem. In other words, the third function played by variables can be thus identified:

c) the function of bound variables is to contribute to a solution of the linkage problem.[108]

More precisely, to each occurrence of a quantifier is exclusively attached its particular grapheme[109] (the exclusivity rights reaching as far as needs be to exclude graphemic homonymy in overlapping scope domains). So that, in a given well-formed formula *wff* of the Predicate Calculus, for any given (possibly zero-) occurrence of a quantifier *Q* having closely concatenated to it an occurrence signalling its hallmark grapheme, the positions governed by *Q* in *wff* are all and only the positions occupied by the other occurrences of that grapheme inside the scope of *Q*.[110]

Nevertheless, if variables are one way of contributing to a solution to the linkage problem, they are not the only one. Since "the only purpose of the [bound] variables is to show which [position] is […] governed by which quantifiers," instead of using the standard algebraic notations we have been examining, the "cross-indexing job […] can be clarified by actually drawing links between quantifiers and the variables they govern," and "all relevant information can be shown in the link notation without using variables—for an example of this, cf. fig. 1-4 and 1-5 *infra*, which show how, starting from the well-formed formulas of i) and ii) *supra*, nothing is lost in the transition from purely algebraic notations [1a and 2a],